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MELTING EFFECT ON STAGNATION POINT FLOW OF CASSON FLUID VIA STRETCHING/SHRINKING SHEET

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Abstract :
This study focuses on the motion of heat transfer via a stretching/shrinking sheet at the stagnation point. The main objective is to investigate the behavior of Casson liquid and its impact on heat transfer in this system. The study aims to enhance thermal systems through technological advancements. The partial differential equations are converted into ordinary differential equations using standard similarity variables. The resulting physical equations are solved using the Runge-Kutta-Fehlberg 4th order iterative process. Numerical solutions are obtained and presented through graphs. It is observed that the Reynolds number has a significant influence on non-Newtonian motion compared to Newtonian motion.
Keywords: Casson Fluid, Melting Effect, Stagnation Point, Stretching Sheet, Shrinking Sheet.

Introduction

The behaviour of the boundary layer in these scenarios is important for optimizing industrial processes, controlling heat transfer rates, and achieving desired material properties. Researchers and engineers often use mathematical models and computational fluid dynamics simulations to study these phenomena and develop practical applications in various industries.Khan and Pop **[1]** developed BL motion of a nanofluid (NFs) via SS. **[2-3]** consider the laminar BL equations at a position of separation. Kausar et al. **[4]** created HT capabilities of conventional NFs via permeable SS. Some of authors **[5-6]** presented the numerical computing on mixed convection BL motion of Casson liquid via a linear elongating surface**.** The strength of feed-forward NN-BLMS developed by Ullah et al. **[7]**. Wang et al. **[8]** developed three-dimensional NFs motion with "nonlinear Thermal Radiation" (NLRT). Khashi'ie et al. [9] presented MHD BL motion of hybrid NFs via moving plate. The Buongiorno NFs model on BL motion of viscoelastic liquid via nonlinear SS by Nadeem et al. [10]. The BL heat transfer and entropy generation of NFs via isothermal linear SS was exhibited [11-12]. The numerical study of BL motion on a moving horizontal flat plate was analyzed Fatima et al. **[13]**. Zhao et al. **[14]** proposed fast precision measurement method. Wu et al. **[15]** revealed information about the size and velocity of droplets in the air-assisted spray, which is valuable in various applications, such as agricultural spraying, coating processes, and combustion systems. Sun et al. **[16]** prepared resulting coatings, known as superhydrophobic coatings, are extremely water-repellent due to their micro- and nano-scale surface roughness and chemical composition.

The dynamic involvement of steady BL motion of a Newtonian or non-Newtonian (NN) ("except zero value of shear stress") liquid via stretching/ shrinking surface continue to develop by Reiner-Philipp off liquid model be a lot of interest for upcoming scientists. The liquid of NN play big imperative role in industrial applications ("biomedicine, the industrial processes dealing with polymer extrusion, polymer and food processing, artificial filaments, fabrications, metal coiling, thermal oil recovery, crystals, paper production, glass blowing and discharge of industrial") **[17-18]**. Zaman et al. **[19]** examines the HMT in the context of blood motion through a tapered artery. Shan et al. **[20]** developed analytical non-Newtonian fluids have viscosity that varies with shear rate, and this paper likely provides mathematical models to describe their behavior in the context of capillary flow. Rathore and Sandeep **[21]** discusses their potential applications in anticancer treatments and their interaction with radiative heat in the presence of blood motion. Samrat et al. **[22]** involves the examination of fluid flow and thermal exchange in a parabolic extending region. Anantha Kumar et al. **[23]** proposed mass and heat transfer on 3D bio convective motion of NFs across slandering surface. Gnaneswara Reddy et al. **[24]** described BL motion across melting surface. Faghiri et al. [25] presented the NN liquid motion via circular tube. The VD on radiative HT of Reiner-Philipp off liquid motion via nonlinearly shrinking sheet was created Khashi'ie et al. [26]. The behaviour of Williamson NFs via porous surface of stretching cylinder was examined Haq et al. **[27]**. Shoaib et al. **[28]** presented Williamson liquid motion field properties. Mishra et al. **[29]** examined the MHD motion of Williamson liquid via a non-Darcy porous medium. Jalili et al. **[30]** dedicated the Lorentz force and variable viscosity impact on NN NFs motion via stretching plate. The "spectral Quasi-Linearization" (SQLM) method implemented on nonlinear mixed convection HT of NN liquid via vertical microchannel was examined Mahanthesh et al. **[31]**. Rashid et al. **[32]** described incompressible Williamson liquid in curved channel. Some of scientists **[33-34]** developed NN NFs motion via various geometrical model. Salahuddin et al. **[35]** assumed thermal energy system is heat exchanger. Waqas et al. **[36]** demonstrated incompressible nonlinear mixed convective motion of Williamson liquid via SS. Almaneea et al. **[37]** exhibited characteristics of Williamson liquid with hybrid nanoparticles Cu and Al_2O_3 on heat transfer. Huang et al. **[38]** explored the motion properties of mass and heat transfer of multilayer motion of phan-Thien-Tanner and Jeffrey liquid via channel. Some authors **[39-40]** morphologies analysis for combination with microstructures, cracks propagation paths and fracture surface. Recently, some authors **[41-42]** analysed composite structures made of 2 mm-thick titanium and 10 mm-thick carbon steel.

