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AN AFFINE ARITHMETICS FOR LARGE SCALE MULTI INPUT DYNAMIC SYSTEM MODEL ORDER REDUCTION

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Abstract
This article presents the strategies for producing reduced order model for a model ofhigher order utilizing modified polynomial derivatives and affine arithmetic (AA).To make system analysis simpler, a large-scale Continuous Time MIMO Dynamic system's reduced order model is necessary.The suggested approach is contrasted with several MIMO dynamic systems described in the literature.Analysis reveals that the proposed technique behaves almost exactly like the original systems of higher order.To demonstrate efficiency of the suggested strategy, The use of well-known numerical examples is considered.

Keywords: Modified Polynomial Derivative, Affine Arithmetic, Model Order Reduction, MIMO Dynamic Systems.

Introduction

Since high order systems are frequently too complex to be applied to actual issues, their analysis is time-consuming and expensive.Mathematical techniques are often used to build the fundamental models for the original systems of higher order. The reduction of a higher order system into its low order system is regarded to be advantageous for practical system analysis, synthesis, and simulation.Large-scale systems or extremely quick processes that need to be managed with low-order controllers are the most frequent sources of engineering issues.

For decreasing the order of the large scale linear SISO systems in frequency domain, a variety of reduction techniques are documented in the literature.Unfortunately, there are very few techniques for linear MIMO system reduction.Numerous techniques for SISO that are now accessible in the international literature can simply be expanded to reduce linear MIMO systems.By combining temporal moment matching with Pade approximations, Bendekas et al. previously presented a solution for the reduction of MIMO systems.On the basis of Pade's approximated generalised temporal moments, Taiwo and colleagues developed certain approaches.Chen has provided a method that uses the matrix continuing fraction technique.Sinha expanded Hutton's Routh approximation method to MIMO systems.

The proposed method consistently produces stable low order models for stable original interval systems of higher order.The time response approximation is improved by the low order model utilising the suggested technique because it keeps the initial interval Markov-parameter and (r - 1) initial interval time moments of the original system.The new method being presented would provide reduced order models that provide good steady state and transient response matching. The reduced order models generated by the proposed method do not need to calculate the time moments beforehand.The suggested approach may be used in systems with matrices that aren't square as well.Very easy to compute because it does not require matrix multiplication, sequential differentiation, reciprocal transformation, or extensive Routh-type differentiation arrays.

In this research, an extension of the modified polynomial derivative approach has been used to achieve the order reduction of MIMO interval systems [1]. Reciprocal transformations [2–5], matrix multiplications, and the creation of extensive Routh-type differentiation arrays are not necessary with the unique technique. The recommended strategy is simple to calculate and uncomplicated.

Problem Formulation

There are relatively few methods available for modelling big continuous time MIMO interval systems. The following significant flaws and restrictions plague Ismail's approaches [6-7]. Ismail et al. advise using this technique to reduce MIMO structured uncertain systems. By using the Pade approximation, the k^{th} order reduced model is generated by this procedure. This approach is more computationally complex and time-consuming since it depends on having to evaluate the original MIMO interval system's time moments in advance. It has been highlighted that the approach, which was developed using the generic Pade approximation methodology, occasionally generates unstable low order models for stable original systems of higher order [6]. According to Ismail's technique, the Routh table, which is created using the denominator D(s) of original system, is used to build reduced order denominator polynomial of the k^{th} order reduced model. By comparing the first "k" moments of reduced model with those from the original system, numerator Nk(s) from the reduced order model is found. This approach, which requires an unreasonable amount of computer work, guarantees the construction of the interval Routh array in its whole, regardless of the order of the necessary model. This technique requires advance calculation of the temporal moments of a high order MIMO interval system, which takes a lot of time [6, 8].

The numerator Nk(s) is created in this approach such that Rk(s) is a Pade approximant of G(s) about 'k' points. The denominator polynomial Dk(s) from k^{th} order reduced model is acquired from the Routh stability array built for system denominator D(s). This technique expands G(s) and RK(s) approximately k points, increasing the number of calculations with the order of the needed model. This approach, which requires a significant amount of computing labour, requires the creation of a Routh type array in its whole, independent of the reduced model's order.

