



HEAT AND MASS TRANSFER OF OSCILLATORY FREE CONVECTION IN A VERTICAL WAVY CHANNEL WITH CHEMICAL REACTION

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Abstract

The effect of chemical reaction on unsteady combined heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel with oscillatory flux. The non-linear governing equations are solved by employing a regular perturbation technique with the slope δ of the wavy wall as a perturbation parameter. The velocity, the temperature and the concentration are analyzed for different variations of the governing parameters. The rate heat and mass transfer are evaluated for different variations.

Keywords: Heat Transfer, Mass Transfer, Chemical reaction, Wavy channel,

1.Introduction

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing .Das et al[1] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy[2] has studied the effects of reaction on a long surface with suction. Radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

Kandaswamy et al[3] have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.

The study of heat transfer and mixed convection flow in enclosures of various shapes has received attention [4] due to its practical applications. Interest in these convection flow and heat transfer in porous medium has been motivated by a broad range of applications to geothermal systems, crude oil production, storage of nuclear waste materials, ground water pollution, fiber and granular insulations solidification of castings. In a wide range of such problems, the physical system can be modeled as a two-dimensional rectangular enclosure with vertical walls held at different temperatures and the connecting adiabatic horizontal walls. Convective heat transfer in a rectangular porous duct whose vertical walls are maintained at two different temperatures and horizontal walls insulated received attention by several investigators [5]. Furthermore, in references [6 and 7] numerical results are being presented.

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage ,evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed .Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[8],Lai and Kulacki[9].The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al[10]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably , Nelson and Wood[11].

Theoretical and experimental investigations of natural convection MHD flow over a vertical porous plate in presence of chemical reaction plays an important role in Agriculture, geophysics and astrophysics. To study the underground water resources, filtration and water purification process in chemical engineering one must know the concepts of the fluid flow through porous medium. The



porous medium is like a non homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid. Oscillatory flows are associated with high rates of heat and mass transfer. Many studies have been done to understand its characteristics in different systems such as pulse combustors and reciprocating engines. Many investigators reported oscillatory flows by involving various physical situations.

2.Mathematical model

We consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls . The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at $y =L$ while the boundary at $y = -L$ is maintained at constant temperature T_1 while both the walls are maintained at uniform concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. We choose a rectangular Cartesian system $0 (x ,y)$ with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at $y=\pm L$.

The equations governing the unsteady flow, heat and mass transfer in terms of stream function ψ .

$$[(\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x] = \nu \nabla^4 \psi - \beta g (T - T_0)_y - \beta^* g (C - C_0)_y - \left(\frac{\nu}{k}\right) \nabla^2 \psi \quad (2.1)$$

$$\rho_e C_p \left(\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \lambda \nabla^2 \theta - Q(T - T_0) + Q_1(C - C_0) \quad (2.2)$$

$$\left(\frac{\partial \phi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} \right) = D \nabla^2 \phi - k_1(C - C_0) \quad (2.3)$$

The boundary conditions for the velocity and temperature fields are

$$\begin{aligned} \frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad T = T_1, \quad C = C_1 \quad \text{on } y = -L \\ \frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad T = T_2 + \Delta T_e \sin(mx + nt), \quad C = C_2 \quad \text{on } y = L \end{aligned} \quad (2.4)$$

Introducing the non-dimensional variables as

$$x' = mx, \quad y' = y/L, \quad t' = t \nu m^2, \quad \Psi' = \Psi/\nu, \quad \theta = \frac{T - T_2}{T_1 - T_2}, \quad \phi = \frac{C - C_2}{C_1 - C_2} \quad (2.5)$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$\delta R (\delta (\nabla_1^2 \psi)_t + \frac{\partial (\psi, \nabla_1^2 \psi)}{\partial (x, y)}) = \nabla_1^4 \psi + \left(\frac{G}{R}\right) (\theta_y + N \phi_y) - D^{-1} \nabla_1^2 \psi - M^2 \frac{\partial^2 \psi}{\partial y^2} \quad (2.6)$$

$$\delta P (\delta \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}) = \nabla_1^2 \theta - \alpha \theta + Q_2 \phi \quad (2.7)$$

$$\delta S c (\delta \frac{\partial \phi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y}) = \nabla_1^2 \phi - \gamma \phi \quad (2.8)$$

where

$$R = \frac{UL}{\nu} \quad (\text{Reynolds number}) \quad G = \frac{\beta g \Delta T_e L^3}{\nu^2} \quad (\text{Grashof number})$$



$$P = \frac{\mu c_p}{k_1} \text{ (Prandtl number),}$$

$$D^{-1} = \frac{L^2}{k} \text{ (Darcy parameter),}$$

$$Sc = \frac{\nu}{D_1} \text{ (Schmidt number)}$$

$$M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{\nu^2} \text{ (Hartmann Number)}$$

$$\alpha = \frac{QL^2}{\lambda} \text{ (Heat source parameter)} \quad Q_2 = \frac{Q_1(C_1 - C_2)L^2}{(T_1 - T_2)} \text{ (Radiation absorption parameter)}$$

$$\gamma_1 = \frac{K_1 L^2}{D_1} \text{ (Chemical reaction parameter)} \quad \delta = mL \text{ (Aspect ratio)}$$

$$\gamma = \frac{n}{vm^2} \text{ (non-dimensional thermal wave velocity)}$$

$$\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = -1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm 1 \tag{2.9}$$

$$\theta(x, y) = 1, \quad C = 1 \quad \text{on } y = -1$$

$$\theta(x, y) = \sin(x + \gamma t), \quad C = 0 \quad \text{on } y = +1$$

$$\theta(x, y) = \sin(x + \gamma t), \quad C = 1$$

$$\frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0 \tag{2.10}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis. Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t .

