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AN ANALYSIS OF VIBRATION AND BUCKLING OF COMPOSITE CANTILEVER BEAM USING FINITE ELEMENT METHOD

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Abstract
Cracks in structural members lead to local changes in their stiffness and consequently their static and dynamic behaviour is altered. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been the subject of many investigations. However studies related to behaviour of composite cracked structures subject to in-plane loads are scarce in literature. Present work deals with the vibration and buckling analysis of a cantilever beam made from graphite fiber reinforced polyimide with a transverse one-edge non-propagating open crack using the finite element method. The undamaged parts of the beam are modelled by beam finite elements with three nodes and three degrees of freedom at the node. An „overall additional flexibility matrix" is added to the flexibility matrix of the corresponding non-cracked composite beam element to obtain the total flexibility matrix, and therefore the stiffness matrix in line with previous studies. The vibration of cracked composite beam is computed using the present formulation and is compared with the previous results. The effects of various parameters like crack location, crack depth, volume fraction of fibers and fibers orientations upon the changes of the natural frequencies of the beam are studied. It is found that, presence of crack in a beam decreases the natural frequency which is more pronounced when the crack is near the fixed support and the crack depth is more. The natural frequency of the cracked beam is found to be maximum at about 45% of volume fraction of fibres and the frequency for any depth of crack increases with the increase of angle of fibres. The static buckling load of a cracked composite beam is found to be decreasing with the presence of a crack and the decrease is more severe with increase in crack depth for any location of the crack. Furthermore, the buckling load of the beam decreased with increase in angle of the fibres and is maximum at 0 degree orientation.

Keywords: Composite beam, Flexibility Matrix, Buckling and Vibration of beam under dynamic loading, MATLAB environment etc.

I. Introduction

Composites as structural material are being used in aerospace, military and civilian applications because of their tailor made properties. The ability of these materials to be designed to suit the specific needs for different structures makes them highly desirable. Improvement in design, materials and manufacturing technology enhance the application of composite structures. The suitability of a particular composite material depends on the nature of applications and needs. The technology has been explored extensively for aerospace and civil engineering applications, which require high strength and stiffness to weight ratio materials.

Preventing failure of composite material systems has been an important issue in engineering design. Composites are prone to damages like transverse cracking, fiber breakage, delamination, matrix cracking and fiber-matrix debonding when subjected to service conditions. The two types of physical failures that occur in composite structures and interact in complex manner are interalaminar and interlaminar failures. Interalaminar failure is manifest in micro-mechanical components of the lamina such as fiber breakage, matrix cracking, and debonding of the fiber-matrix interface. Generally, aircraft structures made of fiber reinforces composite materials are designed such that the fibers carry the bulk of the applied load. Interlaminar failure such as delamination refers to debonding of adjacent lamina. The possibility that interalaminar and interlaminar failure occur in structural components is considered a design limit, and establishes restrictions on the usage of full potential of composites.

Similar to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. The crack in a composite structure may reduce the structural stiffness and strength, redistribute the load in a way that the structural failure is delayed, or may lead to structural collapse. Therefore, crack is not necessarily the ultimate structural failure, but rather it is the part of the failure process which may ultimately lead to loss of structural integrity. As one of the failure modes for the fiber-reinforced composites, crack initiation and propagation have long been an important topic in composite and fracture mechanics.

Thus, the importance of inspection in the quality assurance of manufactured products is well understood. Several methods, such as non- destructive tests, can be used to monitor the condition of a structure. It is clear that new reliable and inexpensive methods to monitor structural defects such as cracks should be explored. These variations, in turn, affect the static and dynamic behavior of the whole structure considerably. In some cases this can lead to failure, unless cracks are detected early enough. To ensure the safe, reliable and operational life of structures, it is of high importance to know if their members are free of cracks and, should they be present, to assess their extent. The procedures that are often used for detection are called direct procedures such as ultrasonic, X-rays, etc. However, these methods have proven to be inoperative and unsuitable in some particular cases, since they require expensive and minutely detailed inspections. To avoid these disadvantages, researchers have focused on more efficient procedures in crack detection based on the changes of modal parameters likes natural frequencies, mode shapes and modal damping values that the crack introduces.

