DETERMINATION OF RELIABILITY AND M.T.T.F BY USING THE BOOLEAN FUNCTION TECHNIQUE

Dr. M. Gayathri, Associate Professor, Department of Mathematics, Government First Grade College, VARTHUR, BANGALORE, KARNATAKA, INDIA

Abstract

The Boolean function technique has been utilized in this paper to calculate the reliability of a considered electrical circuit. The failure rates of each part of the circuit correspond to either a Weibull time distribution or an exponential time distribution. The reliability and mean time to circuit failure have also been calculated. The paper also includes numerical examples, reliability graphs vs. time, and M.T.T.F vs. failure rates.

Keywords: Algebra of logic, Boolean Function Technique, Reliability, M.T.T.F, Weibull Distribution, Exponential Distribution, Circuit, failure rate.

Introduction

Many researchers have contributed their work to determine the Reliability and M.T.T.F of the system which is considered by utilizing the Boolean Function Technique. Shiksha Bansal along with S.C. Agarwal [1] used the Boolean function technique to figure out the reliability of the milk powder manufacturing plant. Pervaiz Iqbal, P.S. Sheik Uduman [2] used the Boolean function as well as the fuzzy logic techniques in a paper plant to evaluate the reliability parameters. Dr. Reena Garg [3] has looked at a refinery unit and evaluated various parameters of reliability. Anil Chandra, Surbhi Gupta, and Anjali Naithani [4] looked at a chocolate manufacturing plant and assessed reliability factors by using the Boolean function technique and neural network. Pooja Dhiman and Amit Kumar [5] assessed the reliability and sensitivity of the thermal plant. Vikas Kumar and Ashish Kumar Arora considered a power plant and calculated the profit of the plant.

In this paper, the author examined a circuit composed of four subsystems which consists of nine units in series and parallel configuration. As illustrated in Fig.1. the four subsystems are as follows: subsystem A composed of three input units U_1,U_2,U_3. Subsystem B has a single unit U_4. Subsystem C is composed of four units U_5,U_6,U_7,U_8 arranged in a series-parallel configuration, Subsystem D has an output unit U_9.

Assumptions

1. The reliabilities of all units of the circuit are priorly known.
2. All the components have independent states.
3. Each unit state and the whole circuit is either bad or good.
4. The time of failure for all the units is arbitrary.
5. No facility of repair.

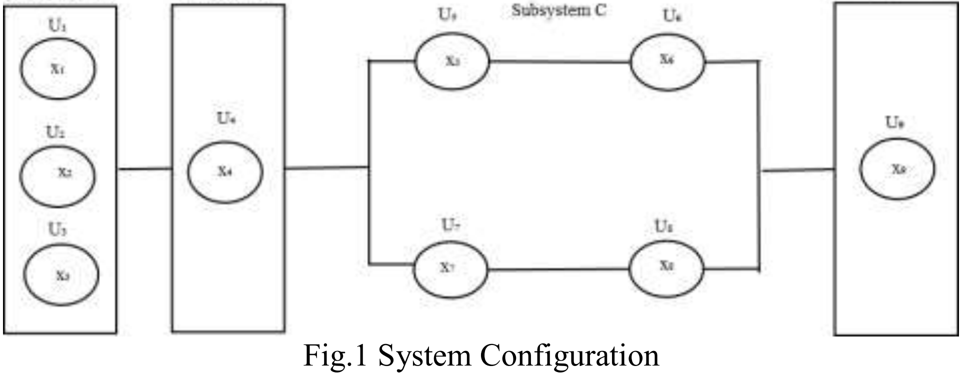

Fig.1 System Configuration

Notations:

x_1, x_2, x_3	States of the input units U_1, U_2, U_3.
x_4	State of the unit U_4.
x_5, x_6, x_7, x_8	States of the units U_5, U_6, U_7, U_8.
x_9	State of the output unit U_9.
x_k^1	Negation of x_k.
$\wedge$	Conjunction
$\vee$	Disjunction

Formation of the mathematical model:

With the help of Boolean function technique, logical matrix expression for considered successful circuit operation “is

$$f\left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\right) = \begin{vmatrix} x_1 & x_4 & x_5 & x_6 & x_9 \\ x_1 & x_4 & x_7 & x_8 & x_9 \\ x_2 & x_4 & x_5 & x_6 & x_9 \\ x_2 & x_4 & x_7 & x_8 & x_9 \\ x_3 & x_4 & x_5 & x_6 & x_9 \\ x_3 & x_4 & x_7 & x_8 & x_9 \end{vmatrix} \tag{1}$$

