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A STUDY ON COLORING IN VARIOUS GRAPHS USING CHROMATIC NUMBER

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ABSTRACT

This paper goal is to assess the chromatic number of various graph types. The advanced tool in Graph Theory known as Coloring of Graphs is becoming more and more popular in a variety of domains. The process of giving the graph color is known as coloring. The graphs vertices, edges, or region area may be given a color. It can be divided into three categories. either region, edge, or vertex coloring. the process of giving each vertex in a graph a label or color so that no edge joins two vertices with the same color. The number of colors required to color the graphs vertices is known as the chromatic number. It examines various graphs chromatic numbers. This concludes the new study about the special types of various graphs are Sun-let Graph, Tadpole Graph, Wheel Graph, Planar Graph, Tree Graph , Friendship Graph . Then mainly focusing on allocating problems by using vertex coloring.

KEYWORDS : Vertex Coloring, Edges, Chromatic Number.

INTRODUCTION

Graph coloring is a problem in graph theory that involves assigning colors to the vertices of a graph in such a way that no two adjacent vertices have the same color. It is a fundamental concept in graph theory that has important applications in various areas, including computer science, operations research, social network analysis, and even art. In graph theory, a graph is a collection of vertices and edges connecting them. Each vertex represents a point or object, and each edge represents a connection between two vertices. Graph coloring is a problem of assigning colors to vertices in such a way that no two adjacent vertices have the same color. This problem can be represented mathematically as finding the minimum number of colors required to color a graph. The concept of graph coloring has important applications in various areas of computer science, such as scheduling, register allocation, and map coloring. In scheduling problems, the graph coloring problem is used to represent the minimum number of time slots needed to schedule a set of tasks without any conflicts. In register allocation, graph coloring is used to allocate registers to variables in a computer program, ensuring that no two variables that are simultaneously in use are allocated to the same register. In map coloring, graph coloring is used to represent the minimum number of colors needed to color a map such that no two adjacent regions have the same color.

GRAPH THEORY

In graph theory, a graph is a collection of vertices and edges connecting them. The vertices represent the objects or entities being studied, and the edges represent the relationships between these objects. Graphs can be directed, meaning that the edges have a specific direction, or undirected, meaning that the edges have no direction. They can also be weighted, meaning that each edge has a numerical weight or cost associated with it.

GRAPH COLORING

Assigning colors to a graphs vertices so that no two adjacent vertices have the same color is known as the graph coloring issue in graph theory. The chromatic number of a graph refers to the minimum number of colors needed to color it. A graph is a collection of vertices and the edges that connect them in graph theory. Each edge represents a relationship between two vertices, and each vertex represents a point or object. The mathematical solution to this problem is to determine the minimum number of colors necessary to color a graph. Vertex coloring, edge coloring, and region coloring are the three categories in which graph coloring is classified.

VERTEX COLORING

In the field of graph theory, the vertex coloring problem involves selecting colors for every vertex of a graph such that **no two adjacent vertex colors are the same.** This problem approaches graph coloring, but it only concerns the graph's vertices rather than its edges. Vertex coloring is frequently used in scheduling challenges, where the vertices represent the tasks that must be completed and the colors represent the possibility of time slots.

EDGE COLORING

In edge coloring, each edge of a graph is assigned a color from a set of available colors, such that no two adjacent edges have the same color. This problem is similar to vertex coloring, but it only involves the edges of the graph, and not the vertices. Edge coloring is commonly used in problems such as scheduling, where the edges represent communication links or transportation routes that need to be scheduled.

REGION COLORING

Region coloring, also known as map coloring, is a problem in graph theory that involves assigning colors to the regions of a map in such a way that no two adjacent regions have the same color. This problem is similar to graph coloring, but it is specifically used to model the problem of coloring a map. The regions of the map represent countries, states, or other areas, and the edges represent the borders between them.

CHROMATIC NUMBER

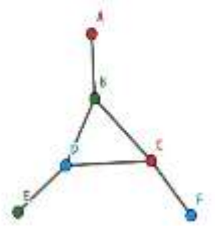
In graph theory, the chromatic number of a graph is the minimum number of colors needed to color its vertices in such a way that no two adjacent vertices share the same color. Equivalently, it is the smallest number of colors needed to color the graph so that no two adjacent edges have the same color.