The liquid "Boundary Layer" (BL), "stagnation point" (SP) motion is basis in several convective cooling procedures. The study of Casson liquid within SP has become an motivative research area via stretching applications. The SP motion represents motion of a liquid via SS. The SP region presents largest deposition "Heat and Mass Transfer" (HMT) and pressure. **Warke et al. [43] d**iscusses the use of counter-motion jets in planes, which could be related to fluid dynamics and propulsion systems in aircraft. **Abbas et al. [44]**mentions the application of SP motion in the polymer industry, particularly in the context of extrusion processes. **Zhang et al. [45]**examine convection and joule heating in the context of Magneto-Hydrodynamic (MHD) 2D SP motion of non-Newtonian fluids via permeable solid surfaces. **Chu et al. [46]** discusses SP motion of NN liquids. **Swain et al. [47]** suggests that High-Temperature (HT) SP motion is important in applications like solar collectors, nuclear reactors, and coolers. **Khashi'ie et al. [48]** presents research on SP hybrid motion with mixed convection, possibly induced by a Riga plate. **Abbasi et al. [49]** analyzing fluid flow problems formulated in cylindrical coordinates. **Zhao et al. [50]** discusses entropy generation in the context of heat and mass transfer during the motion of a fluid with magnetic field influence (MHD SP motion) across a stretchable surface. **Zainal et al. [51]** focus on MHD mixed convection motion of hybrid non-Newtonian fluids past a vertical flat plate. Recently, some of scientists **[52-53]** created related and useful SP motion via SS. Arain et al. **[54]** presented NFs bioconvection via a pair of vertical parallel plates with activation energy. The characteristics of MHD oscillatory oblique SP motion of micropolar NFs was presented Sadiq et al. **[55]**. Dharmendar Reddy et al. **[56]** investigated the numerical results for 2D MHD SP motion of incompressible NFs via stretching cylinder. Nandi et al. **[57]** presented the influence of NLTR and heat generation on MHD SP motion methanol-based NFs via permeable SS. Khan et al. **[58]** provided insights into the behavior of hybrid NFs in the presence of a rotating disk. Zhang et al. **[59]** involved both experimental and numerical investigations to understand how nanoparticles and microorganisms interact within a rotating environment.

The motivation of this research is motivated by the study of the melting effect on stagnation point (SP) motion of Casson liquid. The study involves numerical computations using a built-in MATLAB tool called "bvp4 method." This method is help to solve a system ODEs that describe the fluid motion in this particular scenario. The practical applications highlighted in the research is the potential impact on the cooling rate of refrigerators. The Casson liquid stagnation motion has various practical applications, including in fibrous insulation, electric ovens, geysers, soil pollution, electric kettles, and electric bulbs, among others.

