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MHD FLOW THROUGH AN INFINITE VERTICAL POROUS PLATE THROUGH A POROUS MEDIUM

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ABSTRACT
Magnetohydrodynamic (MHD) flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting, and Boussinesq fluid over a vertical oscillating plate embedded in a Darcian porous medium in the presence of a thermal radiation are presented. The fluid considered here is a gray, absorbing/emitting radiating, but nonscattering medium. At time $t > 0$, the plate temperature and concentration near the plate are raised linearly with time t. The governing partial differential equations are reduced to a system of self-similar equations using the similarity transformations. The resultant equations are then solved numerically using the Runge-Kutta method along with shooting technique. The effects of governing physical parameters on velocity, temperature and concentration as well as skin-friction coefficient, Nusselt number and Sherwood number are computed and presented in graphical and tabular forms.

Keywords*:* MHD, heat generation, variable suction, viscous dissipation

I. Introduction

The study of free convection with heat and mass transfer is very useful in fields such as chemistry, agriculture and oceanography. A few representative fields of interest in which combined heat and mass transfer play an important role are the design of chemical processing equipment, formulation and dispersion of fog, distribution of temperature and moisture over agriculture fields and groves of fruits trees, damage of crops due to freezing, and pollution of the environment. This technique is used in the cooling processes of plastic sheets, polymer fibers, glass materials, and in drying processes of paper.

Magneto convection plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics.

In recent years, the problems of free convective heat and mass transfer flows through a porous medium under the influence of a magnetic field have been attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. In view of these applications, many researchers have studied MHD free convective heat and mass transfer flow in a porous medium. The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Heat generation effects may alter the temperature distribution and this in turn can affect the particle deposition rate in nuclear reactors, electronic chips and semi conductor wafers. Although exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can be used to express its general behavior for most

physical situations. Heat generation or absorption can be assumed to be constant, space-dependent or temperature-dependent.
on the heat transfer flow of a viscous incompressible fluid through a highly porous medium bounded by an infinite, vertical, porous plate, by means of the series solution method. Ahmed [1] studied the effect of transverse periodic permeability oscillating with time on the free convective heat transfer flow of a viscous, incompressible fluid through a highly porous medium bounded by an infinite, vertical, porous plate subjected to a periodic suction velocity. Kumar and Verma [2] presented the problem of an unsteady flow past an infinite, vertical, permeable plate with constant suction and a transverse magnetic field with oscillating plate temperature. If the temperature of the surrounding fluid is high, radiation effects play an important role, and this situation does not exist in space technology. In such cases one has to take into account the effect of thermal radiation and mass diffusion. The effects of radiation and viscous dissipation on the transient natural convection-radiation flow of viscous dissipation fluid along an infinite vertical surface embedded in a porous medium, by means of the network simulation method, is investigated by Zueco [3]. The effect of radiation on the natural convection flow of a Newtonian fluid along a vertical surface embedded in a porous medium was presented by Mahmoud and Chamkha [4]. Soundalgekar and Takhar[5] have considered the radiation free convection flow of an optically thin, gray gas past a semi-infinite vertical plate. Radiation effects on a mixed convection flow along an isothermal vertical plate were studied by Hossain and Takhar [6].

In all the above studies, the vertical plate has been considered as stationary. Raptis and Perdikis [7] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Ahmed [8] studied the effects of radiation and magnetic Prandtl number on the steady mixed convective heat and mass transfer flow of an optically thin, gray gas over an infinite, vertical, porous plate with con- stant suction in the presence of a transverse magnetic field. Ahmed and Kalita [9] in- vestigated the effects of porosity and MHD on a horizontal channel flow of a viscous, incompressible, electrically conducting fluid through a porous medium in the presence of thermal radiation and a transverse magnetic field. Ahmed and Kalita[10] presented the magnetohydrodynamic transient convective radiative heat transfer in an isotropic, homogenous, porous regime adjacent to a hot vertical plate. Ahmed and Kalita [11] investigated the effects of chemical reaction as well as magnetic field on the heat and mass transfer of Newtonian fluids over an infinite, vertical oscillating plate with variable mass diffusion. Ahmed et al. [12] gave a numerical solution for the problem of MHD heat and mass transfer flow past an impulsively started semi-infinite vertical plate in the presence of thermal radiation by an implicit finite-difference scheme of the Crank–Nicolson type. The effects of Darcian drag force and radiation conduction on the unsteady two-dimensional MHD flow of a viscous, electrically conducting, and Newtonian fluid over a vertical plate adjacent to a Darcian regime in the presence of thermal radiation and a transverse magnetic field were reported by Ahmed et al. [13]. Ahmed [14] analyzed the effects of conduction-radiation, porosity, and chemical reaction on the unsteady hydromagnetic free convection flow past an impulsively started semi-infinite verti- cal plate embedded in a porous medium in the presence of a first-order chemical reaction and thermal radiation.