In order to get around the limitations and flaws of the approaches suggested by Ismail & Bandyopadhyay [6-8] for reduction of Interval MIMO systems, this study presents novel strategy [9] for the reduction of Interval Uncertain (Interval) MIMO systems. The newly suggested approach is based on Polynomial Derivative Technique. The recommended approach guarantees that reduced order model [$R_k(s)$] is unique and stable if original transfer function matrix [G(s)] is stable and that the degree of the minimum polynomial of [$R_k(s)$] is smaller than that of [G(s)].

Proposed Modified Polynomial Derivative Technique:

Considerthat the transfer matrix form of initial high order q-input, p-output MIMO interval system be as follows:

$$G_n(s) = \frac{[N_{i,j}(s)]}{D_n(s)}; \text{with } i=1,2,3,\ldots, p; \; j=1,2,3,\ldots, q \text{ and } (m \le n)$$

$$= \begin{bmatrix} N_{11}(s) & N_{12}(s) & \ldots & N_{1q}(s) \\ N_{21}(s) & N_{22}(s) & \ldots & N_{2q}(s) \\ \ldots & \ldots & \ldots & \ldots \\ N_{p1}(s) & \ldots & .. & N_{pq}(s) \end{bmatrix} \qquad \ldots\ldots (1)$$

Where the common denominator polynomial is

$$D_n(s) = [A_n^-, A_n^+]s^n + [A_{n-1}^-, A_{n-1}^+]s^{n-1} + \ldots\ldots\ldots\ldots + [A_1^-, A_1^+]s + [A_0^-, A_0^+] \text{ And}$$

The scalar numerator polynomials are,

For i = 1, j = 1,

$$N_{11}(s) = [B_m^-, B_m^+]_{11}\, s^m + .[B_{m-1}^-, B_{m-1}^+]_{11}\, s^{m-1} + \ldots\ldots\ldots\ldots + [B_1^-, B_1^+]_{11}\, s + [B_0^-, B_0^+]_{11}$$

For i = 1, j = 2,

$$N_{12}(s) = [B_m^-, B_m^+]_{12}\, s^m + [B_{m-1}^-, B_{m-1}^+]_{12}\, s^{m-1} + \ldots\ldots + [B_1^-, B_1^+]_{12}\, s + [B_0^-, B_0^+]_{12}$$

… … ….

And in general,

$$N_{ij}(s)=[B_m^-,B_m^+]_{ij}\, s^m+[B_{m-1}^-,B_{m-1}^+]_{ij}\, s^{m-1}+..........+[B_1^-,B_1^+]_{ij}\, s+[B_0^-,B_0^+]_{ij}$$

For the original high order interval system mentioned above, it is suggested to obtain a low order model as follows:

$$R_r(s)=\frac{[n_{i,j}(s)]}{d_r(s)};\text{with } i=1,2,3,\ldots..,p;\ j=1,2,3,\ldots..,q.$$

$$=\begin{bmatrix} n_{11}(s) & n_{12}(s) & \ldots & n_{1q}(s)\\ n_{21}(s) & n_{22}(s) & \ldots & n_{2q}(s)\\ \ldots & \ldots & \ldots & \ldots\\ n_{p1}(s) & \ldots & \ldots & n_{pq}(s)\end{bmatrix} \quad \ldots\ldots (2)$$

Where the reduced order common denominator polynomial is

$$d_r(s)=[a_r^-,a_r^+]s^r+..........+[a_1^-,a_1^+]s+[a_0^-,a_0^+] \text{ With } [a_r^-,a_r^+]=[1,1]$$

And the scalar numerator polynomials are

For i = 1, j = 1, $n_{11}(s)=[b_{r-1}^-,b_{r-1}^+]_{11}\, s^{r-1}+..........+[b_1^-,b_1^+]_{11}\, s+[b_0^-,b_0^+]_{11}$

For i = 1, j = 2, $n_{12}(s)=[b_{r-1}^-,b_{r-1}^+]_{12}\, s^{r-1}+..........+[b_1^-,b_1^+]_{12}\, s+[b_0^-,b_0^+]_{12}$

… … ….

