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ANALYZING THE IMPACT OF ETHD ON HEAT TRANSFER AROUND A HEXAGONAL CYLINDER WITH MACHINE LEARNING TEMPERATURE PREDICTIONS

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ABSTRACT
Our investigation explores the impact of Electro-Thermo-Convective Hydrodynamics (ETHD) on heat transfer and fluid dynamics by examining the interactions among electric fields, heat, and fluid flow. Using the Lattice Boltzmann Method (LBM) for mesoscopic fluid simulation, the model features a square cavity with a centrally placed hexagonal cylinder. This hexagonal cylinder, maintained at a higher electric potential and temperature than the grounded, cooler cavity, creates an environment for electrochemical reactions within the fluid. The six-sided geometry of the cylinder, with its unique corners and slanting sides, is expected to enhance heat transfer by interacting with the electric field, particularly at high electric Rayleigh numbers where Coulomb forces become dominant, thus increasing the average Nusselt number. Various machine learning algorithms were applied to predict temperature, charge distributions, and average Nusselt numbers. The model achieved a training Mean Absolute Error (MAE) of 0.0679 and R^2 of 0.9934, indicating high accuracy and an excellent fit. However, in testing, the model showed a higher MAE of 0.3003 and an R^2 of 0.7676, suggesting limited generalization, potentially due to dataset constraints. The scattered residuals indicate possible underfitting or noise, highlighting a need for more extensive data to improve predictive capability

Keywords:
Electro-thermo-convective hydrodynamics (ETHD), Lattice Boltzmann Method (LBM), machine learning and hexagonal cylinder.

I. Introduction
Electro-Thermo-Hydro Dynamics (ETHD) focuses on the interaction between electric fields, heat, and fluid movement to improve system performance. By using electric forces to control fluid flow and enhance heat transfer, ETHD eliminates the need for traditional mechanical components like pumps, offering a more efficient and energy-saving solution. This technology is particularly useful in applications such as cooling electronics and nuclear reactors, where conventional methods are often insufficient, and is gaining traction for its simplicity, energy efficiency, and quiet operation.
Thermal effects in ETHD arise from temperature gradients, leading to phenomena like buoyancy forces, thermal expansion, and convection. Buoyancy forces drive fluid motion as hot, less dense fluids rise and cooler, denser fluids sink, promoting convection currents. Thermal expansion alters material properties and can induce thermal stresses, potentially leading to fatigue or failure. Phase changes in fluids also affect thermal and electrical properties, sometimes resulting in energy losses due to

overheating. These effects are crucial in the design of ETHD systems, and advanced modeling tools are used to optimize thermal management.
In ETHD, fluid dynamics are influenced by electromagnetic forces, temperature gradients, and fluid properties such as viscosity and density. These factors work together to improve heat transfer by promoting mixing and disrupting thermal boundary layers. Electromagnetic interactions can also create instabilities in magnetohydrodynamic systems, important in applications like plasma physics and energy generation. Ultimately, ETHD offers enhanced heat management for a wide range of applications, from industrial processes to energy generation and cooling technologies.