Veera Krishna and Chamkha [15] discussed the MHD squeezing flow of a water-based nanofluid through a saturated porous medium between two parallel disks, taking the Hall current into account. Veera Krishna and Chamkha [16] investigated The diffusion-thermo, radiation-absorption and Hall and ion slip effects on MHD free convective rotating flow of nano-fluids past a semi-infinite permeable moving plate with constant heat source. Hall and ion slip effects on Unsteady MHD Convective Rotating flow of Nanofluids have been discussed by Veera Krishna and Chamkha [17]. Veera Krishna and Chamkha [18] investigated the Hall and ion slip effects on the MHD convective flow of elastico-viscous fluid through porous medium between two rigidly rotating parallel plates with time fluctuating sinusoidal pressure gradient. Veera Krishna et al. [19] discussed the MHD flow of an electrically conducting second-grade fluid through porous medium over a semi-infinite vertical stretching sheet. The influence of thermal radiation, Hall and ion-slip impacts on the unsteady MHD free convective rotating flow of Jeffreys fluid past an infinite vertical porous plate with the ramped wall temperature has been investigated by Veera Krishna [20].

II. Formulation of the problem

Consider the unsteady laminar two-dimensional free convection boundary layer flow of an incompressible viscous electrically conducting fluid along a vertical porous plate. Let *x*-axis is taken along the plate and *y*-axis is normal to the plate. Magnetic field of intensity B_0 is applied in y-direction. A homogeneous first order chemical reaction between fluid and the species concentration is considered, in which the rate of chemical reaction is directly proportional to the species concentration. The fluid is assumed to be slightly conducting, and hence the magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation). Under these assumptions along with the boundary layer approximation and considering the viscous dissipation , the governing boundary layer equations for continuity, momentum, heat and mass transfer in the presence of heat generation and chemical reaction take the following form of the governing equations is given by

$$\frac{\partial v}{\partial y}=0 \Rightarrow v \text{ is independent of } y \Rightarrow v=v(t), \tag{1}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K^*}u \tag{2}$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0\left(T-T_\infty\right)}{\rho c_p}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

whereu,v are the velocity components along x – and y – directions, respectively. t is the time variable, ν is the kinematic viscosity, ρ is the fluid density, g is the acceleration due to gravity of the earth, σ is the electrical conductivity, β is the volumetric expansion coefficient for heat transfer, β^* is the volumetric expansion coefficient for mass transfer, K^* is the permeability of the porous medium, c_p is the specific heat at constant pressure, μ is the dynamic viscosity, T is temperature of the fluid in the boundary layer, C is the concentration of the fluid in the boundary layer, κ is the thermal conductivity, the term $Q_0(T-T_\infty)$ is assumed to be amount of heat generated or absorbed per unit volume and Q_0 is a constant, which may take on either positive or negative values, T_∞ is the heat temperature far away from the plate, C_∞ is the mass temperature far away from the plate, D is the molecular diffusivity and the boundary conditions for velocity, temperature and concentration fields for $(t \to 0)$are given by

$$u=0, v=v(t),\ T=T_w,\ C=C_w \text{ at } y=0,$$

$$u \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \tag{5}$$

where T_w be the fluid temperature at plate.

III. Method of solution

Define time dependent similarity parameter h having length scale as,

$$h = \{= h(t)\} = 2\sqrt{\nu t}\,, \qquad (6)$$

Specially used for unsteady boundary layer problems. In terms of $h(t)$, a convenient solution of Equation (1) is given by

$$v = v(t) = -f_w \frac{\nu}{h(t)}\,, \qquad (7)$$

where f_w is suction parameter.

