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RELIABILITY ANALYSIS OF A STANDBY SYSTEM SUBJECT TO DIFFERENT FAILURES AND REPAIR

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ABSTRACT
This paper deals with the stochastic modeling and analysis of Tippler-Conveyor system which is numerically and graphically analyzed by calculating reliability variables like MTSF, system availability, busy period and maintenance analysis using Markov process and regenerative point technique. The system consists of three tipplers with two conveyor belts in standby configuration with two modes of failure, e.g., Electrical and Mechanical. Besides regular repair and scheduled maintenance, emergency repair is also performed for the whole system. The negative exponential and general probability distributions are considered for defining failure, repair and maintenance rates.

Key Words:
Stochastic analysis, MTSF, system availability, reliability, scheduled maintenance, regeneration point technique.

1. INTRODUCTION
Reliability is an important consideration in the planning, design and operation of systems. Engineers dealing with large and diverse projects today, require information on reliability as it affects differing systems. An engineer needing information in these areas generally faces a great deal of difficulty as not much work has been done so far in the field of reliability related to industries. Parallel, series, k out of n: F system, k out of n: G system are widely studied in the field of reliability. However, very little work has been reported by taking industrial systems into consideration. Kumar et al. [1988] have analyzed a feeder system of sugar industry. Dhillon et al. [1987] & Nateson et al. [1984] have analyzed pulverizer systems with common cause failures. Kocher et al. [1983] have analyzed the reliability of the electric motors which are used in irrigation. Recently, Singh et al. [1995] analyzed a stone crushing system having one apron feeder, one grizzly and one gyratory crusher. This group of equipment is used to get iron ore from stones in the mining crushing plants. They obtained various parameters of the system which are useful to system managers and engineers.
For the purpose of analyzing industrial models, Bhilai Steel Plant has been selected taken as working area. Bhilai Steel Plant is one of the leading steel plants of India.
The purpose of the present paper is to study a **Tippler-Conveyor** system of coke-oven area of Bhilai Steel Plant. The aforesaid system is the starting point of coke-oven area. In this area, coal comes from nearest coal mines and other countries. Coal is unloaded here with the help of tipplers from wagons. Peripheral unloading of coal wagons is envisaged to quicken turn round time of wagons. After crushing, coal is transferred to coal stock yard through conveyor belts. There are three rotary tipplers, each is having two roots of conveyor belts. All types of maintenance are performed in case of tipplers such as scheduled maintenance, running maintenance etc. But in case of conveyors, only breakdown maintenance or repairs are performed. Scheduled maintenance is performed as per the requirement of the unit and it is performed in case of tipplers only. Running maintenance is performed during the operation period of the unit. In fact, it has some inspection schedules to overcome the difficulties of heavy breakdowns as this may cause a heavy loss to whole production unit. Simple repair includes electrical repair, mechanical repair and on-line repair. Besides this, emergency repair concept is also imposed in the system to make the system ready as early as possible in case of major break downs. Using regenerative point technique, following measures of system effectiveness are obtained to carry out the profit analysis:

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(i) Steady state transition probabilities and Mean sojourn times in different states;
(ii) Mean time to tippler-conveyor system failure in (0, t] and in steady state;
(iii) Availability of the tippler-conveyor system in (0, t] and in steady state;
(iv) Expected busy period of the repairmen in repair (electrical / Mechanical/ on line) in (0, t] and in steady state;
(V) Expected busy period of the repairman in schedule maintenance in (0, t] and in steady state;
(vi) Expected busy period of the repairman in emergent repair in (0, t] and in Steady state;
(vii) Expected profit earned by the system in (0, t] and in steady state.

At last, some particular cases are discussed and graphs are plotted to highlight the important results.

2. Symbols for states of the system

T_O / T_S : Tippler in operative / standby state

$C_O / C_S / C_r$: Conveyor belt under operation / in standby / under repair

sm/SM : Tippler under scheduled maintenance / under continued scheduled maintenance from the previous state.