Solution of the model:

Equation (1) may be expressed as the following while using the logic algebra.

$$f\left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\right) = \begin{vmatrix} x_4 & x_9 \end{vmatrix} \vee g\left(x_1, x_2, x_3, x_5, x_6, x_7, x_8\right) \tag{2}$$

Where $$g\left(x_1, x_2, x_3, x_5, x_6, x_7, x_8\right) = \begin{vmatrix} x_1 & x_5 & x_6 \\ x_1 & x_7 & x_8 \\ x_2 & x_5 & x_6 \\ x_2 & x_7 & x_8 \\ x_3 & x_5 & x_6 \\ x_3 & x_7 & x_8 \end{vmatrix} \tag{3}$$

Substituting $G_1 = x_1 \quad x_5 \quad x_6$

$G_2 = x_1 \quad x_7 \quad x_8$

$G_3 = x_2 \quad x_5 \quad x_6$

$G_4 = x_2 \quad x_7 \quad x_8$

$G_5 = x_3 \quad x_5 \quad x_6$

$G_6 = x_3 \quad x_7 \quad x_8$

Equation (3) could” be written as

$$g\left(x_1, x_2, x_3, x_5, x_6, x_7, x_8\right) = \begin{vmatrix} G_1 & & & & & \\ G_1^1 & G_2 & & & & \\ G_1^1 & G_2^1 & G_3 & & & \\ G_1^1 & G_2^1 & G_3^1 & G_4 & & \\ G_1^1 & G_2^1 & G_3^1 & G_4^1 & G_5 & \\ G_1^1 & G_2^1 & G_3^1 & G_4^1 & G_5^1 & G_6 \end{vmatrix} \tag{4}$$

Using the algebra of logic, we get

$$G_1^1 = \begin{vmatrix} x_1^1 & & \\ x_1 & x_5^1 & \\ x_1 & x_5 & x_6^1 \end{vmatrix} \qquad G_2^1 = \begin{vmatrix} x_1^1 & & \\ x_1 & x_7^1 & \\ x_1 & x_7 & x_8^1 \end{vmatrix} \qquad G_3^1 = \begin{vmatrix} x_2^1 & & \\ x_2 & x_5^1 & \\ x_2 & x_5 & x_6^1 \end{vmatrix}$$

$$G_4^1 = \begin{vmatrix} x_2^1 & & \\ x_2 & x_7^1 & \\ x_2 & x_7 & x_8^1 \end{vmatrix} \qquad G_5^1 = \begin{vmatrix} x_3^1 & & \\ x_3 & x_5^1 & \\ x_3 & x_5 & x_6^1 \end{vmatrix} \qquad G_6^1 = \begin{vmatrix} x_3^1 & & \\ x_3 & x_7^1 & \\ x_3 & x_7 & x_8^1 \end{vmatrix}$$

$$G_1^1\,G_2 = \left|\begin{array}{lll} x_1^1 & & \\ x_1 & x_5^1 & \\ x_1 & x_5 & x_6^1 \end{array}\right| \wedge \left|x_1 \quad x_7 \quad x_8\right| = \left|\begin{array}{lllll} x_1 & x_5^1 & x_7 & x_8 & \\ x_1 & x_5 & x_6^1 & x_7 & x_8 \end{array}\right| \tag{5}$$

In the same manner, we can get

$$G_1^1\,G_2^1\,G_3 = \left|x_1^1 \quad x_2 \quad x_5 \quad x_6\right| \tag{6}$$

$$G_1^1\,G_2^1\,G_3^1\,G_4 = \left|\begin{array}{llllll} x_1^1 & x_2 & x_5^1 & x_7 & x_8 & \\ x_1^1 & x_2 & x_5 & x_6^1 & x_7 & x_8 \end{array}\right| \tag{7}$$

$$G_1^1\,G_2^1\,G_3^1\,G_4^1\,G_5 = \left|x_1^1 \quad x_2^1 \quad x_3 \quad x_5 \quad x_6\right| \tag{8}$$

$$G_1^1\,G_2^1\,G_3^1\,G_4^1\,G_5^1\,G_6 = \left|\begin{array}{lllllll} x_1^1 & x_2^1 & x_3 & x_5^1 & x_7 & x_8 & \\ x_1^1 & x_2^1 & x_3 & x_5 & x_6^1 & x_7 & x_8 \end{array}\right| \tag{9}$$