And in general, $n_{ij}(s)=[b_{r-1}^-,b_{r-1}^+]_{ij}\, s^{r-1}+..........+[b_1^-,b_1^+]_{ij}\, s+[b_0^-,b_0^+]_{ij}$

Reduced order Denominator, $d_r(s)$:

The following novel procedures are suggested to produce the reduced order model's low order common denominator polynomials, $d_r(s)$ (r<= n):

For r = 1, $d_1'(s)=\left(\frac{{}^{n-1}K_{n-1}}{{}^{n}K_{n-1}}\right)[A_1^-,A_1^+]s+[A_0^-,A_0^+]$

For r = 2, $d_2'(s)=\left(\frac{{}^{n-2}K_{n-2}}{{}^{n}K_{n-2}}\right)[A_2^-,A_2^+]s^2+\left(\frac{{}^{n-1}K_{n-2}}{{}^{n}K_{n-2}}\right)[A_1^-,A_1^+]s+[A_0^-,A_0^+]$

… … ….

And in general,

$$d_r'(s)=\left(\frac{{}^{n-r}K_{n-r}}{{}^{n}K_{n-r}}\right)[A_r^-,A_r^+]s^r+\sum_{j=1}^{r}\left(\frac{{}^{n-j+1}K_{n-r}}{{}^{n}K_{n-r}}\right)[A_{j-1}^-,A_{j-1}^+]s^{j-1} \quad \ldots\ldots (3)$$

Where, $${}^{P}K_Q=\frac{P!}{P!\,(P-Q)!} \text{ and } {}^{P}K_Q=1. \quad \ldots\ldots (4)$$

The reduced order common denominator of "r^th" order is obtained as follows after correct normalisation:

$$d_r(s)=\sum_{i=0}^{r}[a_i^-,a_i^+]s^r;\ \text{ with } [a_r^-,a_r^+]=[1,1]. \quad \ldots\ldots (5)$$

Reduced order Numerator, $n_{ij}(s)$:

The proposed new algorithms toget numerator polynomials that retain first interval Markov parameter and (r-1) initial interval time moments from original interval system are:

For $r=1$, $n_1(s)\ =\ [b_0^-,b_0^+]$

For $r=2$, $n_2(s)\ =\ [b_1^-,b_1^+]s+[b_0^-,b_0^+]$

…. … …..

And in general,

$$n_r(s)=[b_{r-1}^-,b_{r-1}^+]s^{r-1}+......+[b_1^-,b_1^+]s+[b_0^-,b_0^+] \quad \ldots\ldots (6)$$

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where $[b_0^-, b_0^+] = c[a_0^-, a_0^+]$; $\quad [b_{r-1}^-, b_{r-1}^+] = d[a_r^-, a_r^+]$ and
for $i = 1,2,3,\ldots\ldots\ldots\ldots,(r-2)$

$$[b_i^-, b_i^+] = \left[\frac{[B_i^-, B_i^+]}{[A_0^-, A_0^+]}[a_0^-, a_0^+] + \frac{[B_0^-, B_0^+]}{[A_0^-, A_0^+]}\left\{[a_i^-, a_i^+] - \frac{[A_i^-, A_i^+]}{[A_0^-, A_0^+]}[a_0^-, a_0^+]\right\} + \sum_{j=1}^{i-1}(-1)^j\left\{\frac{[A_j^-, A_j^+]}{[A_0^-, A_0^+]}[b_j^-, b_j^+] - \frac{[B_j^-, B_j]}{[B_0^-, B_0^+]}[a_j^-, a_j^+]\right\}\right] \quad \ldots\ldots (7)$$

With 'c' = mean of $\frac{[B_0^-, B_0^+]}{[A_0^-, A_0^+]}$ and'd' = mean of $\frac{[B_m^-, B_m^+]}{[A_n^-, A_n^+]}$