II. Literature

Recent research on Electro-Thermo-Hydro Dynamics (ETHD) has explored a wide range of factors affecting fluid flow and heat transfer under the influence of electric fields and thermal gradients. Kun He et al. [1] demonstrated that higher temperatures improve thermal conductivity and accelerate heat transfer rates in solid walls, emphasizing the importance of material properties for thermal management. Qiang Liu et al. [2] highlighted the potential of ETHD in applications such as cooling, desalination, and microfabrication, pointing to innovative opportunities in these areas. Yifei Guan et al. [3] investigated electro-thermo-hydrodynamic convection with charge injection, revealing oscillating flow patterns and the formation of electro-thermo-convective vortices. Jian Zhao et al. [4] showed that Rayleigh-Benard convection enhanced by electric fields improves heat transfer efficiency, with the three-cell flow pattern being the most effective. Tien-Mo Shih et al. [5] focused on the interplay between electric fields, flow motion, and thermal fields to optimize heat and mass transfer in ETHD systems.
Deepak Selvakumar et al. [6] examined electro-thermo-convection in a square cavity with unipolar ion injection, highlighting the interaction between electric fields and thermal gradients in producing intricate flow patterns and temperature distributions. Jong Hyeon Son and Seouk Park [7] analyzed the heat transfer coefficient, showing it to be over three times higher for vertical electrode pairs compared to horizontal ones at specific Rayleigh and electric Rayleigh numbers. Dantchi et al. [8] explored the impact of electric field strengths on electro-thermo-convection, significantly enhancing understanding of electrohydrodynamic behavior for engineering applications. Zhiming Lu et al. [9] investigated electro-thermo-convection in dielectric liquids, revealing that thermal gradients and electric forces influence complex flow patterns, which could improve thermal performance. J.B. Hull et al. [10] demonstrated that electrohydrodynamic effects significantly enhance natural convection rates, improving thermal performance in practical applications.
Walid Hassen et al. [11] studied the transition to chaos in electro-thermo-convection, showing periodic oscillations, chaotic behavior, and steady-state flows. Tian-Fu Li et al. [12] identified critical parameters leading to chaotic behavior in electro-thermo-convection of dielectric liquids, uncovering the significance of non-linear interactions in heat transfer. In another study, Walid Hassen et al. [13] explored the impact of multi-walled carbon nanotube (MWCNT) dielectric nanofluids on heat transfer, demonstrating that MWCNTs enhance thermal conductivity and improve heat transfer efficiency under electric fields. Yazhou Wang et al. [14] developed a spectral element method to simulate electro-thermally enhanced heat transfer in enclosures, providing valuable insights into optimizing thermal management systems.
Further studies have focused on electro-thermo-convection in complex fluid systems. Zheng-Gang et al. [15] examined the influence of fluid properties on convection stability and intensity in non-Newtonian fluids, revealing their significant impact under electric fields and advancing understanding of electrohydrodynamic processes in industrial applications. Hao-kui et al. [16] investigated electro-thermo-convection instability and bifurcations in a tilted square cavity filled with dielectric liquid, identifying critical bifurcation points and complex convection behaviors. Perez et al. [17] conducted a linear stability analysis of electrohydrodynamic flow in dielectric liquids, highlighting stability thresholds and convection onset, which enhances the understanding of electrohydrodynamic effects in

confined geometries. Mohamed Issam et al. [18] studied the impact of electric fields on heat transfer between two elliptical cylinders, revealing how electric fields enhance convective heat transfer and providing insights for optimizing cylindrical thermal management applications.
W. Hassen et al. [19] explored electroconvection between coaxial cylinders under unipolar injection, showing how cylinder ratios affect flow patterns and heat transfer, which could have implications for advanced cooling applications. Walid Hassen et al. [20] also studied electro-thermo-convection in annular dielectric layers, revealing how injection conditions affect flow dynamics and heat transfer efficiency, providing guidance for optimizing thermal management. In a subsequent study, Walid Hassen et al. [21] examined the impact of cylinder arrangement and injection strength on electro-thermo-convection in concentric and eccentric cylinders, highlighting how these factors influence flow patterns and heat transfer, with significant implications for thermal management systems.

III Physical Model

The purpose of this study is to investigate the electro-thermo-convective hydrodynamics within a square cavity containing a hexagonal cylinder centrally placed. The hexagonal cylinder is maintained at a higher electric potential with a higher temperature than that in the outer grounded square cavity.

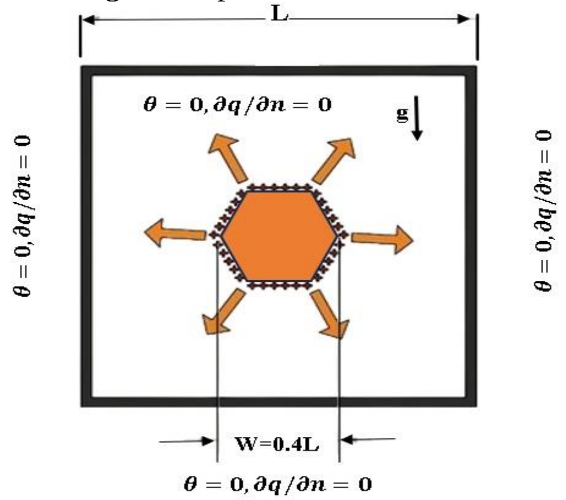

Fig 3.1: Problem statement

Governing Equations:

3.1 Lattice Boltzmann method for flow field:

The generalized lattice Boltzmann equation for flow field with a body force can be expressed as

$$f(x + c\Delta t, t + \Delta t) - f(x,t) = \Omega + \Delta t \times F,$$

Where f is the distribution function, c is the microscopic velocity, Ω is the collision operator, and F is the body force term