Mathematical Formulation:

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We consider a steady 2D ("Two Dimensional") BL motion of a Casson liquid via Stretchable ("such as stretching/shrinking") sheet with melting effect into constant property of thermal liquid as depicted in the **Fig.1**. At the sheet, we consider ambient velocity $U_e = ax$ and $U_w = cx$ is a quantity of stretching rate.

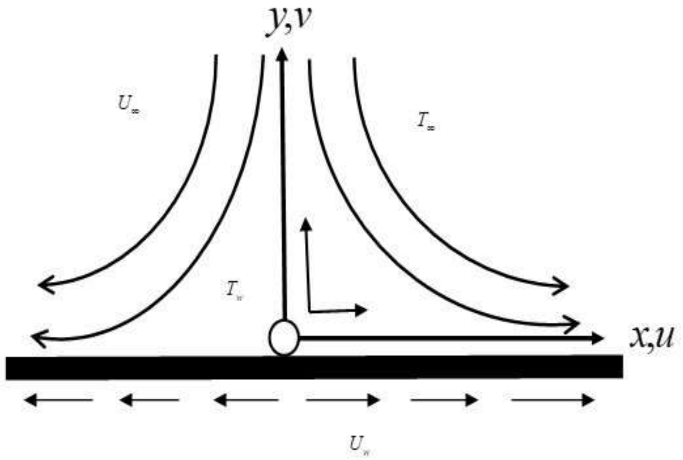


Fig.1 Physical Model of the Problem

The rheological equation of Casson liquid is defined as (**ref. [8]**):

$$\tau_{ij} = \begin{cases} \left(2\mu_0^* + \dfrac{\sqrt{2}p_y^*}{\sqrt{\pi^*}}\right)e_{ij}, & if\ \pi^* \geq \pi_c^* \\ \left(2\mu_0^* + \dfrac{\sqrt{2}p_y^*}{\sqrt{\pi_c^*}}\right)e_{ij}, & if\ \pi^* < \pi_c^* \end{cases} \quad (1)$$

In above consideration the Gov. Eqs (**ref. [60]**) are $u_x + u_y = 0$ (2)

$$uu_x + vu_y = (u_e)_x + \upsilon_1\left(\frac{\Lambda+1}{\Lambda}\right)u_{yy} \quad (3)$$

$$u\Psi_x + v\Psi_x = \alpha\Psi_{yy} \quad (4)$$

B. C's of governing Eq's (1-3) are as below

$$\left.\begin{array}{l} at \quad y = 0 \\ u = U_w = cx \\ T = T_m \\ as \quad y \to \infty \\ u \to U_e \\ T \to T_\infty \end{array}\right\} \quad (5)$$

and

$$\rho\left[\lambda + Cs(T_\infty - T_0)\right]v(x,0) = k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (6)$$

Similarity variables as

$$\left.\psi = x\sqrt{c\upsilon}\, f(\eta),\ \theta(\eta) = \frac{\Psi - \Psi_m}{\Psi_\infty - \Psi_m},\ \eta = \sqrt{\frac{c}{\upsilon}} y \right\} \tag{7}$$

Where $\psi = \psi_y,\quad v = -\psi_s$

Using Eq. (7) to solving Eqs (2)-(3), we get following ODE's

$$f'''\left(\frac{\Lambda+1}{\Lambda}\right) + ff'' - (f')^2 + 1 = 0 \, (8)$$

$$\theta'' + \Pr f\theta = 0 \, (9)$$

With corresponding B. C's as bellow

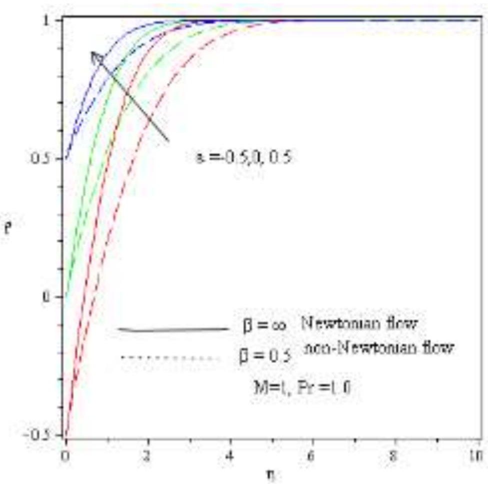
$$\left.\begin{array}{l} f(0) = \varepsilon \\ \Pr f(0) = -M\theta(0) \\ \theta(0) = 0 \\ f(\infty) = 1 \\ \theta(\infty) = 1 \end{array}\right\} (10)$$