3. Nusselt number and Sherwood number

Knowing the temperature & concentration the local rate of heat and mass transfer on the walls have been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y} \right)_{y=\pm 1}$$

where $\theta_m = 0.5 \int_{-1}^1 \theta dy$ and $Sh = \frac{1}{C_m - C_w} \left(\frac{\partial C}{\partial y} \right)_{y=\pm 1}$

where $C_m = 0.5 \int_{-1}^1 C dy$

where d_1, d_2, \dots, d_{14} are constants.

4. Discussion of the numerical results

In this analysis we investigate the effect of Chemical reaction on convective Heat and mass transfer flow of a viscous fluid in a vertical wavy channel.

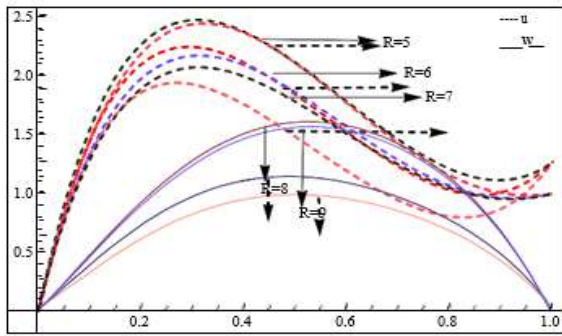


Fig.1 Velocity Profile for various values of Chemical Reaction

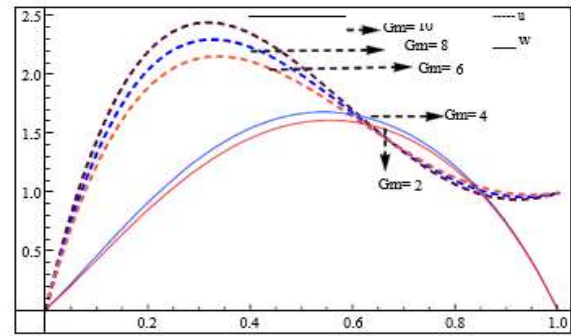


Fig.2 Velocity Profile for various values of Grashof Number

increase of k^2 (rotation parameter) increase the Primary velocity but the reverse process exists for the secondary velocity. At the same time in certain stage after that reverse processes exists for the secondary velocity in the fluid flow. The increasing of permeability parameter K , Grashof number for heat transfer Gr (Figs.1&2)

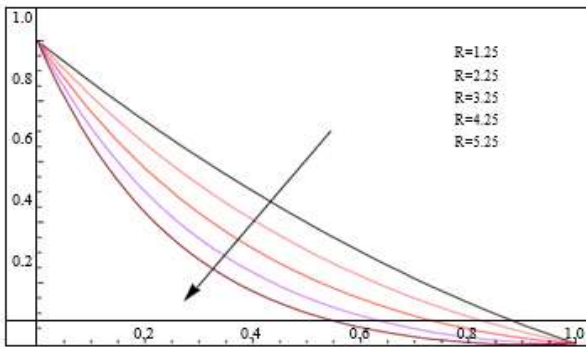


Fig.3 Temperature Profile for various values of Chemical Reaction

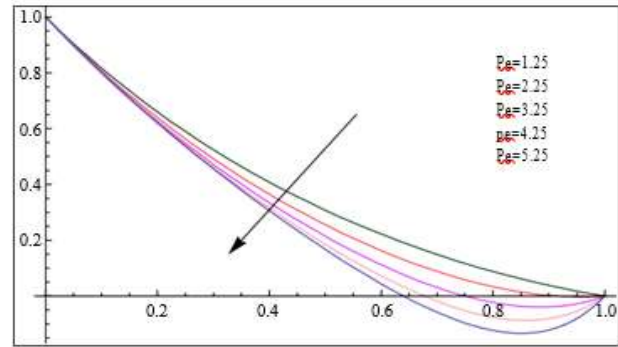


Fig.4 Concentration Profile for various values of Peclet Number

Fig.3 shows that, The increase effects of temperature exists, the reverse processes exists if increase of chemical reaction parameter. Concentration profile shows the decrease effects while increasing of Peclet number (Fig.4).