Cracks found in structural elements have various causes. They may be fatigue cracks that take place under service conditions as a result of the limited fatigue strength. They may also be due to mechanical defects, as in the case of turbine blades of jet turbine engines. In these engines the cracks are caused by sand and small stones sucked from the surface of the runway. Another group involves cracks which are inside the material: they are created as a result of manufacturing processes.

1.1 Scope of the present Investigation

The main aim of this paper is to work out a composite beam finite element with a non- propagating one-edge open crack. It has been assumed that the crack changes only the stiffness of the element whereas the mass of the element is unchanged. For theoretical modeling of cracked composite beam dimensions, crack locations, crack depth and material properties is specified. In this work an “overall additional flexibility matrix”, instead of the “local additional flexibility matrix” is added to the flexibility matrix of the corresponding non cracked composite beam element to obtain the total flexibility matrix, and therefore the stiffness matrix in the line with the other researchers. By using the present model the following effects due to the crack of the cantilever composite beam have been analyzed.

(i) The influence of the volume fraction of fibers, magnitude, location of the crack, angle of fibers upon the bending natural frequencies of the cantilever cracked composite beam.

(ii) The effects of above parameters on buckling analysis of cracked composite beam.

II. Literature

In the present investigation an attempt has been made to the reviews on composite beam in the context of the present work. This problem has been a subject of many papers, but only a few papers have been devoted to the changes in the static characteristics of cracked composite elements.

Przemieniecki and **Purdy** (1968) presented the general analysis for large deflections of frame structures using concept of discrete element idealizations. They were presented the results for deflections of a six-bay truss and buckling of columns with either constant axial load or gravity loading.

Ozturk & **Sabuncu** (2005) examined the static and dynamic stabilities of a laminated composite cantilever beam having linear translation spring and torsional spring as elastic supports subjected to periodic axial loading. The beam was assumed to be an Euler beam and modelled by using the finite element method. The model was considered to have symmetric and asymmetric lay-ups. The effects of the variation of cross-section in one direction, the ratio of length to thickness, translation spring and torsional spring, and position of the elastic support on stability were examined. In addition, the obtained results of the fundamental natural frequency and critical buckling load parameters were compared with the results of other investigators in existing literature.

Goyal & **Kapania** (2008) performed a stability analysis of laminated beam structures subject to sub-tangential loading, a combination of conservative and non-conservative tangential follower loads, using the dynamic criterion. These loads were characterized using a non-conservativeness loading parameter. This parameter allows them to study the effect of the level of load conservativeness on the stability of laminated beams. The element tangent stiffness and mass matrices were obtained using analytical integration through the dynamic version of the principle of virtual work for laminated composites.

Akbulut, **Gundogdu** & **Sengul** (2010) studied on the theoretical prediction of buckling loads for symmetric angle-ply and cross-ply laminated flat composite columns, consisting of two portions of different widths connected by fillets. They obtained the buckling loads of the column under axial compression for the following end conditions: simply supported, simply-clamped, clamped-clamped, and clamped free.Discussions are limited to the cracked composite beam for area of buckling analysis.

Nikpour (1990) studied the buckling of an edge-notched beam for isotropic and anisotropic composites. The local compliance due to the presence of cracks in an anisotropic medium was formulated as a function of the crack-tip stress intensity factors and the elastic constants of the material. The effect of reducing rigidity on the load-carrying capacity and the post-buckling behavior of the beam was discussed.

Yang & **Chen** (2008) presented a theoretical investigation in free vibration and elastic buckling of beams made of functionally graded materials (FGMs) containing open edge cracks by using Bernoulli–Euler beam theory and the rotational spring model. Analytical solutions of the natural frequencies, critical buckling load, and the corresponding mode shapes were obtained for cracked FGM beams with clamped–free, hinged–hinged, and clamped–clamped end supports. A detailed parametric study was conducted to show the influences of the location and total number of cracks, material properties, slenderness ratio, and end supports on the flexural vibration and buckling characteristics of cracked FGM beams.