Numerical Examples

The results of applying the following numerical examples from the literature to the suggested approach are successfully validated in order to determine the method's adaptability and efficacy.
Consider the following transfer matrix for a stable 5-order, single-input, 2-output stable interval system:

$$\mathbf{G}_s(s) = \frac{[\mathbf{N}_i(s)]}{D_s(s)}(i = 1, 2) = \frac{1}{D_s(s)}\begin{bmatrix} N_1(s) \\ N_2(s) \end{bmatrix}$$

$$= \frac{\begin{bmatrix}[3.83, 4.06] \\ [3.78, 4.00]\end{bmatrix}s^4 + \begin{bmatrix}[118.0, 125.0] \\ [95.8, 101.6]\end{bmatrix}s^3 + \begin{bmatrix}[339.60, 360.10] \\ [267.86, 283.9]\end{bmatrix}s^2 + \begin{bmatrix}[275.50, 280.10] \\ [233.53, 238.54]\end{bmatrix}s + \begin{bmatrix}[66.34, 70.32] \\ [66.34, 70.32]\end{bmatrix}}{[1.0, 1.03]s^5 + [24.6, 25.34]s^4 + [136.14, 140.23]s^3 + [282.72, 291.2]s^2 + [236.51, 243.61]s + [66.34, 70.32]}$$

For the aforementioned original high order internal system, it is advised to employ the following procedure to obtain a second order reduced model:

$$\mathbf{R}_2(s) = \frac{[\mathbf{n}_i(s)]}{d_2(s)}(i = 1,2) = \frac{1}{d_2(s)}\begin{bmatrix} n_1(s) \\ n_2(s) \end{bmatrix}$$

Reduced order denominator, $d_2(s)$:

By using the suggested technique, the following 2nd order common reduced denominator is obtained:

$$d_2'(s) = \left(\frac{{}^3K_3}{{}^5K_3}\right)[282.72, 291.2]s^2 + \left(\frac{{}^4K_3}{{}^5K_3}\right)[236.51, 243.61]s + [66.34, 70.32]$$

$$= [28.272, 29.12]s^2 + [94.604, 97.444]s + [66.34, 70.32]$$

The 2nd order reduced common denominator, $d_2(s)$, is derived after correct normalisation as:

$$d_2(s) = [1,1]s^2 + [3.2488, 3.4467]s + [2.2782, 2.4873]$$

Reduced order numerator, $n_i(s)$:

The simplified model's scalar coefficients for the numerator polynomial are as follows:

$$n_1(s) = [b_1^-, b_1^+]s + [b_0^-, b_0^+]$$

$$\text{where, } [b_0^-, b_0^+] = c[a_0^-, a_0^+]; \text{ with } c = \text{mean of } \frac{[B_0^-, B_0^+]}{[A_0^-, A_0^+]} = 1$$

$$[b_0^-, b_0^+] = [2.2782, 2.4873] \text{ and}$$

$$[b_1^-, b_1^+] = d[a_r^-, a_r^+];\ \text{with } d = \text{mean of } \frac{[B_m^-, B_m^+]}{[A_n^-, A_n^+]} = 3.89$$

$$[b_1^-, b_1^+] = [3.89, 3.89]$$

$$\text{similarly},\quad n_2(s) = [b_1^-, b_1^+]s + [b_0^-, b_0^+]$$

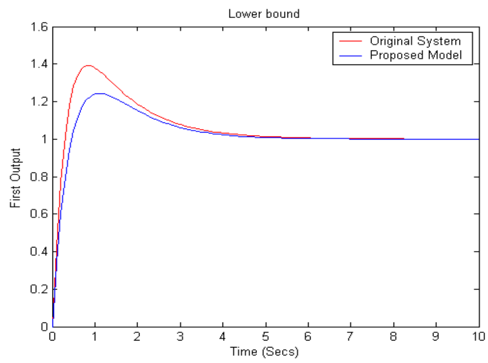
$$\text{where},\qquad [b_0^-, b_0^+] = [2.2782, 2.4873]$$

$$\text{and } [b_1^-, b_1^+] = d[a_r^-, a_r^+];\text{with } d = \frac{[3.78, 4.0]}{[1.0, 1.03]} = 3.84$$

$$[b_1^-, b_1^+] = [3.84, 3.84]$$

Consequently, the transfer matrix form of reduced 2nd order model that was generated from the suggested technique is provided by

$$\mathbf{R}_2(s) = \frac{\begin{bmatrix}[3.89, 3.89]\\ [3.84, 3.84]\end{bmatrix} s + \begin{bmatrix}[2.2782, 2.4873]\\ [2.2782, 2.4873]\end{bmatrix}}{[1,1]s^2 + [3.2488, 3.4467]\,s + [2.2782, 2.4873]}\quad (\textit{Proposed Method})(\textit{Stable})$$



