LUCAS ANTIMAGIC LABELING OF SOME TYPES OF LADDER GRAPHS

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ABSTRACT

A (p, q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \rightarrow \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \rightarrow \{1,2, \ldots \sum_{i=1}^{q} L_i\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).

In this paper the Lucas Antimagic Labeling of Closed Ladder, Open Ladder, Slanting Ladder, Open Triangular Ladder, Closed triangular Ladder, Mobius Ladder are found.

KEYWORDS: Closed Ladder, Open Ladder, Slanting Ladder, Open Triangular Ladder, Closed triangular Ladder, Mobius Ladder.

1.INTRODUCTION

In this paper, graph $G(V, E)$ is considered as finite, simple and undirected with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges. Labeled graphs are used in radar, circuit design, communication network, astronomy, cryptography etc. For detailed survey on graph labeling we refer to Gallian[1]. The notion of Antimagic labeling was introduced by N.Hartsfield and G.Ringel in the year 1990 in [2].

A graph with q edges is called Antimagic if its edges can be labeled with 1,2,…q without repetition such that the sum of the labels of the edges incident to each vertex are distinct[1].

Here we introduce a **new notion called Lucas Antimagic labeling** and investigated the Lucas antimagic labeling behavior of different types of Ladder graphs. For all other terminology and notations Harary is followed[3].

2.PRELIMINARIES

Definition 2.1:[5] A ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with n vertices and $\times$ denotes the Cartesian product and K_2 is a complete graph with two-vertices.

Definition 2.2:[5] An Open ladder OL_n is acquired from two paths of length n-1 with

$V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and

$E(G) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1 , u_i v_i : 2 \leq i \leq n-1\}$.

Definition 2.3: [5] A slanting ladder SL_n is the graph acquired from two paths $u_1 u_2 \ldots u_n$ and $v_1 v_2 \cdots v_n$ by joining each u_i with $v_{i+1}, 1 \leq i \leq n-1$.

Definition 2.4: [5] A triangular ladder TL_n is a graph acquired from L_n by including the edges $u_i v_{i+1}, 1 \leq i \leq n-1$.

Definition 2.5: [5] An open Triangular ladder $O(TL_n)$ is obtained from an open ladder OL_n by including the edges $u_i v_{i+1}, 1 \leq i \leq n - 1$.

Definition 2.6: [5] A Mobius ladder graph M_n is a graph obtained from the ladder $P_n \times P_2$ by joining the opposite end points of the two copies of P_n.

Definition 2.7: Lucas number is defined by

$L_1 = 2, L_2 = 1, L_n = L_{n-1} + L_{n-2}\ , if\ n > 2$

The first few Lucas numbers are 2,1,3,4,7,11,18,29,47,…

Definition 2.8: A (p, q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \to \{1,2, \ldots \sum_{i=1}^{q} L_i\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct(where $E(u)$ is the set of edges incident to u).

3.MAIN RESULTS

THEOREM 3.1 :

The Ladder $L_n (n \geq 3)$ is Lucas antimagic graph.

Proof:

Let $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$

$E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n - 1\ , u_i v_i : 1 \leq i \leq n\}$