SUR : Subsystem under repair

ER : System under emergency repair

UP States

$S_0 = \{T_o, C_o, C_s / T_o, C_o, C_s / T_s, C_s, C_s\}$; $S_1 = \{SUR/T_o, C_o, C_s / T_o, C_o, C_s\}$

$S_2 = \{T_o, C_r, C_o / T_o, C_o, C_s / T_s, C_s, C_s\}$; $S_3 = \{sm/T_o, C_o, C_s / T_o, C_o, C_s\}$

$S_4 = \{SUR/T_o, C_r, C_o / T_o, C_o, C_s\}$; $S_6 = \{T_o, C_r, C_o / T_o, C_r, C_o / T_s, C_s, C_s\}$

$S_7 = \{SM/T_o, C_r, C_o / T_o, C_o, C_s\}$; $S_3 = \{SM/T_o, C_o, C_s / T_o, C_o, C_s\}$

Down State

$S_5 = \{ER\}$

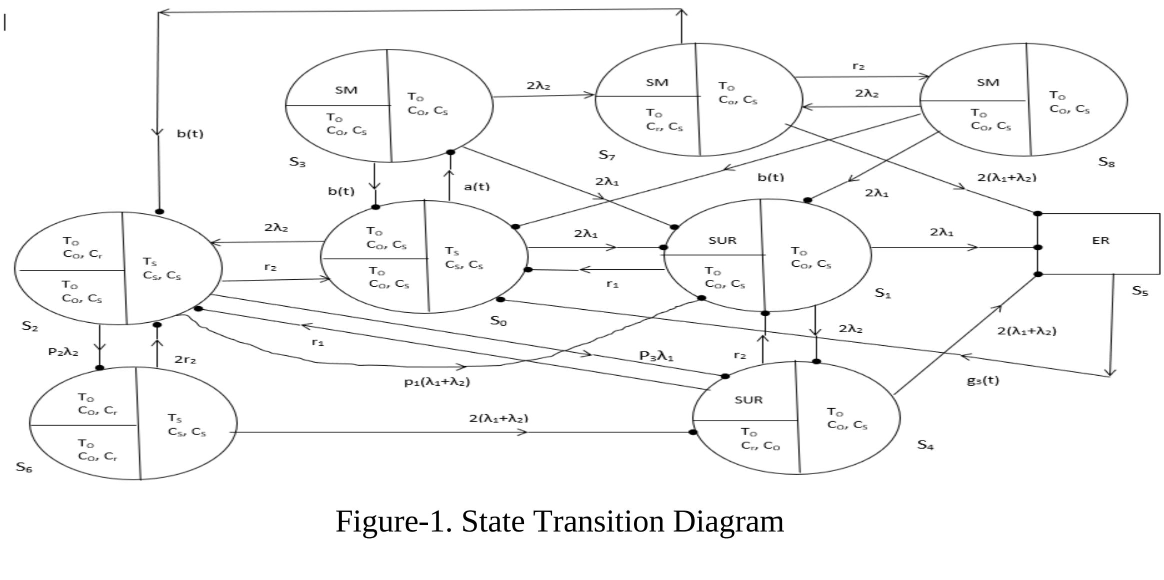

Figure-1. State Transition Diagram

3. Description of the system

(1) System consists of the three subsystems. Each subsystem is having one tippler and two conveyor belts in standby configuration.

(2) After a random time, any of the tipplers leaves for scheduled maintenance. The time of scheduled maintenance varies according to the requirement of the operative unit i.e., if operation time increases, scheduled maintenance time can be adjusted accordingly.

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(3) Any subsystem undergoes repair if any of the tippler fails or any tippler or conveyor belt of the same subsystem fails provided one conveyor belt is already under repair.

(4) Besides, scheduled maintenance which is planned maintenance, emergency repair is also there for unpredictable breakdowns etc.