By utilizing the equations (5) to (9) in the equation (4) we “obtain,

$$g\left(x_1, x_2, x_3, x_5, x_6, x_7, x_8\right) = \left|\begin{array}{lllllll} x_1 & x_5 & x_6 & & & & \\ x_1 & x_5^1 & x_7 & x_8 & & & \\ x_1 & x_5 & x_6^1 & x_7 & x_8 & & \\ x_1^1 & x_2 & x_5 & x_6 & & & \\ x_1^1 & x_2 & x_5^1 & x_7 & x_8 & & \\ x_1^1 & x_2 & x_5 & x_6^1 & x_7 & x_8 & \\ x_1^1 & x_2^1 & x_3 & x_5 & x_6 & & \\ x_1^1 & x_2^1 & x_3 & x_5^1 & x_7 & x_8 & \\ x_1^1 & x_2^1 & x_3 & x_5 & x_6^1 & x_7 & x_8 \end{array}\right| \tag{10}$$

From (2) and (10) we have

$$f\left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\right) = \left|\begin{array}{lllllllll} x_1 & x_4 & x_5 & x_6 & & & & & \\ x_1 & x_4 & x_5^1 & x_7 & x_8 & x_9 & & & \\ x_1 & x_4 & x_5 & x_6^1 & x_7 & x_8 & x_9 & & \\ x_1^1 & x_2 & x_4 & x_5 & x_6 & x_9 & & & \\ x_1^1 & x_2 & x_4 & x_5^1 & x_7 & x_8 & x_9 & & \\ x_1^1 & x_2 & x_4 & x_5 & x_6^1 & x_7 & x_8 & x_9 & \\ x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6 & x_9 & & \\ x_1^1 & x_2^1 & x_3 & x_4 & x_5^1 & x_7 & x_8 & x_9 & \\ x_1^1 & x_2^1 & x_3 & x_4 & x_5 & x_6^1 & x_7 & x_8 & x_9 \end{array}\right|$$

(11)

The” reliability of the circuit is determined by the probability of successful operation, which is provided by

$$\begin{aligned} R_s &= P_r\left\{f\left(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\right)\right\} \\ &= R_1R_4R_5R_6 + R_1R_4Q_5R_7R_8R_9 + R_1R_4R_5Q_6R_7R_8R_9 + Q_1R_4R_5Q_6R_7R_8R_9 + Q_1R_2R_4Q_5R_7R_8R_9 \\ &\quad + Q_1R_2R_4R_5Q_6R_7R_8R_9 + Q_1Q_2R_3R_4R_5R_6R_9 + Q_1Q_2R_3R_4Q_5R_7R_8R_9 + Q_1Q_2R_3R_4R_5Q_6R_7R_8R_9 \end{aligned} \tag{12}$$

$$\begin{aligned} R_S = {} & R_1R_4R_5R_6 + R_2R_4R_5R_6R_9 - R_1R_2R_4R_5R_6R_9 + R_3R_4R_5R_6R_9 - R_1R_3R_4R_5R_6R_9 - R_2R_3R_4R_5R_6R_9 + R_1R_2R_3R_4R_5R_6R_9 \\ & + R_1R_4R_7R_8R_9 + R_2R_4R_7R_8R_9 - R_1R_2R_4R_7R_8R_9 + R_3R_4R_7R_8R_9 - R_1R_3R_4R_7R_8R_9 - R_2R_3R_4R_7R_8R_9 \\ & + R_1R_2R_3R_4R_7R_8R_9 - R_1R_4R_5R_6R_7R_8R_9 - R_2R_4R_5R_6R_7R_8R_9 + R_1R_2R_4R_5R_6R_7R_8R_9 - R_3R_4R_5R_6R_7R_8R_9 \\ & + R_1R_3R_4R_5R_6R_7R_8R_9 + R_2R_3R_4R_5R_6R_7R_8R_9 - R_1R_2R_3R_4R_5R_6R_7R_8R_9 \end{aligned} \tag{13}$$

