Industrial Engineering Journal
ISSN: 0970-2555
Volume : 52, Issue 5, No. 3, May : 2023

A REVIEW STUDY ON VIBRATION AND BUCKLING OF COMPOSITE CANTILEVER BEAM

Mr. Rajbahadur, Research Scholar, Department of Civil Engineering, Maharishi University of Information Technology Lucknow U.P.
Dr. Gaurav Shukla, Assistant Professor, Department of Civil Engineering, Maharishi University of Information Technology Lucknow U.P.

Abstract
Cracks in structural members lead to local changes in their stiffness and consequently their static and dynamic behaviour is altered. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been the subject of many investigations. However studies related to behaviour of composite cracked structures subject to in-plane loads are scarce in literature. Present work deals with the vibration and buckling analysis of a cantilever beam made from graphite fiber reinforced polyimide with a transverse one-edge non-propagating open crack using the finite element method. The undamaged parts of the beam are modelled by beam finite elements with three nodes and three degrees of freedom at the node. An „overall additional flexibility matrix" is added to the flexibility matrix of the corresponding non-cracked composite beam element to obtain the total flexibility matrix, and therefore the stiffness matrix in line with previous studies. The vibration of cracked composite beam is computed using the present formulation and is compared with the previous results. The effects of various parameters like crack location, crack depth, volume fraction of fibers and fibers orientations upon the changes of the natural frequencies of the beam are studied. It is found that, presence of crack in a beam decreases the natural frequency which is more pronounced when the crack is near the fixed support and the crack depth is more. The natural frequency of the cracked beam is found to be maximum at about 45% of volume fraction of fibres and the frequency for any depth of crack increases with the increase of angle of fibres. The static buckling load of a cracked composite beam is found to be decreasing with the presence of a crack and the decrease is more severe with increase in crack depth for any location of the crack. Furthermore, the buckling load of the beam decreased with increase in angle of the fibres and is maximum at 0 degree orientation.
Composites as structural material are being used in aerospace, military and civilian applications because of their tailor made properties. The ability of these materials to be designed to suit the specific needs for different structures makes them highly desirable. Improvement in design, materials and manufacturing technology enhance the application of composite structures. The suitability of a particular composite material depends on the nature of applications and needs. The technology has been explored extensively for aerospace and civil engineering applications, which require high strength and stiffness to weight ratio materials.

Keywords: Composite beam, Flexibility Matrix, Buckling and Vibration of beam under dynamic loading, MATLAB environment etc.

I. Introduction

Preventing failure of composite material systems has been an important issue in engineering design. Composites are prone to damages like transverse cracking, fiber breakage, delamination, matrix cracking and fiber-matrix debonding when subjected to service conditions. The two types of physical failures that occur in composite structures and interact in complex manner are interalaminar and interlaminar failures. Interalaminar failure is manifest in micro-mechanical components of the lamina such as fiber breakage, matrix cracking, and debonding of the fiber matrix interface. Generally, aircraft structures made of fiber reinforces composite materials are designed such that the fibers carry the bulk of the applied load. Interlaminar failure such as delamination refers to debonding of adjacent lamina. The possibility that interalaminar and interlaminar failure occur in structural components is considered a design limit, and establishes restrictions on the usage of full potential of composites.

Similar to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. The crack in a composite structure may reduce the structural stiffness and strength, redistribute the load in a way that the structural failure is delayed, or may lead to structural collapse. Therefore, crack is not necessarily the ultimate structural failure, but rather it is the part of the failure process which may ultimately lead to loss of structural integrity.

Thus, the importance of inspection in the quality assurance of manufactured products is well understood. Several methods, such as non- destructive tests, can be used to monitor the condition of a structure. It is clear that new reliable and inexpensive methods to monitor structural defects such as cracks should be explored. These variations, in turn, affect the static and dynamic behaviour of the whole structure considerably. In some cases this can lead to failure, unless cracks are detected early enough. To ensure the safe, reliable and operational life of structures, it is of high importance to know if their members are free of cracks and, should they be present, to assess their extent. The procedures that are often used for detection are called direct procedures such as ultrasonic, X-rays, etc. However, these methods have proven to be inoperative and unsuitable in some particular cases, since they require expensive and minutely detailed inspections. To avoid these disadvantages, researchers have focused on more efficient procedures in crack detection based on the changes of modal parameters likes natural frequencies, mode shapes and modal damping values that the crack introduces.