Karaagac, Ozturk & **Sabuncu** (2011) investigated the effects of a single-edge crack and its locations on the buckling loads, natural frequencies and dynamic stability of circular curved beams using the finite element method, based on energy approach. This study consists of three stages, namely static stability (buckling) analysis, vibration analysis and dynamic stability analysis. The governing matrix equations were derived from the standard and cracked curved beam elements combined with the local flexibility concept. Results showed that the reductions in buckling load and natural frequency depend not only on the crack depth and crack position, but also on the related mode shape. Analyses also showed that the crack effect on the dynamic stability of the considered

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curved beam was quite limited**.**

III. The Methodology

The governing equations for the vibration analysis of the composite beam with an open one-edge transverse crack are developed. An additional flexibility matrix is added to the flexibility matrix of the corresponding composite beam element to obtain the total flexibility matrix and therefore the stiffness matrix is obtained by Krawczuk & Ostachowicz (1995).

The assumptions made in the analysis are:

i. The analysis is linear. This implies constitutive relations in generalized Hook’s law for the materials are linear.
ii. The Euler–Bernoulli beam model is assumed.
iii. The damping has not been considered in this study.
iv. The crack is assumed to be an open crack and have uniform depth a.

3.1 Buckling analysis studies

Mass and stiffness matrices of each beam element are used to form global mass and stiffness matrices. The dynamic response of a beam for a conservative system can be formulated by means of Lagrange’s equation of motion in which the external forces are expressed in terms of time-dependent potentials and then performing the required operations the entire system leads to the governing matrix equation of motion.

3.2 Derivation of Element Matrices

In the present analysis three nodes composite beam element with three degree of freedom (the axial displacement, transverse displacement and the independent rotation) per node is considered. The characteristic matrices of the composite beam element are computed on the basis of the model proposed by Oral (1991). The stiffness and mass matrices are developed from the procedure given by Krawczuk & Ostachowicz (1995).

IV. Result and Discussion

In order to check the accuracy of the present analysis, the case considered in Krawczuk & Ostachowicz (1995) is adopted here. The beam assumed to be made of unidirectional graphite fiber-reinforced polyamide. The geometrical characteristics and material properties of the beam are chosen as the same of those used in Krawczuk & Ostachowicz (1995).

4.1 Vibration analysis of results of composite beam with multiple cracks

Quantitative results on the effects of various parameters on the vibration of composite $\frac{l_i}{L}$ cracked beams with multiple cracks are presented. The results are compared with previous studies and the present method is validated with published papers. The Finite element analysis is carried out for free vibration of a composite cracked beam for various crack positions and crack depth=0.2 for the example problem considered by Ozturk & Sabuncu (2005). The variation of the first three lowest natural frequencies of the composite beam with multiple cracksis shown as a function of fiber orientation (α) for different crack locations l_3. In these figures, two cases, labeled as X and Y, are considered. In the model, the number of the cracks assumed to be three. The crack location’s for the cases X and Y are chosen as (0.1, 0.2, 0.4), (0.6, 0.8, 0.9), respectively. For numerical calculations the volume fraction of fiber (V) and the crack depth (a/H) are assumed to be 0.10 and 0.2, respectively. The non-dimensional natural frequencies are normalized according to Equation.

When the cracks are placed near the fixed end the decreases in the first natural frequencies are highest, whereas when the cracks are located near the free end, the first natural frequencies are almost unaffected. It is concluded that the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free

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end, respectively.

Similarly the study is extended for buckling analysis of a cantilever composite beam with a crack for the same beam taken by Ozturk & Sabuncu (2005).