The collision term can be written as

$$\Omega = -[f(x,t) - f^{eq}(x,t)]/\tau_v$$

The equilibrium distribution function can be written as

$$f_j^{eq} = \rho\omega_j\left(1 + \frac{c_j.u}{c_s^2} + \frac{(c_j.u)^2}{2c_s^4} - \frac{u^2}{2c_s^2}\right),$$

$$F_j = \omega_j\left(1 - \frac{1}{2\tau_v}\right)\frac{c_j\left[qE + \rho g\beta\left(\theta - \theta_{ref}\right)\right]}{c_s^2}$$

3.2. Lattice Boltzmann method for electric potential and electric field

The distribution function of electric potential can be written as

$$g_j(x + c_j\Delta t, t + \Delta t) - g_j(x,t) = -\frac{1}{\tau_\emptyset}\left[g_j(x,t) - g_j^{eq}(x,t)\right] + \Delta t S_j$$

The relaxation time $\boldsymbol{\tau_\varphi}$ can be determined as,

$$\boldsymbol{\tau_\varphi = \frac{3\gamma}{c^2\Delta t} + \frac{1}{2}}$$

The electric potential $\boldsymbol{\varphi}$ can be evaluated as,

$$\boldsymbol{\varphi = \sum_j g_j}$$

3.3 Lattice Boltzmann method for charge density distribution

The Lattice Boltzmann Equation (LBE) to determine the charge density distribution,

$$\boldsymbol{h_j}(x + \boldsymbol{c_j}\Delta t, t + \Delta t) - \boldsymbol{h_j}(x,t) = -\frac{1}{\tau_q}\left[\boldsymbol{h_j(x,t) - h_j^{eq}(x,t)}\right],$$

The relaxation time $\boldsymbol{\tau_q}$ can be determined as,

$$\tau_q = \frac{3D}{c^2\Delta t} + \frac{1}{2}$$

The charge density q can be calculated as,

$$\boldsymbol{q = \sum_j h_j}$$

3.4 Lattice Boltzmann method for thermal field

The Lattice Boltzmann Equation (LBE) to determine the temperature distribution,

$$\boldsymbol{l_j}(x + \boldsymbol{c_j}\Delta t, t + \Delta t) - \boldsymbol{l_j}(x,t) = -\frac{1}{\tau_\theta}\left[\boldsymbol{l_j(x,t) - l_j^{eq}(x,t)}\right],$$

The relaxation time $\boldsymbol{\tau_\theta}$ can be determined as,

$$\tau_\theta = \frac{3\chi}{c^2\Delta t} + \frac{1}{2}$$

The equilibrium distribution function for temperature $\boldsymbol{\theta}$ can be expressed as,

$$\boldsymbol{\theta = \sum_j l_j}$$

3.5 Lattice Boltzmann method for flow field:

The generalized lattice Boltzmann equation for flow field with a body force can be expressed as

$$f(x + c\Delta t, t + \Delta t) - f(x,t) = \Omega + \Delta t \times F,$$

Where f is the distribution function, c is the microscopic velocity, Ω is the collision operator, and F is the body force term

The collision term can be written as

$$\Omega = -[f(x,t) - \boldsymbol{f^{eq}}(x,t)]/\boldsymbol{\tau_v}$$

The equilibrium distribution function can be written as

$$\boldsymbol{f_j^{eq} = \rho\omega_j\left(1 + \frac{c_j.u}{c_s^2} + \frac{(c_j.u)^2}{2c_s^4} - \frac{u^2}{2c_s^2}\right)},$$

$$\boldsymbol{F_j = \omega_j\left(1 - \frac{1}{2\tau_v}\right)\frac{c_j\left[qE + \rho g\beta\left(\theta - \theta_{ref}\right)\right]}{c_s^2}}$$

3.6. Lattice Boltzmann method for electric potential and electric field

The distribution function of electric potential can be written as

$$\boldsymbol{g_j(x + c_j\Delta t, t + \Delta t) - g_j(x,t) = -\frac{1}{\tau_\emptyset}\left[g_j(x,t) - g_j^{eq}(x,t)\right] + \Delta t S_j}$$