The momentum, energy and concentration equations can be transformed into the corresponding differential equations by introducing the following similarity variables and non-dimensional parameters:

$$\eta = \frac{y}{h},\; u = Uf(\eta),\; \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\; \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \qquad (8)$$

into the equations (2) to (4), we get

$$f'' + (2\eta + f_w) f' + Gr\theta + Gc\phi - \left(M + \frac{1}{K}\right) f = 0 \qquad (9)$$

$$\theta'' + (2\eta + f_w)\Pr\theta' + \Pr Q\theta + \Pr Ec\left(f''\right)^2 = 0 \qquad (10)$$

where η is the similarity variable,
U is the uniform characteristic velocity,
f is the dimensionless stream function,

$Gr = \dfrac{g\beta h^2 (T_w - T_\infty)}{\nu U}$ is the Grashof number for heat transfer,

$Gc = \dfrac{g\beta^* h^2 (C_w - C_\infty)}{\nu U}$ is the Grashof number for mass transfer,

$M = \dfrac{\sigma B_0^2 h^2}{\nu\rho}$ is the magnetic parameter,

$K = \dfrac{K^*}{h^2}$ is the permeability parameter,

$\Pr = \dfrac{\mu c_p}{\kappa}$ is the Prandtl number,

$Q = \dfrac{h^2 \kappa Q^*}{\mu c_p}$ is the heat generation parameter ,

$Ec = \dfrac{U^2}{c_p (T_w - T_\infty)}$ is the Eckert number,

$Sc = \dfrac{\nu}{D}$ is the Schmidt number and The reduced corresponding boundary conditions are

$$f(0) = 0,\quad \theta(0) = 1,\quad \phi(0) = 1$$
$$f(\infty) = 0,\, \theta(\infty) = 0,\, \phi(\infty) = 0 \qquad (11)$$

The governing equations (9) &(10) are second ordered linear differential equations and solved under the boundary conditions (11) using Rugne-Kutta fourth order method along with shooting technique.

The parameters of engineering interest for the present problem are the skin-friction coefficient, the Nusselt number and the Sherwood number, which are given respectively by the following expressions. Knowing the velocity field the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$C_f = \frac{2\nu}{Uh} f'(0) \tag{12}$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form, in terms of Nusselt number, is given by

$$Nu = \frac{2q\sqrt{\nu t}}{\kappa\left(T_w - T_\infty\right)} = -\theta'(0), \tag{13}$$

where$q$ is heat flux per unit area.

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form, in terms of Sherwood number, is given by

$$Sh = \frac{J\sqrt{\nu t}}{D\left(C_w - C_\infty\right)} = -\phi'(0), \tag{14}$$

where J is mass transfer coefficient.

Where $\mathrm{Re} = \frac{U_0 x}{\upsilon}$ is the Reynold's number.

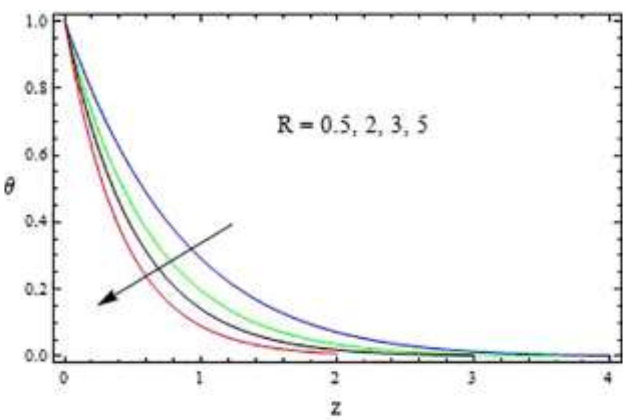
IV. Analysis of the numerical results

The governing equations (9) & (10) with the boundary conditions (11) are solved using Runge-Kutta fourth order method along with shooting technique for different values of the parameters taking step size 0.005. The numerical calculations are presented in the form of graphs for different values of parameters.