Define a function $f: E(L_n) \to \{L_1, L_2, \cdots L_q\}$ by

$f(u_i u_{i+1}) = L_i\ , 1 \leq i \leq n - 1$

$f(v_i v_{i+1}) = L_{n-1+i}\ , 1 \leq i \leq n - 1$

$f(u_i v_i) = L_{2(n-1)+i}\ , 1 \leq i \leq n$

The induced function $f^*: V(L_n) \to \{1,2, \ldots \sum L_q\}$ is given by

$f^*(u_1) = L_{2n-1} + L_1$

$f^*(u_i) = L_{i-1} + L_i + L_{2(n-1)+i}\ , 2 \leq i \leq n - 1$

$f^*(u_n) = L_{n-1} + L_{3n-2}$

$f^*(v_1) = L_n + L_{2n-1}$

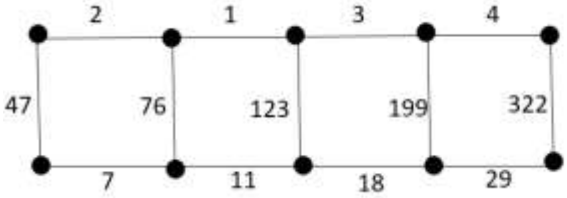
$f^*(v_i) = L_{n-2+i} + L_{n-1+i} + L_{2(n-1)+i}\ , 2 \leq i \leq n - 1$

$f^*(v_n) = L_{2n-2} + L_{3n-2}$

It is observed that the labeling of vertices are all distinct.

Hence L_n is Lucas antimagic graph.

Example 3.2: The Closed Ladder graph L_5 and its Lucas Antimagic Labeling.



THEOREM 3.3:

The Open Ladder $OL_n (n \geq 3)$ is Lucas antimagic graph.

Proof:

Let $V(OL_n) = \{u_i, v_i : 1 \leq i \leq n\}$

$E(OL_n) = \{u_i u_{i+1}, v_i v_{i+1} \; : 1 \leq i \leq n-1, u_i v_i : 2 \leq i \leq n-1\}$

Define a function $f: E(OL_n) \to \{L_1, L_2, \cdots L_q\}$ by

$f(u_i u_{i+1}) = L_i, 1 \leq i \leq n-1$

$f(v_i v_{i+1}) = L_{n-1+i}, 1 \leq i \leq n-1$

$f(u_i v_i) = L_{2n-3+i}, 2 \leq i \leq n-1$

The induced function $f^* : V(OL_n) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_1$

$f^*(u_i) = L_{i-1} + L_i + L_{2n-3+i}, 2 \leq i \leq n-1$

$f^*(u_n) = L_{n-1}$

$f^*(v_1) = L_n$

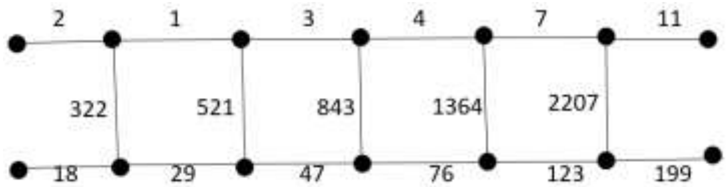
$f^*(v_i) = L_{n-2+i} + L_{n-1+i} + L_{2n-3+i}, 2 \leq i \leq n-1$

$f^*(v_n) = L_{2n-2}$

It is observed that the labeling of vertices are all distinct.

Hence OL_n is Lucas antimagic graph.

Example 3.4: The Open Ladder Graph OL_7 and its Lucas Antimagic Labeling.



THEOREM 3.5:

The Slanting Ladder $SL_n (n \geq 3)$ is Lucas antimagic graph.

Proof:

Let $V(SL_n) = \{u_i, v_i : 1 \le i \le n\}$

$E(SL_n) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} : 1 \le i \le n-1\}$

Define a function $f: E(SL_n) \to \{L_1, L_2, \cdots L_q\}$ by

$f(u_i u_{i+1}) = L_{2(n-1)+i}, 1 \le i \le n-1$

$f(v_i v_{i+1}) = L_i, 1 \le i \le n-1$

$f(u_i v_{i+1}) = L_{n-1+i}, 1 \le i \le n-1$

The induced function $f^* : V(SL_n) \to \{1, 2, \ldots \sum L_q\}$ is given by

$f^*(v_1) = L_1$

$f^*(v_i) = L_{i-1} + L_i + L_{n-2+i}, 2 \le i \le n-1$

$f^*(v_n) = L_{n-1} + L_{2n-2}$

$f^*(u_1) = L_n + L_{2n-1}$

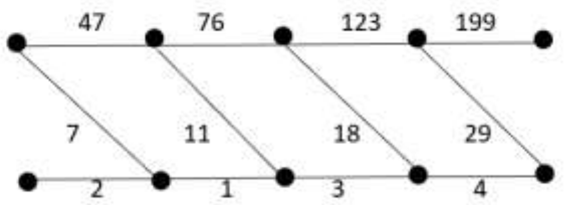
$f^*(u_i) = L_{2n-3+i} + L_{n-1+i} + L_{2(n-1)+i}, 2 \le i \le n-1$

$f^*(u_n) = L_{3n-3}$

It is observed that the labeling of vertices are all distinct.