The relaxation time $\boldsymbol{\tau_\varphi}$ can be determined as,

$$\boldsymbol{\tau_{\varphi}} = \frac{\mathbf{3}\boldsymbol{\gamma}}{\boldsymbol{c}^{\mathbf{2}}\boldsymbol{\Delta t}} + \frac{\mathbf{1}}{\mathbf{2}}$$

The electric potential $\boldsymbol{\varphi}$ can be evaluated as,

$$\boldsymbol{\varphi} = \sum_{j} \boldsymbol{g_j}$$

3.7 Lattice Boltzmann method for charge density distribution

The Lattice Boltzmann Equation (LBE) to determine the charge density distribution,

$$\boldsymbol{h_j}(x + \boldsymbol{c_j}\Delta t, t + \Delta t) - \boldsymbol{h_j}(x,t) = -\frac{\mathbf{1}}{\boldsymbol{\tau_q}}\left[\boldsymbol{h_j}(\boldsymbol{x},\boldsymbol{t}) - \boldsymbol{h_j^{eq}}(\boldsymbol{x},\boldsymbol{t})\right],$$

The relaxation time $\boldsymbol{\tau_q}$ can be determined as,

$$\tau_q = \frac{3D}{c^2\Delta t} + \frac{1}{2}$$

The charge density q can be calculated as,

$$\boldsymbol{q} = \sum_{j} \boldsymbol{h_j}$$

3.8 Lattice Boltzmann method for thermal field

The Lattice Boltzmann Equation (LBE) to determine the temperature distribution,

$$\boldsymbol{l_j}(x + \boldsymbol{c_j}\Delta t, t + \Delta t) - \boldsymbol{l_j}(x,t) = -\frac{\mathbf{1}}{\boldsymbol{\tau_\theta}}\left[\boldsymbol{l_j}(\boldsymbol{x},\boldsymbol{t}) - \boldsymbol{l_j^{eq}}(\boldsymbol{x},\boldsymbol{t})\right],$$

The relaxation time $\boldsymbol{\tau_\theta}$ can be determined as,

$$\tau_\theta = \frac{3\chi}{c^2\Delta t} + \frac{1}{2}$$

The equilibrium distribution function for temperature $\boldsymbol{\theta}$ can be expressed as,

$$\boldsymbol{\theta} = \sum_{j} \boldsymbol{l_j}$$

IV Results and discussions

Fluid flow within the enclosure results from the combination of temperature difference, and potential difference along with charge distribution within the enclosure. The temperature gradient creates thermal convection where warmer fluid rises and cooler fluid sinks, leading to circulating currents. The applied potential difference leads to electric forces that will influence charged particles to shape the flow pattern. Charge distribution within the enclosure also leads to localized Coulomb forces, further affecting fluid movement and heat transfer.

At *T*=100, all motion and heat transfer are due to thermal convection, electric field effect, and charge distribution. The potential difference produces an electric field, though these forces acting on the charged fluid particles are quite insignificant, it does make the flow pattern a bit more involved. The variation in density due to temperature differences between the fluid and the cylinder causes the warmer fluid to rise and cooler fluids to sink creating circulating currents. Moreover, the unequal charge distribution results in local Coulomb forces that affect how the fluid interacts with the boundaries, especially near the bottom where charge voids can occur.

At *T*=350, thermal expansion and buoyant forces create strong vortex fields around the hexagonal body. Stronger vortices create increased kinetic energy in the fluid flow. These vortices cause better mixing, hence optimizing heat transfer rates with a relatively uniform temperature field distribution throughout the fluid. This increases the interaction between the fluid and the surfaces of the hexagonal body, therefore maximizing heat transfer efficiency.

Furthermore, in the case of a higher temperature, the "charge voids" regions extend in the charge distribution plot. These depleted areas of charges are the result of the interaction of thermal and electric fields; thus, the areas of this redistribution of charge carriers change the electrohydrodynamic forces

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in the fluid. Fluid motion is therefore made stronger, and this redistribution reduces the thickness of the thermal boundary layer, thus enhancing efficiency in heat transfe