The skin friction coefficient, HT are defined as:

$$C_f = \frac{\tau_w}{\rho u_e^2}\ Nu_x = \frac{x q_w}{k(\Psi_\infty - \Psi_w)} \, (12)$$

Eq'(12) can reduced as below

$$(\mathrm{Re}_x)^{0.5} \mathrm{Re}_x = f''(0),\ (\mathrm{Re}_x)^{-0.5} Nu_x = -\theta'$$



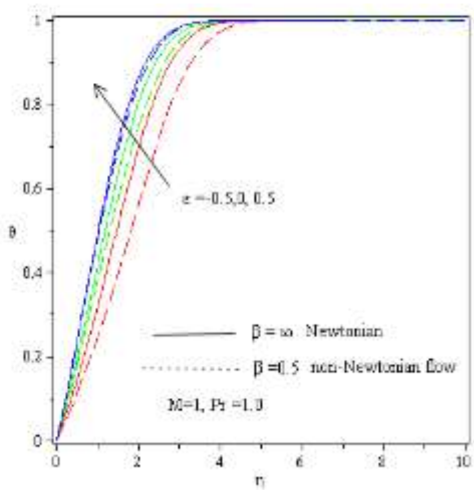
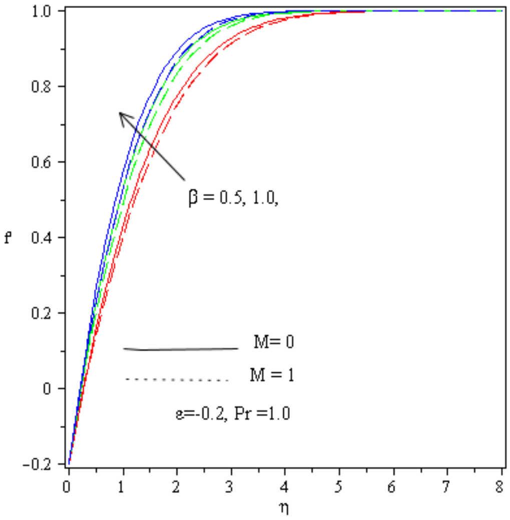


Fig. 2(a) The Impact of ε on f' **Fig. 2(b)** The Impact of ε on θ



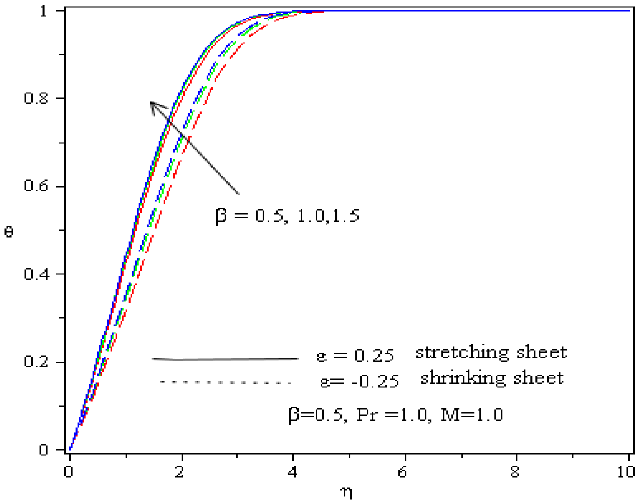
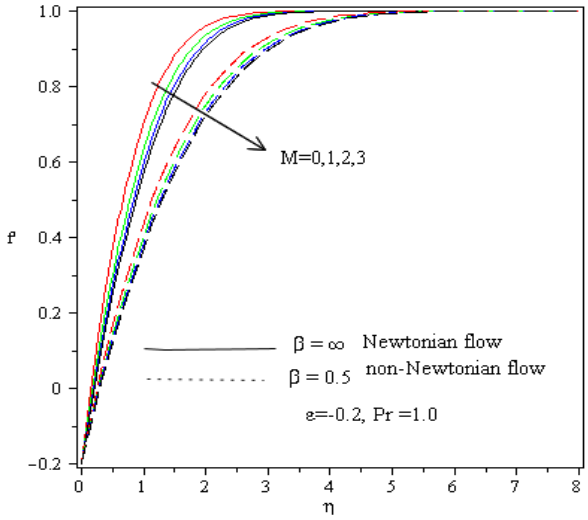