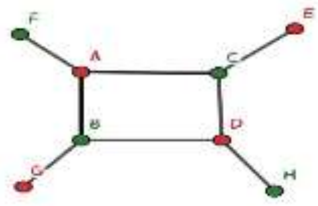
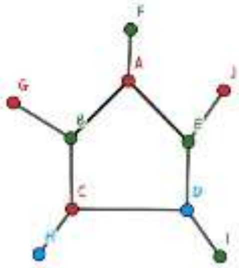
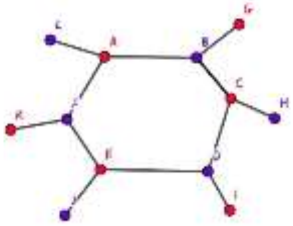
SPECIAL TYPES OF VARIOUS GRAPH USING CHROMATIC NUMBER

SUNLET GRAPH

The graph on vertices generated by attaching pendant edges to a cycle graph is known as a sunlet graph.

TABLE 1: Chromatic number of Sunlet graph

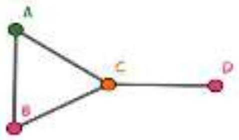
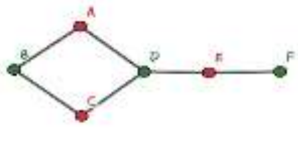
ITERATION	GRAPHS	NO.OF VERTICES	CHROMATIC NUMBER
1		6	3

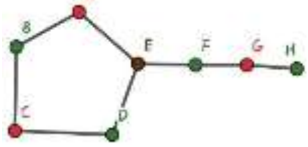
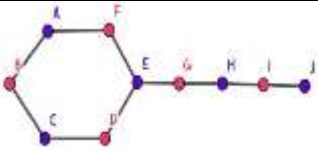
2		8	2
3		10	3
4		12	2
5		14	3

TADPOLE GRAPH

The (m,n) tadpole graph is a unique type of graph in the field of graph theory that consists of a bridge connecting a cycle graph with m (at least 2) vertices and a path graph with n vertices.

TABLE 2: Chromatic number of Tadpole graph

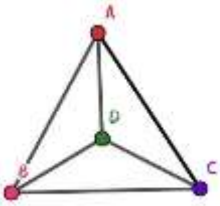
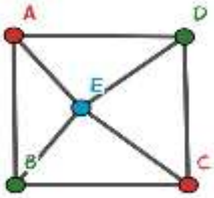
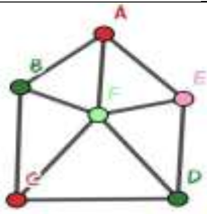
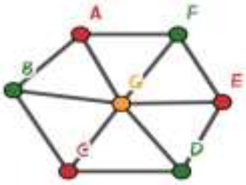
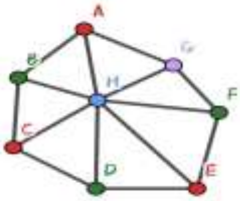
ITERATION	GRAPHS	NO.OF VERTICES	CHROMATIC NUMBER
1		4	3
2		6	2

3		8	2
4		10	2

WHEEL GRAPH

A wheel graph is a type of graph that is formed by connecting a central vertex to all the vertices of a cycle of length n, where n is the number of vertices in the cycle. The resulting graph has n+1 vertices, with one vertex (the hub) having degree n, and the remaining n vertices (the rim) having degree 2.

TABLE 3: Chromatic number of Wheel graph

ITERATION	GRAPHS	NO.OF VERTICES	CHROMATIC NUMBER
1		4	4
2		5	3
3		6	4
4		7	3
5		8	4

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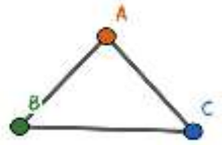
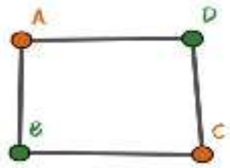
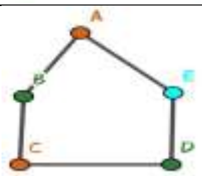
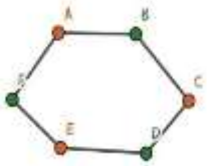
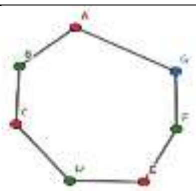
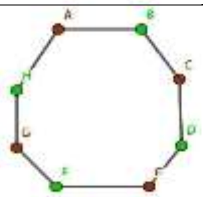
PLANAR GRAPH

In graph theory, a planar graph is a graph that can be drawn in the plane without any edges crossing. Formally, a planar graph is a graph that can be embedded in the plane, meaning that its vertices can be placed on distinct points in the plane, and its edges can be drawn as non-crossing curves between their endpoints.