4.2 Buckling analysis of results of composite beam with single crack

The buckling analysis is done for the same beam as considered by Ozturk & Sabuncu (2005) and Reddy (1997) for different crack locations and crack depths. The results are tabulated in table 4.2.1 and plotted in the Figure 4.2.1 to 4.2.3. The calculations have been carried out for 10%volume fraction of the fiber and the angle of fibers varying from 0 to 90 degrees. From the table 4.16, it is clear that the non-dimensional buckling load of the beam reduces from 4.9984 to 4.7154 with introduction of a crack at 0.1L and relative crack depth of 0.2. The non-dimensional buckling load decreases substantially from 4.7154 to 3.1330 with increase of relative crack depth from 0.2 to 0.6 due to decrease in stiffness. Similarly the non-dimensional buckling load decreases with increase of relative crack depth from 0.2 to 0.6 for other cases of the crack position i.e. 0.1L, 0.2L, 0.4L, 0.6L and 0.8L. The variation of non-dimensional buckling loads of cantilever composite beam with crack location for different relative crack depth (0.2 to 0.6), when angle of fiber = 0 degree is shown in Figure 4.25. It is observed that the non-dimensional buckling load increases from 4.7154 to 4.9640 with increase of x/L from 0.1 to 0.8 for relative crack depth = 0.2. For relative crack depth 0.6 the non-dimensional buckling load increases from 3.1330 to 4.3844 when the location of crack shifts from 0.1L to 0.8L. It means the non-dimensional buckling load of a cracked cantilever composite beam is higher if the crack is near the free end than near the fixed end and non-dimensional buckling load decreases with increase in relative crack depth. For a given crack depth it increases as crack location moves from fixed end to free end. Buckling load decreases with increase in angle of fibers and is maximum at 0 degree. This is due to the fact that for 0 degree orientation the buckling plane normal to the fibers is of maximum stiffness and for other orientations stiffness is less hence buckling load is less.

Table-4.2.1 Buckling loads for different crack locations for a composite cracked beam for angle of fiber = 0, 30, 60 and 90 degrees

Relative Cracked depth (a/H)	Angle of fibers (degree)	Crack position				
		x = 0.1L	x = 0.2L	x = 0.4L	x = 0.6L	x = 0.8L
0.2	0	4.7154	4.7649	4.8753	4.9241	4.9640
0.4		4.0126	4.1812	4.4494	4.6547	4.8769
0.6		3.1330	3.2406	3.5529	3.9873	4.3844
0.2	30	1.1461	1.2740	1.4347	1.5828	1.6142
0.4		0.4037	0.5562	0.7016	1.0428	1.3763
0.6		0.2961	0.4376	0.6292	0.8714	1.1906
0.2	60	0.3507	0.3591	0.3733	0.3816	0.3871
0.4		0.2159	0.2479	0.2743	0.3193	0.3539
0.6		0.1687	0.2100	0.2343	0.2741	0.3243
0.2	90	0.1892	0.1909	0.1957	0.1976	0.1994
0.4		0.1369	0.1452	0.1565	0.1718	0.1849
0.6		0.1101	0.1178	0.1321	0.1484	0.1709

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V. Conclusion

The following conclusions can be made from the present investigations of the composite beam finite element having transverse non-propagating one-edge open crack. This element is versatile and can be used for static and dynamic analysis of a composite or isotropic beam.

1) From the present investigations it can be concluded that the natural frequencies of vibration of a cracked composite beam is not only the functions of the crack locations and crack depths but also the functions of the angle of fibers and the volume fraction of the fibers. The presence of a transverse crack reduces the natural frequencies of the composite beam.

2) The rate of decrease in the natural frequency of the cracked composite beam increases as the crack position approaches the fixed end.

3) The intensity of the reduction in the frequency increases with the increase in the crack depth ratio. This reduction in natural frequency along with the mode shapes of vibrations can be used to detect the crack location and its depth.

4) When, the angle of fibers (α) increase the values of the natural frequencies also increase.

5) The most difference in frequency occurs when the angle of fiber (α) is 0 degree. This is due to the fact that the flexibility of the composite beam due to crack is a function of the angle between the crack and the reinforcing fibers.

6) The effect of cracks is more pronounced near the fixed end than at far free end. It is concluded that the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively.

7) The decrease of the non-dimensional natural frequencies depends on the volume fraction of the fibers. The non-dimensional natural frequency is maximum when the volume fraction of fiber is approximately 45%. This is due to the fact that the flexibility of a composite beam due to crack is a function of the volume fraction of the fibers.

8) Buckling load of a cracked composite beam decrease with increase of crack depth for crack at any particular location due to reduction of stiffness.

9) When, angle of fibers increase the values of the buckling loads decrease. This is due to the fact that for 0 degree orientation of fibers, the buckling plane normal to the fibers is of maximum stiffness and for other orientations stiffness is less hence buckling load is less.