Fig. 3(a) The Impact of β on f' **Fig. 3(b)** The Impact of β on θ



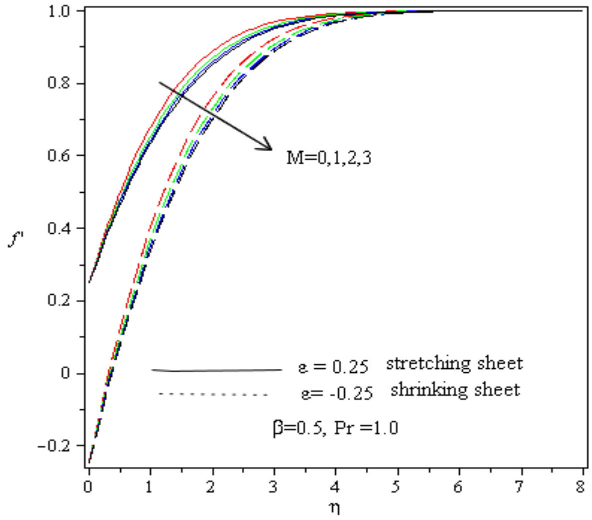


Fig. 4(a) The Impact of β on f' **Fig. 4(b)** The Impact of β on θ

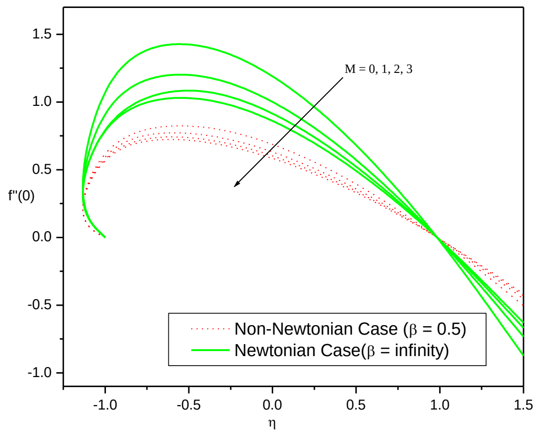
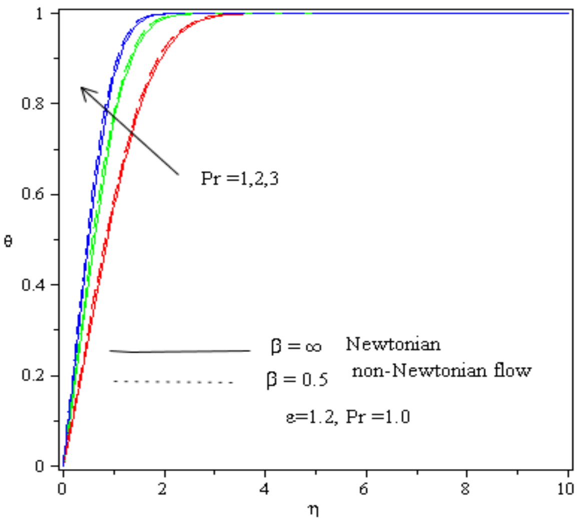