Particular Cases:

Case1:

Equation (13) becomes true if each complex system component has a reliability of R.

$$R_S = R^4 + 5R^5 - 6R^6 - R^7 + 3R^8 - R^9 \tag{14}$$

Case2: When failure rates follow Weibull distribution

When failure rates exhibit a Weibull distribution, we may calculate the reliability in case of complex system at a given instant defined as (t) by using λ_i which is the failure rate for the component corresponding to state x_i .

$$R_S^w(t) = \sum_{i \in A} \exp\left(-a_i t^p\right) - \sum_{j \in B} \exp(-a_j t^p) \tag{15}$$

Where positive parameter is p and a_i 's is provided by

$a_1 = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6$	$a_2 = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$	$a_3 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$
$a_4 = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$	$a_5 = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$	$a_6 = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$
$a_7 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9$	$a_8 = \lambda_1 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$	$a_9 = \lambda_2 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$
$a_{10} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{11} = \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{12} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$
$a_{13} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{14} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{15} = \lambda_1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
$a_{16} = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{17} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{18} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
$a_{19} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{20} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$	$a_{21} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$

The sets A and B are as follows $A = \{1,2,4,7,8,9,11,14,17,19,20\}$ $B = \{3,5,6,10,12,13,15,16,18,21\}$

Case3: When failure rates follow an exponential distribution

A specific kind of Weibull distribution, exponential distribution is highly helpful in a variety of specific issues. The circuit's reliability in this instance at time t is determined “by

$$R_S^E(t) = \sum_{i \in A} \exp\left(-a_i t\right) - \sum_{j \in B} \exp(-a_j t) \tag{16}$$

In this case, M.T.T.F is expressed as

$$M.T.T.F. = \int_0^\infty R_S^E(t)\,dt = \sum_{i \in A} \frac{1}{a_i} - \sum_{j \in B} \frac{1}{a_j} \tag{17}$$

Numerical Computation:

Setting $\lambda_i = \lambda = 0.1$ (for $i = 1-9$) and $p = 2$ in equations (15), (16) and (17) one can get

$$R_S^W(t) = \exp(-0.4t^2) + 5\exp(-0.5t^2) - 6\exp(-0.6t^2) - \exp(-0.7t^2) + 3\exp(-0.8t^2) - \exp(-0.9t^2) \tag{18}$$

$$R_S^E(t) = \exp(-0.4t) + 5\exp(-0.5t) - 6\exp(-0.6t) - \exp(-0.7t) + 3\exp(-0.8t) - \exp(-0.9t) \tag{19}$$

$$M.T.T.F = \frac{1}{a_1} + \frac{1}{a_2} - \frac{1}{a_3} + \frac{1}{a_4} - \frac{1}{a_5} - \frac{1}{a_6} + \frac{1}{a_7} + \frac{1}{a_8} + \frac{1}{a_9} - \frac{1}{a_{10}} + \frac{1}{a_{11}} - \frac{1}{a_{12}} - \frac{1}{a_{13}} + \frac{1}{a_{14}} - \frac{1}{a_{15}} - \frac{1}{a_{16}} + \frac{1}{a_{17}} - \frac{1}{a_{18}} + \frac{1}{a_{19}} + \frac{1}{a_{20}} - \frac{1}{a_{21}}$$

(20)

Table 1

S.No.	Time t	$R_S^W(t)$	$R_S^E(t)$
1	0	1	1
2	1	0.8549354553	0.8549354553
3	2	0.3684180412	0.6753546009
4	3	0.0558691330	0.5075436017
5	4	0.0029265487	0.3684180412
6	5	0.0000621787	0.2606784029
7	6”	$6.310329827 \times 10^{-7}$	0.1808880439
8	7	$3.188342338 \times 10^{-9}$	0.1236342220
9	8	$7.685059735 \times 10^{-12}$	0.0835021384