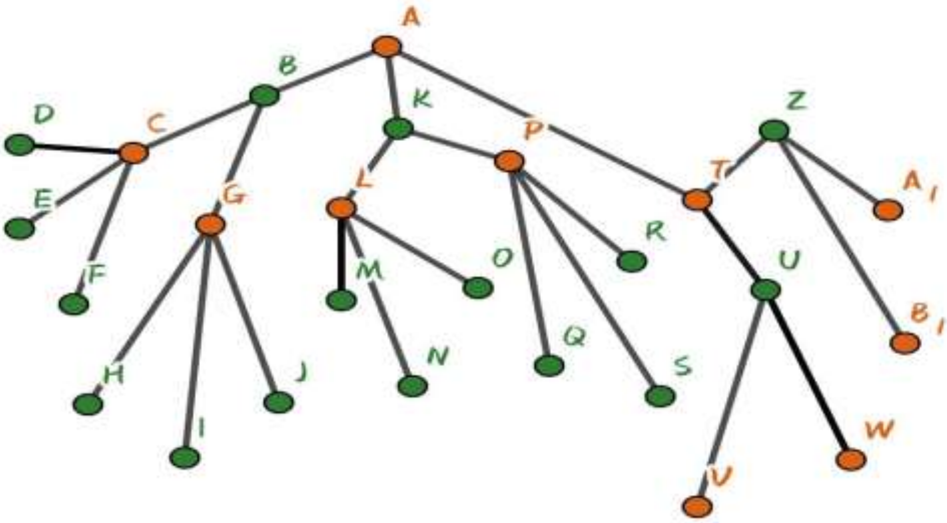
TABLE 4: Chromatic number of Planar graph

ITERATION	GRAPHS	NO.OF VERTICES	CHROMATIC NUMBER
1		3	3
2		4	2
3		5	3
4		6	2
5		7	3
6		8	2

TREE GRAPH

A tree graph, also known as a tree or a rooted tree, is a type of graph that is made up of nodes connected by edges in a branching, hierarchical structure. In a tree graph, there is a single node, called the root, which has no parent, and all other nodes in the graph have exactly one parent. This results in a tree-like structure that is often used to represent hierarchical relationships, such as family trees or organizational charts. Each node in the tree graph can have zero or more child nodes, and the edges connecting nodes are directed and acyclic, meaning that there are no loops or cycles in the graph.

TABLE 5: Chromatic number of Tree graph

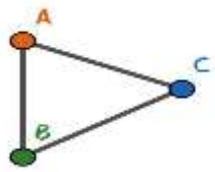
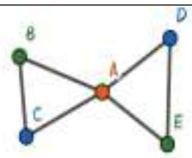
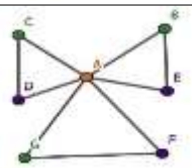


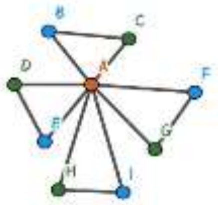
CN(G) = 2 (Coloring)

FRIENDSHIP GRAPH

A friendship graph, also known as a social network graph, is a type of graph that represents social relationships between individuals. In a friendship graph, each individual is represented by a node, and the connections between individuals are represented by edges. The edges in a friendship graph typically indicate a friendship or social relationship between two individuals.

TABLE 6: Chromatic number of Friendship graph

ITERATION	GRAPHS	NO.OF VERTICES	CHROMATIC NUMBER
1		3	3
2		5	3
3		7	3

4		9	3

ALLOCATING PROBLEMS

Allocation problems involve distributing allocated resources among different possibilities in an effort to decrease total costs or increase overall return. Vertex coloring is used in this particular case to allocate. Vertex coloring is used to solve the following problems so that resources are allocated properly.