References

[1] Adams R. D., Cawley, P. C., Pye J. and Stone J. (1978). A vibration testing for non-destructively assessing the integrity of the structures. Journal of Mechanical Engineering Sciences 20, 93-100.
[2] Banerjee J. R. (2001). Frequency equation and mode shape formulae for composite.
[3] . Timoshenko beams. Composite Structures 51 (2001) 381-388.
[4] Binici B. (2005) Vibration of beams with multiple open cracks subjected to axial force. J Sound Vib; 287(1-2):277–95.
[5] Broek D. Elementary Engineering Fracture Mechanics. Martinus Nijhoff; 1986.
[6] Chandrupatla T. R., Belegundu A. D. Introduction to finite elements in engineering. New Jersey: Prentice-Hall; 1991.
[7] Dimarogonas A.D. (1996), Vibration of Cracked Structures: A State of the Art Review. Engineering Fracture Mechanics, 55(5), 831-857.
[8] Ghoneam S. M. (1995). Dynamic analysis of open cracked laminated composite beams. Composite Structures 32 (1995) 3-11.
[9] Goyal Vijay K., Kapania Rakesh K. (2008). Dynamic stability of laminated beams subjected to non-conservative loading. Thin-Walled Structures 46 (2008) 1359– 1369.
[10] Hamada A. Abd El-Hamid (1998). An investigation into the eigen-nature of Composite Cantilever Beam s. Composite Structure Vol. 38, No. l - 4, pp. 45-55.

[11] Gounaris G.D., Papadopoulos CA, Dimarogonas AD. (1996). Crack identification in beams by coupled response measurement. Comput Struct; 58(2):299–305.

[12] Manivasagam S. & Chandrasekharan K. (1992). Characterization of damage progression in layered composites. *Journal of Sound and Vibration* 152, 177-179.

[13] Nikpour K. and Dimarogonas A.D. (1988). Local compliance of composite cracked bodies. *Journal of Composite Science and Technology* 32,209-223.

[14] Nikpour K. (1990). Buckling of cracked composite columns. *Journal of Solids Structures* 26, 371-1386.

[15] Oral S. (1991). A shear flexible finite element for non-uniform laminated composite beams. *Computers and Structures* 38, 353-360.

[16] Ozturk Hasan, Sabuncu Mustafa (2005). Stability analysis of a cantilever composite beam on elastic supports. *Composites Science and Technology* 65 (2005) 1982–1995.

[17] Przemieniecki J. S. *Theory of Matrix Structural Analysis*. London: McGraw Hill first edition (1967).

[18] Przemieniecki J. S. and Purdy D. M. (1968). Large deflection and stability analysis of two dimensional truss and frame structure. AFFDL-TR-68-38.

[19] Reddy J N. Mechanics of laminated composite plates theory and analysis. New York: CRS Press; 1997.

[20] Sih G. C. and Chen E. P. (1981) in Mechanics of Fracture. *Cracks in composite materials.*London: Martinus Nijho first edition.

[21] Timoshenko S. (1940), *Theory of Plates and Shells*, McGraw Hill Book Company Inc., New York & London.

[22] Vinson J. R. and Sierakowski R. L. *Behaviour of Structures Composed of Composite.Materials.* Dordrecht: Martinus Nijho first edition (1991).

[23] Wauer J. (1991). Dynamics of cracked rotors: a literature survey. Applied Mechanics.

[24] Wang Kaihong and Inman Daniel J. (2002). Coupling of Bending and Torsion of a Cracked Composite Beam. Center for Intelligent Material Systems and Structures Virginia Polytechnic Institute and State University Blacksburg, VA 24061-0261.

[25] Yang J., Chen Y. (2008). Free vibration and buckling analyses of functionally graded beams with edge cracks. *Composite Structures* 83 (2008) 48–60.

[26] Zak A., Krawczuk M. and Ostachowicz W. M. (2000). Numerical and experimental investigation of free vibration of multilayered delaminated composite beams and plates. *Composite Mechanics* 26 (2000) 309-315.

[27] Zhou L., Huang Y. (2006). Crack effect on the elastic buckling behavior of axially and eccentrically loaded columns. Structure Eng. Mech. 2006; 22(2):169–84.