To expand a viewpoint of the physics of the flow regime, we have numerically evaluated the effects of the Hartmann number (M), Grashof number (Gr), radiation conduction parameter (R), dimensionless time (t), and porosity pa- rameter (K) on the velocity components u and v, temperature θ, concentration φ, shear stress, Nusselt number, and Sherwood number. Here we consider Gr = 3, Gm = 5 > 0 (cooling of the plate), i.e., free convection currents convey heat away from the plate into the boundary layer, and t = 1, R = 1 throughout the discussion. This is due to the fact that the thermal conductivity of fluid decreases with increasing Pr, resulting in a decrease in thermal boundary layer thickness. As R increases, considerable reduction is observed in temperature profiles from the peak value at the wall (z = 0) across the boundary layer regime to the free stream, at which temperature is negligible for any value of R (Fig.1). It is also observed that the reduction in temperature is accompanied by simultaneous reductions in the thermal boundary layer. Temperature increases with an increase in time parameter t (Fig. 2). We noticed from Fig. 3 that both velocity components u and v retard with increasing the intensity of the magnetic field (Hartmann number M). It is obvious that an increase in the local magnetic parameter M results in a decrease in the velocity at all points. The application of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type of force called the Lorentz force. This force has the propensity to reduce speed for the motion of the fluid in the boundary layer. Figures 4 and 5 depict the variation of the velocity components with permeability parameter K. Both velocity components u and v enhance with the increase of permeability parameter K. The lower the permeability of the porous medium, the lesser the fluid speed in the entire fluid medium.

The presence of a porous medium increases the resistance to flow, resulting in a decrease in the flow velocity. Figure.6 show the effect of mass Grashof number (Gm) on the velocity field. We noticed that an increase in Gr results in an increase in the velocity. Similar behavior is achieved with the solutal Grashof number (Gm) on velocity.Figs. 7–9 reveal the effects of Sc, and R on the velocity profiles. It is evident It is evident that velocity components u and v reduce with increasing Schmidt number (Sc) or radiation conduction parameter (R) (Figs. 7– 9).

We also noticed from Table 1 that the Sherwood number (Sh) enhances as Schmidt number (Sc) and time t increase. The Nusselt number (Nu) increases with Prandtl number (Pr) and decreases with radiation conduction parameter (R) and time (t) (Table 2). The skin friction components τx and τy enhance with increasing Prandtl number (Pr) and decrease with thermal Grashof number (Gr) and mass Grashof number (Gm). The component τx increases,and τy decreases, as Hartmann number (M) and permeability parameter (K) increase, whereas the reverse behavior is observed with increasing Sc, radiation conduction parameter (R), and frequency of oscillation ωt (Table 3).



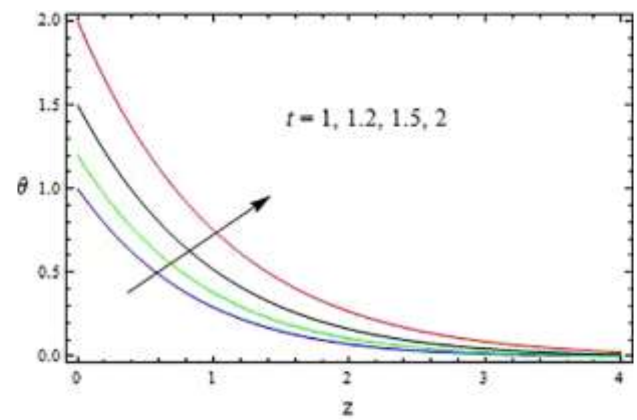
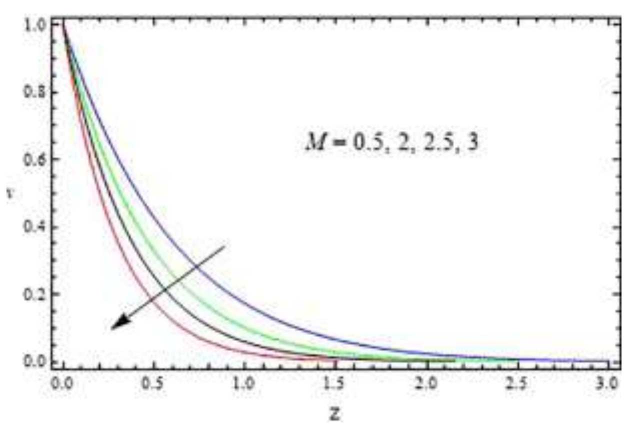