Hence SL_n is Lucas antimagic graph.

Example 3.6: The Slanting Ladder Graph SL_5 and its Lucas Antimagic Labeling.



THEOREM 3.7:

The Triangular Ladder $TL_n (n \ge 3)$ is Lucas antimagic graph.

Proof:

Let $V(TL_n) = \{u_i, v_i : 1 \le i \le n\}$

$E(TL_n) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} : 1 \le i \le n-1, u_i v_i : 1 \le i \le n,\}$

Define a function $f: E(TL_n) \to \{L_1, L_2, \cdots L_q\}$ by

$f(u_i u_{i+1}) = L_i, 1 \le i \le n-1$

$f(v_i v_{i+1}) = L_{n-1+i}, 1 \le i \le n-1$

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$f(u_i v_i) = L_{2(n-1)+i}$, $1 \leq i \leq n$

$f(u_i v_{i+1}) = L_{3n-2+i}$, $1 \leq i \leq n-1$

The induced function $f^* : V(TL_n) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_{2n-1} + L_{3n-1} + L_1$

$f^*(u_i) = L_{i-1} + L_i + L_{2(n-1)+i} + L_{3n-2+i}$, $2 \leq i \leq n-1$

$f^*(u_n) = L_{n-1} + L_{3n-2}$

$f^*(v_1) = L_n + L_{2n-1}$

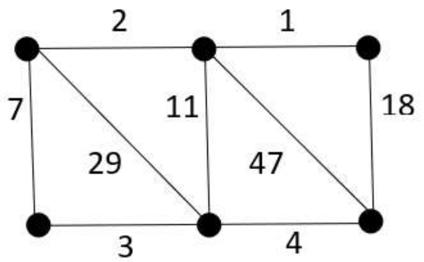
$f^*(v_i) = L_{n-2+i} + L_{n-1+i} + L_{2(n-1)+i} + L_{3n-3+i}$, $2 \leq i \leq n-1$

$f^*(v_n) = L_{2n-2} + L_{3n-2} + L_{4n-3}$

It is observed that the labeling of vertices are all distinct.

Hence TL_n is Lucas antimagic graph.

Example 3.8: The Closed Triangular Ladder Graph TL_3 and its Lucas Antimagic Labeling.



THEOREM 3.9:

The Open Triangular Ladder $O(TL_n)(n \geq 3)$ is Lucas antimagic graph.

Proof:

Let $V(O(TL_n)) = \{u_i, v_i : 1 \leq i \leq n\}$

$E(O(TL_n)) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} : 1 \leq i \leq n-1, u_i v_i : 2 \leq i \leq n-1\}$

Define a function $f: E(O(TL_n)) \to \{L_1, L_2, \cdots L_q\}$ by

$f(u_i u_{i+1}) = L_i$, $1 \leq i \leq n-1$

$f(v_i v_{i+1}) = L_{n-1+i}$, $1 \leq i \leq n-1$

$f(u_i v_i) = L_{2n-3+i}$, $2 \leq i \leq n-1$

$f(u_i v_{i+1}) = L_{3n-4+i}$, $1 \leq i \leq n-1$

The induced function $f^* : V(O(TL_n)) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_{3n-3}$