EXAMPLE-1

ALLOCATING RACK FOR THE THINGS TO BE ARRANGED BY USING VERTEX COLORING:

A store has 10 different types of things which includes Carrot, Plate, Chocolate, Fish, Orange, Apple, Potato, Chicken, Tumbler and Bread which shall henceforth designed by A, B, C, D, E, F, G, H, I and J respectively. Here we need to find that which things can be put together and how many racks needed for them to be arranged. The following table shows that the things that cannot be put together.

TABLE 1:

TYPE	A	B	C	D	E
CANNOT PUT TOGETHER	BCDE FHIJ	ACDE FGHJ	ABDE FGHI	ABCE FGIJ	ABCD GHIJ

TABLE 2:

TYPE	F	G	H	I	J
CANNOT PUT TOGETHER	ABCD GHIJ	BCDE FHIJ	ABCE FGIJ	ACDE FGHJ	ABDE FGHI

How many Racks needed for the things to be arranged? To answer the question draw ten points one representing each type of thing and then draw a graph where each edge joints the vertices representing any thing that are incompatible.Next determine the chromatic number of the graph.

SOLUTION :

A graph G is constructed with vertex set V(G) = A,B,C,D,E,F,G,H,I,J.

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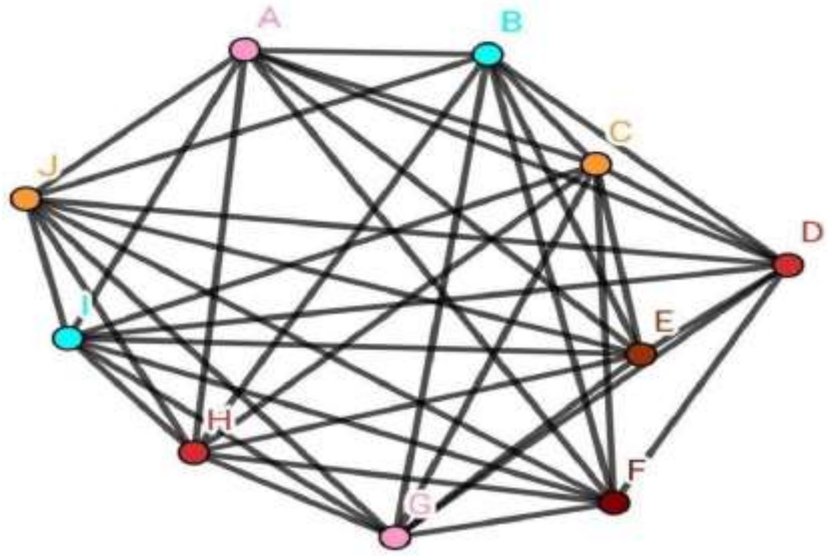

From the above graph, by using vertex coloring the chromatic number of the graph is 5. Hence five racks are required for the things in which it can be arranged by the following:

- Rack 1 : AG.
- Rack 2 : BI.
- Rack 3 : CJ.
- Rack 4 : DH.
- Rack 5 : EF

This problem shows the 5-color problem in graph theory by using vertex coloring.

EXAMPLE-2

ALLOCATING TABLES FOR CUSTOMERS USING VERTEX COLORING:

A Restaurant has has five different types of food varieties which includes juice, vegetarian, non-vegetarian, fast food and dessert which shall henceforth designed by A, B, C, D and E respectively. Due to lot of customers they are in need to combine the tables so that no two customer have no problem in combining or sharing the table with another customer. The following table shows that no two customer cannot share the same table.

TABLE 1:

TYPE	A	B	C	D	E
CANNOT PUT TOGETHER	BCD	ACDE	ABE	ABE	BCD

How many tables needed for the customers? To answer the question draw five points one representing each type of food and then draw a graph where each edge joints the vertices representing any food that are incompatible. Next determine the chromatic number of the graph.