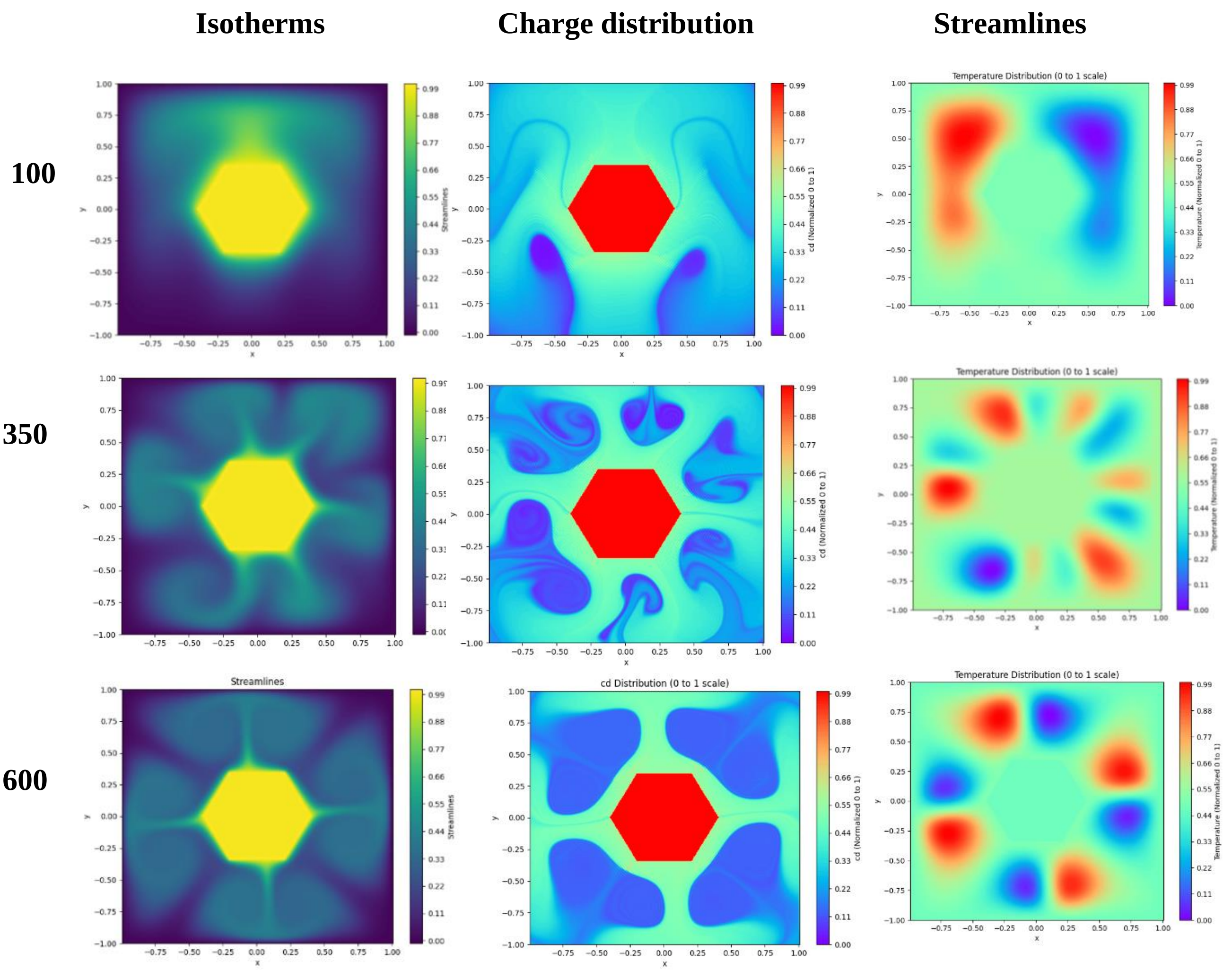

Fig 4.1: Influence of Electric Rayleigh number on Streamlines, isotherms, and Charge distribution at T = 100, T = 350 and T = 600 and Prandtl number (Pr)=10

4.1 Time Evaluation of Nu_M

At T = 100 the heat transfer around the hexagonal body is steady with time. This is reflected in a nearly time-invariant Nusselt number as shown. The steadiness indicates that the temperature gradient and fluid flow balance each other and do not vary appreciably in the system. Because the flow is in a steady state, the heat transfer rate is uniform because the lower temperature does not cause chaotic motion and other vortices; therefore, it maintains its flow pattern without changes. Thus, the heat transfer is also predictable and constant at this temperature level.