Fig. 4(c) The Impact of M on Skin-friction Coefficient



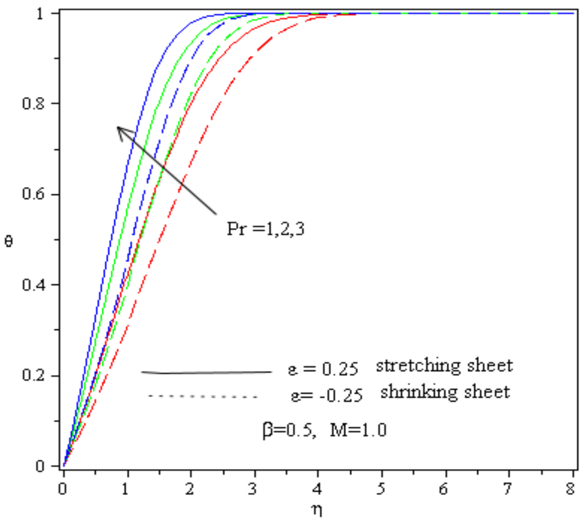


Fig. 5(a) The Impact of Pr on f' **Fig. 5(b)** The Impact of Pr on θ

Table 1. Comparative Values of $f''(0)$ for stretching sheet problem when M=0 and $\beta \to \infty$

ε	Bachok et al. **[60]**	Present Study
0	1.2325877	1.2325878
0.1	1.1465610	1.1465609
0.2	1.0511300	1.0511299
0.5	0.7132949	1.713295
1	0	0
2	-1.8873066	-1.8873056

Table 2. Camparative values of $f''(0)$ for different values of M when $\beta \to \infty$

ε	**Bachok et al. [60]**			**Present Study**		
	M =1	M =2	M =3	M =1	M =2	M=3
0	1.0370034	0.9468506	0.8913811	1.0370034	0.9468505	0.8913810
0.1	0.964	0.880	0.828	0.964	0.880	0.8289483

	2521	4416	9482	2521	4412	
0.2	0.8836747	0.8068762	0.7597516	0.8836747	0.8068762	0.7597515
0.5	0.5990895	0.5043333	0.5151721	0.5990895	0.5043334	0.5151720
1	0	0	0	0	0	0
2	-1.5804839	-1.4427473	-1.3592105	-1.5804839	-1.4427474	-1.359210

Table 3. Camparative values of $-\theta(0)$ for different values of Pr and M when ε=1 and $\beta \to \infty$

Pr	M	**Bachok et al. [53]**	Present Study
1	0	-0.7978846	-0.79788465
	1	-0.5060545	-0.50605450
	2	-0.3826383	-0.38263831
	3	0.3119564	0.31195643
7	0	-2.1110042	-2.11100424
	1	-1.3388943	-1.33889433
	2	-1.0123657	-1.0123657
	3	-0.8253591	-0.82535911

Table 4. Values of $\theta^{'}(0)$ for different values of governing parameters

Governing parameters				$\theta^{'}(0)$
Pr	M	ε	β	
1	0	0.5	0.5	0.56713
	1			0.36377
	2			0.27664
	3			0.22638
1	1	0.5	0.5	0.420979
		1		0.506054
		1.5		0.581487

1	1	0.5	0.5 1.0 1.5	0.420979 0.427700 0.4307125
1 5 7	1	0.5	0.5	0.420979 0.886350 1.0372329

Table 5. Critical values for λ_c for different values of M

M = 0	-1.2465
M = 1	-1.24311
M = 2	-1.2401
M = 3	-1.2366

Results and Discussion:

Figs. 2(a)-2(b) Shows the characteristic of ε ("Moving Parameter"), β ("Casson Liquid Parameter") on $f'(\eta)$, θ respectively with cases of Newtonian and non-Newtonian. It is seen that, the both $f'(\eta)$, θ enhances with expand numerical values of ε. The BL is converging very near to SS ("Stretching Sheet") in case of NN liquid while comparing to Newtonian liquid case.

The impact of β ("Casson liquid parameter") with various cases of M on $f'(\eta)$ as shown in **Figs. 3(a)**. It is perceived that; the $f'(\eta)$ is high for increasing numerical values of β . We noticed that; the liquid BL converges very fast in presence of melting parameter and also, enhances temperature is more when liquid is motion in "stretching sheet" while comparing to "shrinking sheet" as predicted in **Fig. 3(b)**. Physically, the dynamic viscosity is to enhance the temperature and reduces to velocity in SS.