II. Literature

Cracks occurring in structural elements are responsible for local stiffness variations, which in consequence affect their dynamic characteristics. This problem has been a subject of many papers, but only a few papers have been devoted to the changes in the dynamic characteristics of composite constructional elements. In the present investigation an attempt has been made to the reviews on composite cracked beam in the context of the present work and discussions are limited to the following area of analysis.

Lu & Law (2009) studied such effect from multiple cracks in a finite element in the dynamic analysis and local damage identification. The finite beam element was formulated using the composite element method with a one-member–one-element configuration with cracks where the interaction effect between cracks in the same element was automatically included. The accuracy and convergence speed of the proposed model in computation were compared with existing models and experimental results. The parameter of the crack model was found needing adjustment with the use of the proposed model.

Wang, Inmana & Farrar (2004) investigated the coupled bending and torsional vibration of a fiber-reinforced composite cantilever with an edge surface crack. The model was based on linear fracture mechanics, the Castiglione’s theorem and classical lamination theory. The crack was modeled with a local flexibility matrix such that the cantilever beam was replaced with two intact beams with the crack as the additional boundary condition. The coupling of bending and torsion can result from either the material properties or the surface crack.

Dimarogonas (1996) reported a comprehensive review of the vibration of cracked structures. This author covered a wide variety of areas that included cracked beams, coupled systems, flexible rotors, shafts, turbine rotors and blades, pipes and shells, empirical diagnoses of machinery cracks, and bars and plates with a significant collection of references.

Ghoneam (1995) presented the dynamic characteristics laminated composite beams (LCB) with various fiber orientations and different boundary fixations and discussed in the absence and presence of cracks. A mathematical model was developed, and experimental analysis was utilized to study the effects of different crack depths and locations, boundary conditions, and various code numbers of laminates on the dynamic characteristics of CLCB. The analysis showed good agreement between experimental and theoretical results.

Krawczuk & Ostachowicz (1995) investigated eigen frequencies of a cantilever beam made from graphite-fiber reinforced polyimide, with a transverse on-edge non-propagating open crack. Two models of the beam were presented. In the first model the crack was modeled by a massless substitute spring Castiglione's theorem. The second model was based on the finite element method. The undamaged parts of the beam were modeled by beam finite elements with three nodes and three degrees of freedom at the node. The damaged part of the beam was replaced by the cracked beam finite element with degrees of freedom identical to those of the non-cracked done. The effects of various parameters the crack location, the crack depth, the volume fraction of fibers and the fibers orientation upon the changes of the natural frequencies of the beam were studied. Computation results indicated that the decrease of the natural frequencies not only depends on the position of the crack and its depth as in the case of isotropic material but also that these changes strongly depend on the volume fraction of the fiibers and the angle of the fibers of the composite material.

Krawczuk (1994) formulated a new beam finite element with a single non-propagating one edge open crack located in its mid-length for the static and dynamic analysis of Composite Cantilever Beam -like structures. The element includes two degrees of freedom at each of the three nodes: a transverse deflection and an independent rotation respectively. He presented the exemplary numerical calculations illustrating variations in the static deformations and a fundamental bending natural frequency of a composite cantilever beam caused by a single crack.

III. The Methodology

The governing equations for the vibration analysis of the composite beam with an open one-edge transverse crack are developed. An additional flexibility matrix is added to the flexibility matrix of the corresponding composite beam element to obtain the total flexibility matrix and therefore the stiffness matrix is obtained by Krawczuk & Ostachowicz (1995).

The assumptions made in the analysis are:

i. The analysis is linear. This implies constitutive relations in generalized Hook's law for the materials are linear.
ii. The Euler–Bernoulli beam model is assumed.
iii. The damping has not been considered in this study.
iv. The crack is assumed to be an open crack and have uniform depth a.

3.1 Buckling Analysis Studies

Mass and stiffness matrices of each beam element are used to form global mass and stiffness matrices. The dynamic response of a beam for a conservative system can be formulated by means of Lagrange's equation of motion in which the external forces are expressed in terms of time-dependent potentials.