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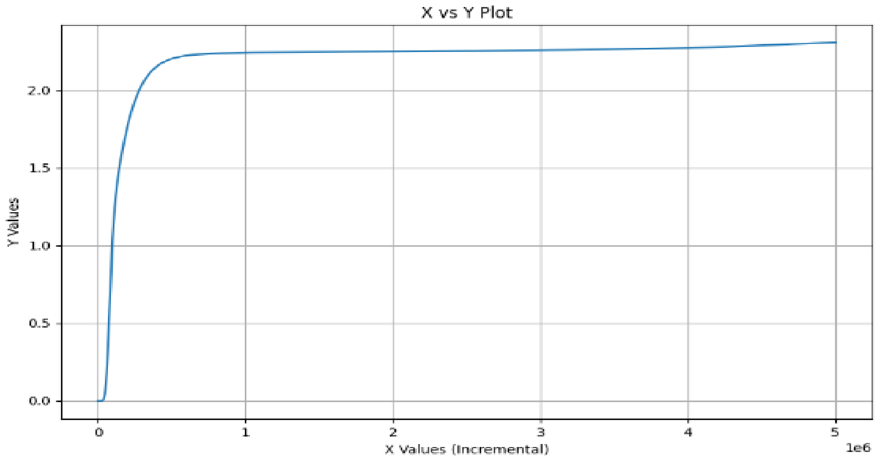

Fig 4.2 Time evolution at $T = 100$

At T=350, the Nusselt number displays a chaotic pattern, with irregular fluctuations indicating unsteady heat transfer around the hexagonal body. This chaotic behavior suggests increased fluid motion and thermal mixing, resulting in variable heat transfer rates.

When the temperature rises to 600(T), these fluctuations intensify, reflecting even greater instability and enhanced heat transfer dynamics due to stronger thermal and electrohydrodynamic effects in the fluid.

4.2 Effect of Electric Rayleigh number (T) on Mean Nusselt Number (Nu_M)

It is observed that the mean Nusselt number, (Nu_M) with an increase in electric Rayleigh number, (T). It increases due to which the heat transfer improves. The same trends are also obtained for all the values of the Prandtl number, (Pr).

The Prandtl number, (Pr), is dimensionless. It can be described as the ratio of momentum diffusivity to thermal diffusivity in a fluid. As long as (Pr) grows, the ratio of the fluid's diffusivity for heat over its momentum diffusivity is smaller, and thus convective heat transfer increases. This causes an increase in the Nusselt number. Hence, increased electric Rayleigh number and Prandtl number help increase (Nu_m) more strongly, and their mutual impact on the conjugate convective heat transfer in the enclosure is highlighted.

Generally, the greater electric Rayleigh number values (T) and Prandtl number values (Pr) imply higher Nusselt numbers and greater efficiency in the heat transfer within the system. The correlation will help employ ETHD whenever these parameters need adjustment to achieve optimal thermal management and heat transfer.

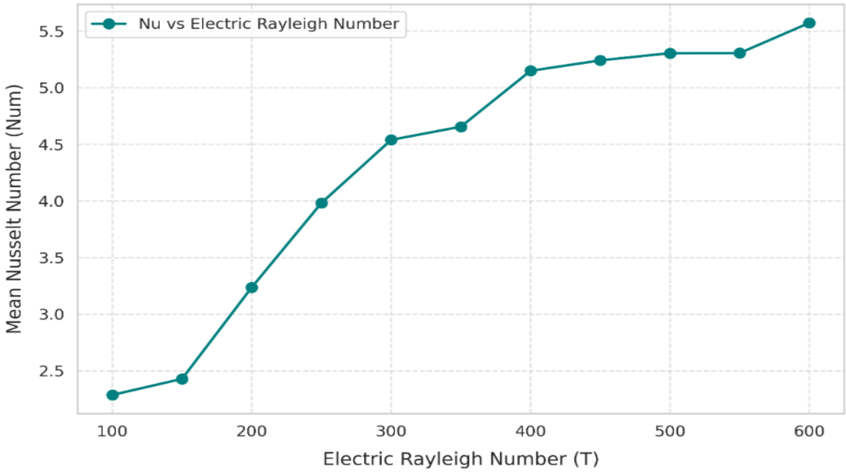

Fig 4.3 Effect of Electric Rayleigh number (T) on Mean Nusselt Number (Nu_M) for Pr = 10

4.3 Mean Nusselt Number Prediction by Machine Learning Model

Ensemble in machine learning is a technique that combines multiple models, or learners, to solve a single problem. By aggregating the predictions of these models, it reduces errors and improves overall accuracy and robustness compared to the performance of any individual model.