10	9	$8.501923127\times10^{-15}$	0.0558691330

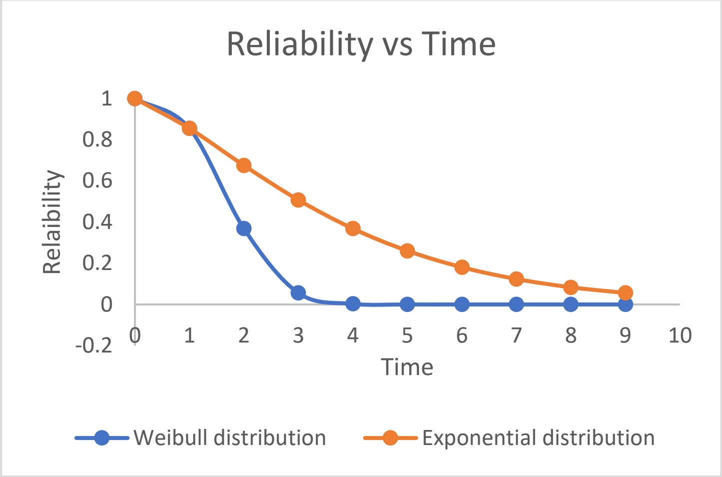

Figure 2: Reliability v/s Time

Table 2

S.No.	λ	M.T.T. F
1	0	∞
2	0.1	3.710317460
3	0.2	1.855158730
4	0.3	1.236772487
5	0.4	0.927579365
6	0.5	0.742063462
7	0.6	0.618386244
8	0.7	0.530045352
9	0.8	0.463789683
10	0.9	0.412257496
11	1.0	0.371031746

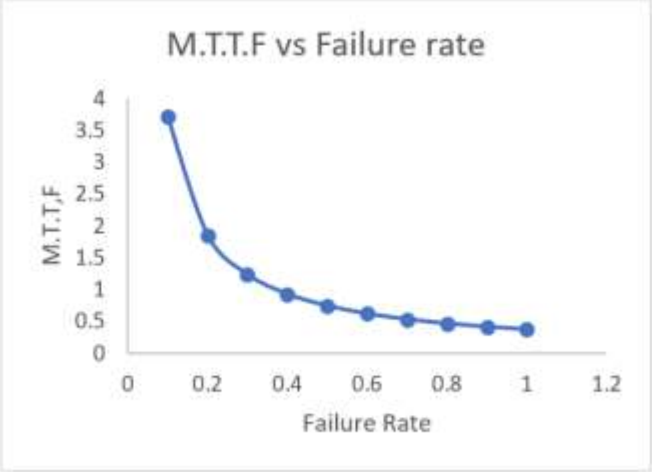

Fig. 3 M.T.T.F v/s Failure rate

Result Interpretation:

The reliability of the considered circuit is illustrated in Table 1 at the instant ‘t’, where failure rates follow both the exponential distribution as well as Weibull distribution, as shown in Figure 2. In the event of an exponential distribution, it is not hard to observe that the reliability of a circuit will degrade approximately in the same manner throughout the process. However, it drops by a significant amount very quickly when the failure followed a Weibull distribution: Table 2 displays, for a variety of different values λ, how long it takes for the circuit to fail. We can see from Fig.3 that the MTTF decreases in a rather uneven manner at first, but that it eventually approaches a more uniform decrease.

References:

[1]. Shikha Bansal, S.C.Agarwal, Kuldeep Sharma, Evaluation of Reliability factors using Boolean function technique in Milk powder manufacturing plant, IJRRAS 4(4),2010, 416-424.
[2]. Pervaiz Iqbal, P.S. Sheik Uduman, Reliability Analysis of Paper Plant using Boolean function with Fuzzy Logic Technique, International Journal of Applied Engineering Research, Vol.11, Number 1, (2016), PP 573-577.
[3]. Dr. Reena Garg, Analysis of various factors of reliability using the Boolean function technique in refinery unit, International Journal of Statistics and Applied Mathematics,2017,2(5), 35-40.
[4]. Anil Chandra, Surbhi Gupta, Anjali Naithani, Assessment of Reliability factors in Chocolate manufacturing plant using Boolean function technique and Neural networking, International Journal of Innovative Technology and Exploring Engineering, Volume-8, Issue-12,2019.
[5]. Pooja Dhiman and Amit Kumar, Reliability and sensitivity assessment of a Thermal power Plant using Boolean function technique, Journal of Physics: Conference Series, 1531(2020)012075.
[6]. Vikas Kumar, Ashish Kumar Arora, Profit Analysis and Modelling of a Power Plant with Boolean Function Algorithm, Journal of Science and Technology, Vol.06, Issue 04, July-August 2021.