FIG. 1: The temperature profile for θ against *R*

FIG. 2: The temperature profile for θ

FIG. 3: The velocity profile for *v* against *M*



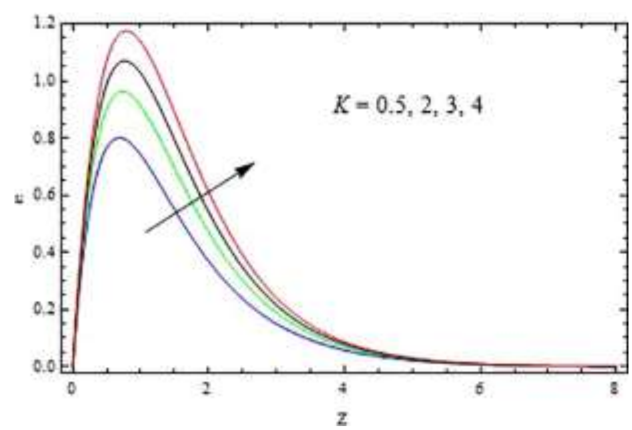


FIG.4: The velocity profile for *u* against *K*

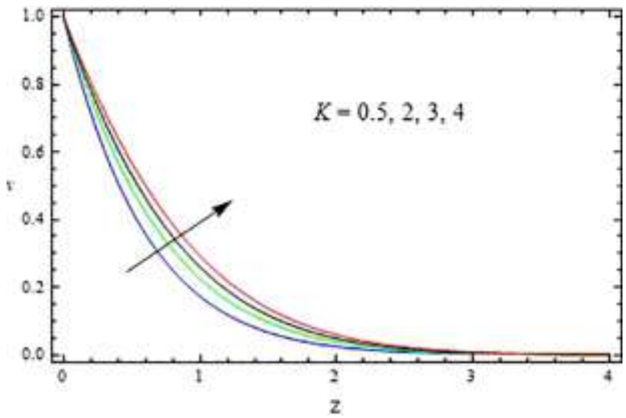


FIG. 5: The velocity profile for *v* against *K*

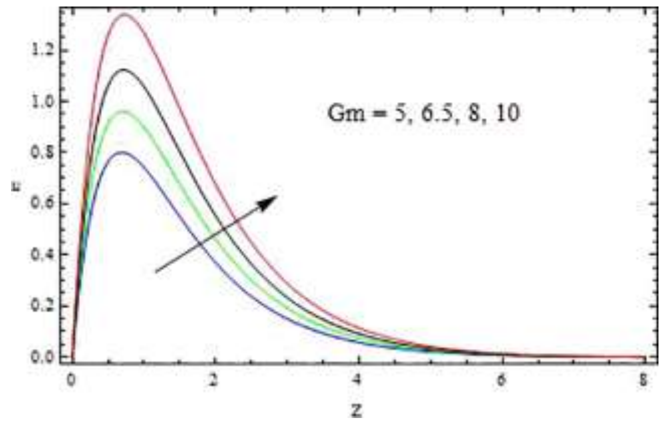


FIG. 6: The velocity profile for *u* against Gm

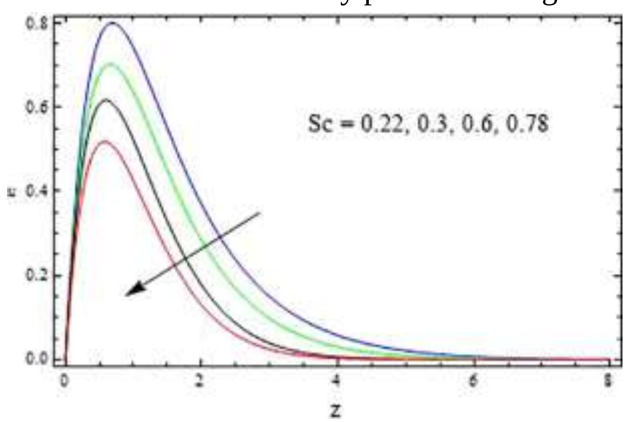


FIG. 7: The velocity profile for *u* against Sc

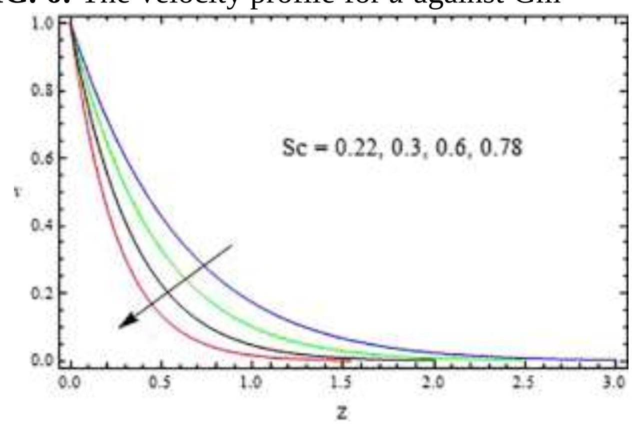


FIG. 8: The velocity profile for *v* against Sc

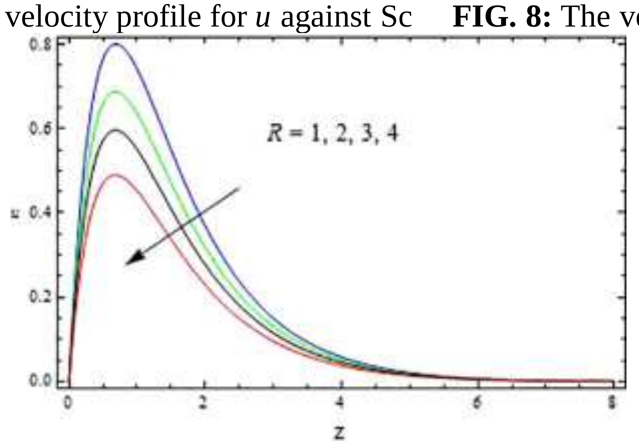


FIG. 9: The velocity profile for *u* against *R*

Table.1 Sherwood number and Nusselt number

Sc	*t*	**Pr**	*R*	*t*	**Sh**	**Nu**
0.22	1	—	—	—	0.742937	—
0.3	—	—	—	—	0.831719	—
0.6	—	—	—	—	0.890123	—
0.78	—	—	—	—	1.210237	—
—	1.2	—	—	—	0.793452	—
—	1.5	—	—	—	0.861884	—