Performance Evolution

The model's performance is evaluated by several metrics:

Train MAE: 0.06792169752320835

Train R^2: 0.9933869259768943

Test MAE: 0.3002843891898894

Test R^2: 0.7676483393386498

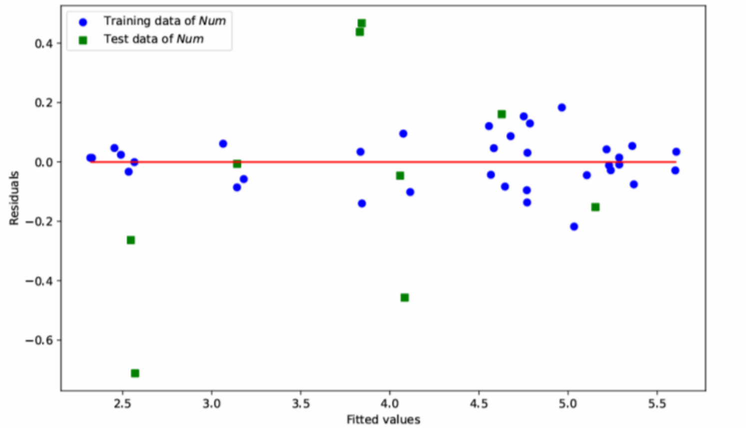

Fig 4.4 Residual Analysis for Ensemble Learning Model Performance

Train Mean Absolute Error (MAE): 0.0679, which measures the average absolute difference between predicted and actual values on the training data. A lower MAE suggests better prediction accuracy.

Train R^2: 0.9934, an indicator of how well the model explains the variance in the training data. An R^2 value close to 1 implies a strong fit for the training set.

Test MAE: 0.3003, representing the error magnitude on unseen data, where a higher error here shows reduced predictive accuracy due to limited training data.

Test R^2: 0.7676, indicating how much of the variance in the test data is explained by the model. While reasonable, this value suggests the model struggles with generalizing beyond the training set, partly due to the small dataset.

The Residual Analysis for Ensemble Learning Model Performance plot Figure 4.6 illustrates how residuals are scattered across fitted values. Ideally, the residuals should be scattered and disperse around zero, which will reflect an ideal model fitting condition. In this plot, we have a spread, which indicates underfitting or noise in the data due to lesser data. Also, the difference of residuals for the training and test data infers that perhaps the model does not generalize well, further reiterating the constraints mentioned above from the model evaluation metrics.

V Conclusion

This study is important for elucidating complex interactions between electro-thermo-convective forces and heat transfer phenomena. Machine learning was used for predicting the temperature distribution, charge distribution, and Nusselt numbers, while LBM was used to simulate fluid dynamics to serve as the work for demonstrating the ETHD's possibility to have an efficient heat management technique. Results are very important, especially for an Electric Rayleigh number equal to 100,350 and 600 revealing significant information about the behavior of fluctuations of electric and thermal fields and temperature patterns around the central hexagonal cylinder.

On the training data, the model exhibited very promising results, with a low Train MAE of 0.0679 and an impressive (R^2) of 0.9934. The test data metrics (Test MAE of 0.3003 and Test (R^2) of 0.7676) indicate that there may be a need for the model to improve its generalization abilities, which may be due to the sparseness in the input data. Fitted values vs. residuals plot together with the residuals spread

around fitted values might indicate underfitting, noise, and, therefore, a deeper need for tuning or additional data.
The coupling of machine learning with ETHD simulation might improve predictive accuracy when we have sufficient data for this model. The study gives a basis for ongoing research into ETHD under more complex geometries and varied conditions, opening up theoretical development and possibly new practical applications.

5.1 Future Scope
The complexity of the model could be increased further through the implementation of a complex neural network or even ensemble methods, which will improve the accuracy in the prediction of temperature and charge distribution. Beyond such a scope, this may include real-time industrial applications where the optimization of heat management is much more enhanced in scenarios such as electronic cooling, nuclear reactors, and microfluidic devices that are based on real-time predictions which allow much better control mechanisms. Much higher electric Rayleigh numbers could therefore reveal intricate and intense convective behaviors, providing useful insights into high-voltage and high-temperature systems. Overall Outcome of the integration of the project with experimental verification into real-world settings: The simulations would introduce more strength to bring them into alignment with practical applications. It will consolidate a foundation for further research within the ambit of ETHD.

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