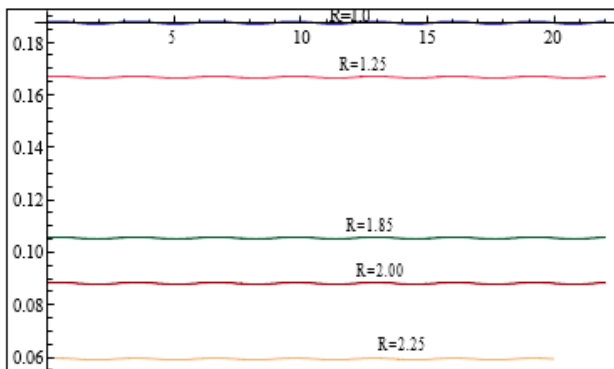


Fig.5 Mass flux for various value of Chemical Reaction

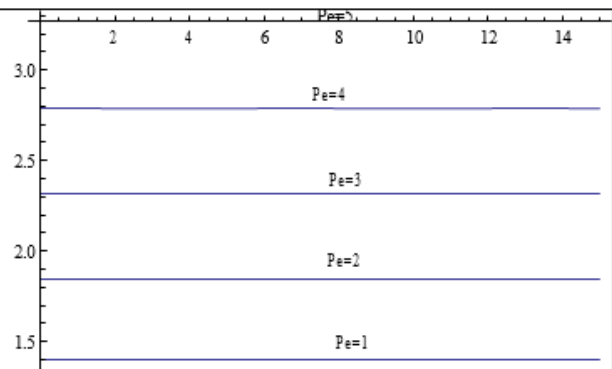


Fig.6 Heat flux for various value of Peclet Number

Mass flux shows decrease effects while increasing of chemical reaction (Fig.5). Heat flux shows increasing effects while increase of Peclet Number (Fig.6).

The average Nusselt number (Nu) which represents the rate of heat transfer at $y=\pm 1$ is shown in tables.1 and 2 for different values of parameters. The Sherwood Number (Sh) which measures the rate of mass transfer at $y=\pm 1$ is shown in Tables.3 and 4 for different variations.



Table-1 Nusselt Number Nu_1 at $y = 1$

G	I	II	III	IV	V	VI
10^3	-3.8684	-4.0856	-3.9793	-4.1008	-3.2383	-3.58102
3×10^3	-3.8775	-4.0935	-3.9906	-4.1126	-3.4217	-3.75541
10^3	-3.9058	-4.1184	-4.0266	-4.1495	-4.2173	-4.49246
3×10^3	-3.8962	-4.11002	-4.014	-4.1369	-3.8988	-4.20121
N	1	2	-0.5	-0.8	1	1
γ_1	2	2	2	2	4	6

Table-2 Nusselt Number Nu_2 at $y = -1$

G	I	II	III	IV	V	VI
10^3	7.9198	7.5106	8.1655	8.1188	6.1796	6.9601
3×10^3	7.9369	7.5256	8.1866	8.1409	6.4816	7.2184
10^3	7.9904	7.5724	8.2528	8.21	7.792	8.3097
3×10^3	7.972	7.5565	8.2303	8.1865	7.2674	7.8784
N	1	2	-0.5	-0.8	1	1
γ_1	2	2	2	2	4	6

Table-3 Sherwood number (Sh) at $y = 1$

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-0.1943	-0.28289	-0.28362	0.2176	6.80664	10.55639	0.60233	0.2843	0.245
3×10^3	-0.19536	-0.28381	-0.28497	-0.219	-3.089	12.66275	0.60341	0.2859	0.246
10^3	-0.19866	-0.28668	-0.28915	0.2234	0.55764	-0.98815	0.60676	0.2908	-0.25
3×10^3	-0.19754	-0.28571	-0.28773	3.2219	0.77307	-1.61405	0.60563	0.2893	0.248
N	1	2	-0.5	-0.8	1	1	1	1	1
γ_1	2	2	2	2	4	6	2	2	2
$x+\gamma t$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/2$	π	2π

Table-4 Sherwood number (Sh) at $y = -1$

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-1.1823	-1.2738	-1.2635	-1.1956	-2.0873	-3.115	-1.8842	-1.9775	-1.1089
3×10^3	-1.1827	-1.2743	-1.2639	-1.1959	-2.0435	-3.2747	-1.8822	-1.9718	-1.1104
10^3	-1.1841	-1.2758	-1.2649	-1.1969	-1.8512	-5.1627	-1.8766	-1.9555	-1.1146
3×10^3	-1.1836	-1.2865	-1.2646	-1.1966	-1.9286	-4.0781	-1.8784	-1.9609	-1.1132
N	1	2	-0.5	-0.8	1	1	1	1	1
γ_1	2	2	2	2	4	6	2	2	2
$x+\gamma t$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/2$	π	2π

An increase in the chemical reaction parameter $\gamma_1 \leq 4$ reduces $|Nu|$ and enhances with higher $\gamma_1 \geq 6$ in the heating case at both the walls. An increase in $x+\gamma t$ reduces the rate of heat transfer at $y=+1$ and fluctuates at $y=-1$. (Tables.1 and 2)

An increase in the chemical reaction parameter γ_1 leads to an enhancement in $n|Sh|$ at both the walls. With reference to the phase $x+\gamma t$ we find that the rate of mass transfer enhances with increase in $x+\gamma t \leq \pi$ and depreciates with $x+\gamma t \geq 2\pi$ at both the walls. (Tables.3 and 4)



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