—	2	—	—	—	0.962162	—
—	—	0.71	0.5	1	—	0.073005
—	—	3	—	—	—	0.790766
—	—	7	—	—	—	2.624421
—	—	—	2	—	—	0.024472
	—	—	2.5	—	—	0.023879
—	—	—	3	—	—	0.023677
—	—	—	—	1.2	—	0.061323
—	—	—	—	1.5	—	0.049386
—	—	—	—	2	—	0.037907

Table.2 Skin friction

M	K	Gr	Gm	Pr	Sc	R	ωt	τ_x	τ_y
0.5	0.5	3	5	0.71	0.22	1	$\pi/6$	0.653252	0.077374
1	—	—	—	—	—	—	—	0.696701	0.066888
2	—	—	—	—	—	—	—	0.810363	0.057916
—	1	—	—	—	—	—	—	0.933254	0.057616
—	2	—	—	—	—	—	—	1.170247	0.049518
—	—	5	—	—	—	—	—	0.499222	0.039912
—	—	7	—	—	—	—	—	0.389322	0.033914
—	—	—	7	—	—	—	—	0.474365	0.060886
—	—	—	10	—	—	—	—	0.359185	0.050145
—	—	—	—	3	—	—	—	0.836947	0.110839
—	—	—	—	7	—	—	—	0.970685	0.166614
—	—	—	—	—	0.3	—	—	0.574365	0.110887
—	—	—	—	—	0.6	—	—	0.510285	0.151912
—	—	—	—	—	—	2	—	0.408355	0.269883
—	—	—	—	—	—	3	—	0.326954	0.478245
—	—	—	—	—	—	—	$\pi/4$	0.443598	0.122049
—	—	—	—	—	—	—	$\pi/3$	0.329958	0.279838

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