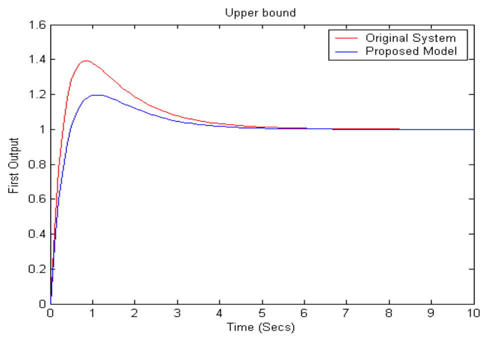
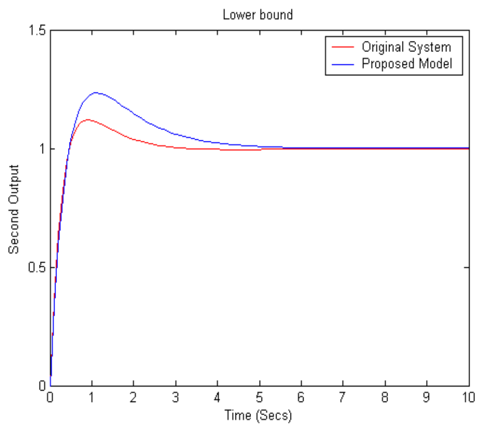


Fig.1a: Step replies are compared. Fig.1b: Comparing the results of the steps.



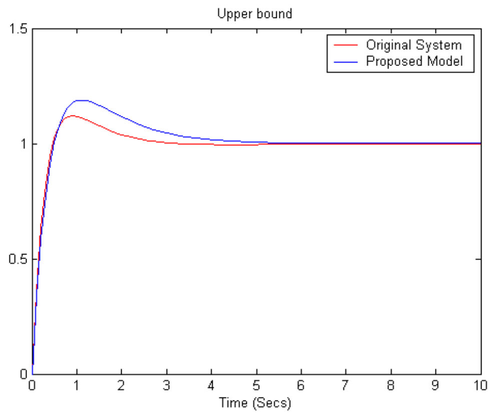


Fig.1c: Step reactions are compared. Fig.1d: Step replies are compared.

Figures 1a–1d compares step responses of original system of higher order with the 2nd order model of suggested method.

Consider a stable interval system of third order with a single input and two outputs that is provided by [125]:

$$G(s) = \frac{1}{D(s)}\begin{bmatrix} N_{11}(s) & N_{12}(s) \\ N_{21}(s) & N_{22}(s)\end{bmatrix}$$

Where

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$$N_{11}(s) = [11,13]s^2 + [30,32]s + [22,23]; \quad N_{12}(s) = [3,5]s^2 + [18,19]s + [9,10]$$
$$N_{21}(s) = [1,2]s^2 + [15,18]s + [12,13]; \quad N_{22}(s) = [7,8]s^2 + [26,29]s + [20,21]$$

And

$$D(s) = [2,3]s^3 + [34,36]s^2 + [75,78]s + [38,39]$$

Applying the suggested strategy yields the following 2nd order reduced model:

$$R_2(s) = \frac{1}{d_2(s)}\begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix} \qquad (\textit{Proposed method})$$

Where

$$n_{11}(s) = [1.5083,1.5083]s + [1.8215,2.0220]$$
$$n_{12}(s) = [1.0750,1.0750]s + [0.7521,0.8871]$$
$$n_{21}(s) = [0.6666,0.6666]s + [1.0089,1.1480]$$
$$n_{22}(s) = [1.1667,1.1667]s + [1.676,1.853]$$
$$d_2(s) = [1,1]s^2 + [4.1667,4.5882]s + [3.1667,3.4412] \qquad (\textit{Stable})$$