Then performing the required operations the entire system leads to the governing matrix equation of motion

$$M \quad q \; + \quad K_e \; - P(t) \; K_g \quad q \; = 0$$

where „q" is the vector of degree of freedoms. M, K_e and K_g are the mass, elastic stiffness and geometric stiffness matrices of the beam. The periodic axial force where Ω is the disturbing frequency, the static and time dependent component of the load can be $P \; t \; = P_o + P_t Cos\Omega t$ represented as a fraction of the fundamental static buckling load P_{cr} hence putting $P \; t \; = \alpha P_{cr} + \beta P_{cr} Cos\Omega t$

In this analysis, the computed static bucking load of the composite beam is considered the reference load. Further the above equation reduces to other problems as follows.

i. Free vibration with α = 0, β = 0 and ω = Ω/2 the natural frequency

$$K_e \;\; - \omega^2 \; M \quad q \;\; = 0$$

ii. Static stability with α = 1, β = 0, Ω = 0

$$K_e \;\; - P_{cr} \; K_g \quad q \;\; = 0$$

3.2 Derivation of Element Matrices

In the present analysis three nodes composite beam element with three degree of freedom (the axial displacement, transverse displacement and the independent rotation) per node is considered. The characteristic matrices of the composite beam element are computed on the basis of the model proposed by Oral (1991). The stiffness and mass matrices are developed from the procedure given by Krawczuk & Ostachowicz (1995).

3.3 Computational procedure for a Composite Cantilever Beam

A computer program is developed to perform all the necessary computations in MATLAB environment. In the initialization phase, geometry and material parameters are specified. For example for a Euler–Bernoulli composite beam model with localized crack, material parameters like modulus of elasticity, the modulus of rigidity, the Poisson ratio and the mass density of the composite beam material and geometric parameters like dimensions of the composite beam, also the specifications of the damage like size of the crack, location of the crack and extent of crack are supplied as input data to the computer program. The beam is divided into n number of elements and n+1 number of nodes. The elements of the mass matrix, elastic stiffness matrix and geometric stiffness matrix are formulated according to above expression and are obtained the non-dimensional natural frequencies and buckling load for non-cracked and Composite Cantilever Beam element. The program uses the MATLAB function, "Gauss Quadrature" to carry out the integration part. Element matrices are assembled to obtain the global matrices. Boundary conditions are imposed by elimination method. For Euler–Bernoulli composite beam with fixed- free end conditions the first three rows and columns of the global matrices are eliminated to obtain the reduced matrices. The non-dimensional natural frequencies are calculated by solving the Eigen value problems in eq. The built in MATLAB function "eig" is used to calculate the eigen values, eigenvectors and mode shape diagram.

IV. Conclusion

The following conclusions can be made from the present investigations of the composite beam finite element having transverse non-propagating one-edge open crack. This element is versatile and can be used for static and dynamic analysis of a composite or isotropic beam.

1. From the present investigations it can be concluded that the natural frequencies of vibration of a Composite Cantilever Beam is not only the functions of the crack locations and crack depths but also the functions of the angle of fibers and the volume fraction of the fibers. The presence of a transverse crack reduces the natural frequencies of the composite beam.
2. The rate of decrease in the natural frequency of the Composite Cantilever Beam increases as the crack position approaches the fixed end.
3. The intensity of the reduction in the frequency increases with the increase in the crack depth ratio. This reduction in natural frequency along with the mode shapes of vibrations can be used to detect the crack location and its depth.

4. When, the angle of fibers (α) increase the values of the natural frequencies also increase.
5. The most difference in frequency occurs when the angle of fiber (α) is 0 degree. This is due to the fact that the flexibility of the composite beam due to crack is a function of the angle between the crack and the reinforcing fibers.
6. The effect of cracks is more pronounced near the fixed end than at far free end. It is concluded that the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively.
7. The decrease of the non-dimensional natural frequencies depends on the volume fraction of the fibers. The non-dimensional natural frequency is maximum when the volume fraction of fiber is approximately 45%. This is due to the fact that the flexibility of a composite beam due to crack is a function of the volume fraction of the fibers.
8. Buckling load of a Composite Cantilever Beam decrease with increase of crack depth for crack at any particular location due to reduction of stiffness.
9. When, angle of fibers increase the values of the buckling loads decrease. This is due to the fact that for 0 degree orientation of fibers, the buckling plane normal to the fibers is of maximum stiffness and for other orientations stiffness is less hence buckling load is less.

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