SOLUTION:

A Graph G is constructed with vertex set V(G) = A,B,C,D,E

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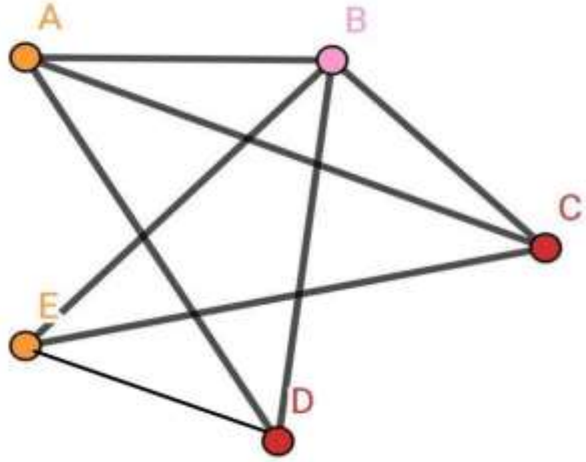


From the above graph , by using vertex coloring the chromatic number of the graph is 3. Hence three tables are required for the customers to be combined by the following:

- Table 1 : AE.
- Table 2 : B.
- Table 3 : CD.

This problem shows the 3-color problem in graph theory by using vertex coloring.

EXAMPLE-3

ALLOCATING REQUIRED DAYS FOR SOME PHYSICAL THERAPIST TO WORK ON WEEKEND BY USING VERTEX COLORING:

There are eight children (described as c1, c2,..., c8) in a remote region who require weekly physical therapy sessions. They are A, B, C, D, E, F, G, and H. Eight physical therapists from an adjacent town are going to visit some of those children times a week, but no child may receive more than one visit on any given day. A tour is the group of children that a physical therapist sees on any given day. It is decided that six children is the ideal amount to do a tour together. Below the table, the following eight tours are decided upon:

TABLE 1:

TYPE	A	B	C	D
CANNOT PUT TOGETHER	BCD EFH	ACD EFG	ABD FGH	ABC EGH

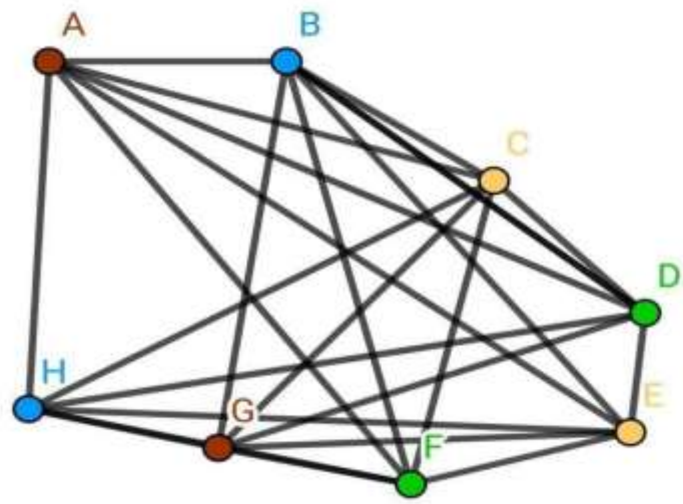
TABLE 2:

TYPE	E	F	G	H
CANNOT PUT TOGETHER	ABD FGH	ABC EGH	BCD EFH	ACD EFG

It would be preferred if all ten tours can take place during Monday through Friday but the physical therapists are willing to work on the weekend if necessary. Is it necessary for someone to work on the weekend ?

SOLUTION:

A Graph G is constructed V(G) = A,B,C,D,E,F,G,H



From the above graph, by using vertex coloring the chromatic number of the graph is 4.Hence four days are required and it is necessary for some physical therapist to work on the weekend.

- Day 1 : AG.
- Day 2 : BH.
- Day 3 : CE.
- Day 4 : DF.

This problem shows the 4-color problem in graph theory by using vertex coloring.

CONCLUSION

Real life involves a lot of coloring. It is appropriate everywhere. Many applications that use vertex coloring determine the minimum number of colors required. A fundamental concept in computer science and mathematics, graph coloring has an extensive variety of real-world applications. Graph coloring approaches have been used to solve a broad variety of real-world problems, and the Four Color Theorem is an essential contribution to the discipline of graph theory. It is expected that graph coloring will continue to be an important field of research and develop as the field of computer science advances. In this paper coloring is being used to find the chromatic number of some graphs like Sunlet graph, Tadpole graph, Wheel graph, Planar graph, Tree graph , Friendship graph and mainly focusing on alloacating problems by using vertex coloring.

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