$f^*(u_i) = L_{i-1} + L_i + L_{2n-3+i} + L_{3n-4+i}$, $2 \le i \le n-1$

$f^*(u_n) = L_{n-1}$

$f^*(v_1) = L_n$

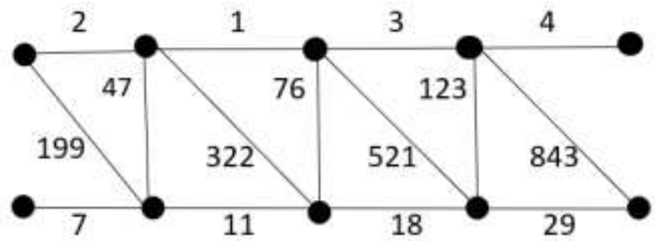
$f^*(v_i) = L_{n-2+i} + L_{n-1+i} + L_{2n-3+i} + L_{3n-5+i}$, $2 \le i \le n-1$

$f^*(v_n) = L_{2n-2} + L_{4n-5}$

It is observed that the labeling of vertices are all distinct.

Hence $O(TL_n)$ is Lucas antimagic graph.

Example 3.10: The Open Triangular Ladder Graph $O(TL_5)$ and its Lucas Antimagic Labeling.



THEOREM 3.11:

The Mobius Ladder $M_n (n \ge 3)$ is Lucas antimagic graph.

Proof:

Let $V(M_n) = \{u_i, v_i : 1 \le i \le n\}$

$E(M_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n-1, u_i v_i : 1 \le i \le n, u_1 v_n, u_n v_1\}$

Define a function $f: E(M_n) \to \{L_1, L_2, \cdots L_q\}$ by

$f(u_1 v_n) = L_1$

$f(u_n v_1) = L_2$

$f(u_i u_{i+1}) = L_{2+i}$, $1 \le i \le n-1$

$f(v_i v_{i+1}) = L_{n+1+i}$, $1 \le i \le n-1$

$f(u_i v_i) = L_{2n+i}$, $1 \le i \le n$,

The induced function $f^* : V(M_n) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_3 + L_{2n+1}$

$f^*(u_i) = L_{i+1} + L_{i+2} + L_{2n+i}$, $2 \le i \le n-1$

$f^*(u_n) = L_2 + L_{n+1} + L_{3n}$

$f^*(v_1) = L_2 + L_{n+1+i} + L_{2n+i}$

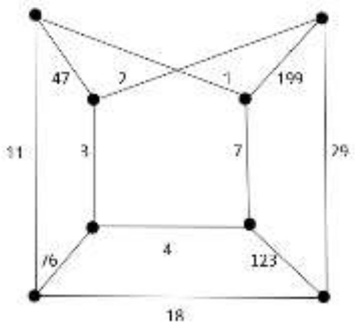
$f^*(v_i) = L_{n+i} + L_{n+1+i} + L_{2n+i}$, $2 \le i \le n-1$

$f^{*}(v_n) = L_1 + L_{2n} + L_{3n}$

It is observed that the labeling of vertices are all distinct.

Hence M_n is Lucas antimagic graph.

Example 3.12: The Mobius Ladder Graph M_4 and its Lucas Antimagic Labeling.



4.CONCLUSION:

In this paper, the notion of Lucas antimagic labeling is instigated and the Lucas antimagic labeling behavior of various ladder graphs are studied. Similar investigations are in process.

5.REFERENCES:

1. J.A.Gallian , “A dynamic survey of Graph labeling” , Electronic Journal of combinatorics,[2019]
2. N.Hartsfield and G.Ringel ,Pearls in graph theory, Academic press, San Diego,1990.
3. F.Harary , “Graph Theory” , Addision Wisely , New Delhi,1969.
4. S.K.Vaidya and N.B.Vyas “Antimagic labeling of some Path and Cycle Related graphs”,Annals of Pure and Applied Mathematics, Vol.3,No.2,2013,pp.119-128.
5. P.Sumathi and A.Rathi "Quotient Labeling of Some Ladder Graphs", American Journal of Engineering Research (AJER),vol.7,no.12,2018,pp.38-42.