The effect of M ("Melting Parameter") as seen in **Fig. 4(a)** for cases of $\beta = \infty$ ("Newtonian") and $\beta = 0.5$ ("non-Newtonian"). The $f'(\eta)$ ("Velocity Profile") BL is declined with enlarge values of M . It is clear, the velocity declined in "non-Newtonian liquid motion" when compared to "Newtonian" liquid motion and also, the velocity of Casson liquid motion in SS is high when compared to shrinking sheet is observed in **Fig. 4(b)**. In **Fig. 4(c)** predicts that, the "Skin-friction coefficient" is declined with high numerical values of M while compared to "Newtonian liquid". Physically, The Lorenz force has applicable in opposite direction of liquid motion, due to opposite force the liquid motion declined for Casson liquid motion via SS.

Fig. 5(a)-5(b) shows the effect of $\Pr$ on $f'(\eta)$ with "Newtonian and non-Newtonian liquids", θ ("Temperature Profile") with "stretching sheet and shrinking sheet" respectively. It is clear that, the $f'(\eta)$ and θ is more in case of "non-Newtonian" liquid and also, more active in motion of stretching sheet while compared to shrinking sheet. Physically, the Pr is directly proportional to TC, but rather, the Prandtl number describes the relative importance of TC and dynamic viscosity (DV) in a fluid, and a low Prandtl number can lead to slower temperature variations in the presence of low TC.

To find solutions of present study for a diverse set of numerical values of governing physical parameters to validate our results as predicted in **Table-1, 2**.

In **Table-3** and **Table- 4** are shown HT rate comparing present results into Bachok et al. **[2]** and **Table-5** is critical values for different values of M.

Conclusion

The conclusions of this study are

- The $f'(\eta)$ and $\theta(\eta)$ are high in case of non-Newtonian liquid motion in stretching sheet while comparing to Newtonian liquid in shrinking sheet with enlarge numerical values of ε.
- The $f'(\eta)$, $\theta(\eta)$ and skin-friction co-efficient are high in case of "non-Newtonian" liquid motion in stretching sheet while comparing to "Newtonian" liquid in shrinking sheet with enlarge numerical values of M.
- The $f'(\eta)$ and $\theta(\eta)$ are high in case of non-Newtonian liquid motion in stretching sheet while comparing to Newtonian liquid in shrinking sheet with enlarge numerical values of $\Pr$.

Data Availability:

- The datasets generated and/or analysed during the current study are not publicly available but are available from the corresponding author on reasonable request.

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Nomenclature

(x, y)	Cartesian coordinate's
a, c	Constants
u, v	velocity components along x_1, y_1, z_1
$U_e = ax$	Velocity of ambient fluid
$U_w = cx$	Stretching Rate
C_s	Solid Surface Heat Capacity
f	Non-Dimensional Motion function
c_p	Specific heat
C_∞	Uniform ambient concentration
p_y	Stress Value of Casson liquid
T_w	Temperature at Wall
Nu_x	Local Nusselt Number
$\Pr = \frac{\upsilon}{\alpha}$	Prandtl number
$\varepsilon = \frac{c}{a}$	Moving Parameter
M	Melting Parameter
T	Temperature
T_m	Temperature of Melting Surface
T_∞	Temperature of Casson Liquid
Re_x	Local Reynold's Number
U_w	Stretching velocity
U_∞	Free stream velocity

Greek symbols

μ_B	Dynamic Viscosity of the Casson liquid
$\pi = e_{ij}e_{ij}$, e_{ij}	Component Product Rate of $(i, j)^{th}$ component
π_c	Critical Value

$\tau_w = \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) \left(\frac{\partial u}{\partial y} \right)_{y=0}$ Stress tensor at Wall

λ Latent Heat of the Casson Liquid

ψ Function of Stream

θ Non-Dimensionless temperature

Subscripts

∞ condition at free stream

Abbreviations

HT	Heat Transfer
SS	Stretching Sheet
TR	Thermal Radiation
SP	Stagnation Point
NFs	Nanofluids