The approach "Stable Multi point Pade Approximation for linear MIMO Structured Uncertain systems" proposed by Ismail [125] is used to get the reduced second order model, which is obtained as:

$$R'_2(s) = \frac{1}{d_2(s)}\begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix} \qquad (\textit{Ismail method})$$

Where

$$n_{11}(s) = [38.69,42.77]s + [22,23]$$
$$n_{12}(s) = [19.9,22.88]s + [9,10]$$
$$n_{21}(s) = [11.98,16.06]s + [12,13]$$
$$n_{22}(s) = [31.06,35.1]s + [20,21]$$
$$d_2(s) = [34,36]s^2 + [71.558,75.88]s + [38,39]$$

Figures 2a–2d illustrate the step response comparison of reduced order models obtained using the suggested method and the method "Stable Multi point Pade Approximation for linear MIMO Structured Uncertain systems" proposed by Ismail [125].

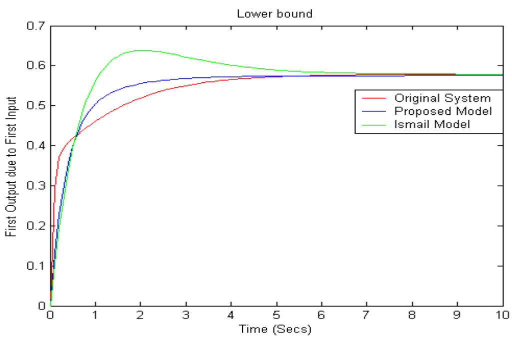

Fig.2.a: Step replies are compared.

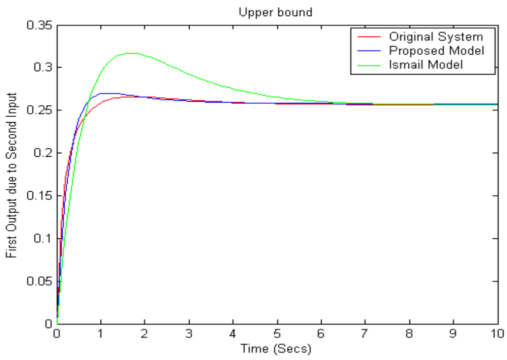

Fig.2b: Comparison of step responses.

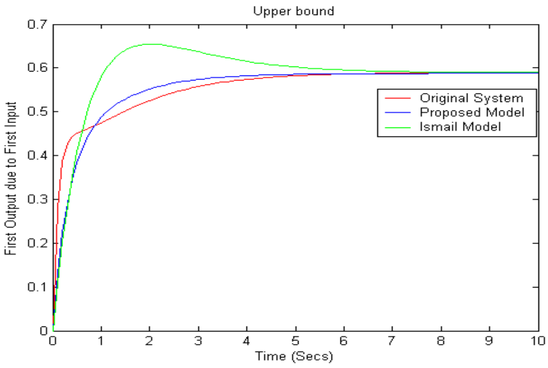

Fig.2c: Comparison of step responses.

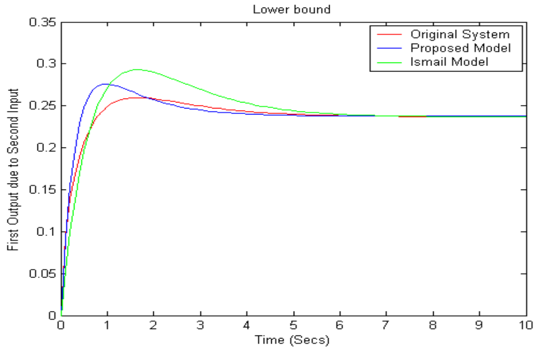

Fig.2d: Comparison of step responses.

Conclusion

The study and design of large Continuous Time MIMO Uncertain Systems are found to be complicated and expensive in the literature. The recommended approach has improved for the analysis and simulation of high-order uncertain systems. There shouldn't be any transition from a system of higher-order to a loworder model that affects primary components of the original system. Costly lower-order systems that were constructed from the original systems to describe their performance. Simulating higher-order systems involves reducing the model order. Since it offers superior stable intervals than interval arithmetic, affine arithmetic preserves the relationships between those values.

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