(5) Emergency repair takes place in case when two subsystems are in failed condition i.e., system will not work with a single subsystem in working mode.

(6) Further, scheduled maintenance is performed in case of tipplers only and in such a way that whenever any sub-system undergoes repair, then the subsystem which is kept in scheduled maintenance immediately replaces the failed subsystem in order to continue the work.

(7) There are sufficient repairmen for maintenance purpose. However, in case of emergency repair, few more repairmen may be called to make the system ready at earliest.

(8) Failure time distributions of tippler and conveyor belts along with repair time distribution of conveyor belt and that of any subsystem are also taken as exponentially distributed and repair time distribution of tippler and maintenance time distributions are arbitrarily distributed.

(9) After repair units work as good as new.

4. Notations

E_0 : State of the system at t=0

E_1 : Set of regenerative states $\{S_0,S_1,S_2,S_3,S_4,S_5,S_6\}$

E_2 : Set of non-regenerative states $\{S_7,S_8\}$

a(t), A(t) : pdf and cdf of scheduled maintenance time.

b(t), B(t) : pdf and cdf of completion of schedule maintenance time.

SUR : Subsystem under repair due to failure of Tippler only or due to tippler or conveyor belt provided one conveyor belt is already under repair in the same subsystem.

p_1 : P [Probability that subsystem in state S_2 fails due to failure of tippler or conveyor belt and goes to state S_1].

p_2 : P [Probability that subsystem in state S_2 fails due to failure of conveyor belt only and goes to state S_6]

p_3 : P [Probability that subsystem in state S_2 fails due to failure of tippler only and goes to state S_4]

λ_1 / λ_2 : Constant failure rate of Tippler / Conveyor Belt.

r_1 / r_2 : Constant repair rate of SUR / Conveyor Belt.

$g_3(t), G_3(t)$: pdf and cdf of emergency repair time.

$p_{i,j;k}$: Transition from state i to j visiting state k only once.

$p_{ij}^{(k,1)}$: Transition from state i to j visiting state k &1 any number of times

$$\overline{a}_{ij} = 1 - p_{ij}p_{ji}$$

s = Laplace-Stieltjes Convolution, **c** = Laplace Convolution

5. Transition probabilities and sojourn times

Simple probabilistic considerations yield the following expressions for non-zero transition probabilities p_{ij} :

$p_{01} = (2\lambda_1)\{\overline{A}^*(2\lambda_1 + 2\lambda_2)(t)\}$; $p_{02} = (2\lambda_2)\{\overline{A}^*(2\lambda_1 + 2\lambda_2)(t)\}$;

$p_{03} = \{\overline{A}^*(2\lambda_1 + 2\lambda_2)(t)\}$; $p_{10} = r_1 / X_2$; $p_{15} = 2\lambda_1 / X_1$;

$p_{14}=2\lambda_2/X_2$; $p_{20}=r_2/X_7$; $p_{21}=p_1(\lambda_1+\lambda_2)/X_7$

$$p_{24}=\{p_3\lambda_1/X_7\};p_{26}=p_2\lambda_2/X_7;p_{30}=\{b^*(2\lambda_1+2\lambda_2)(t)\};$$

$$p_{32}^{(7)}=(2\lambda_2)\{(2\lambda_1+2\lambda_2)^*(u)\}\{b^*(r_2+2\lambda_1+2\lambda_2)(v/u)$$

$$p_{32}^{(7,8)}=(2\lambda_2)\{(2\lambda_1+2\lambda_2)^*(u)\}\sum_{n=1}^{\infty}\{\{r_2(r_2+2\lambda_1+2\lambda_2)^*(v/u)\}\{2\lambda_2(2\lambda_1+2\lambda_2)^*(w/v)\}\}^n]$$
$$\{b^*(r_2+2\lambda_1+2\lambda_2)(x/w)\}$$

$$p_{31}=(2\lambda_1)\{\bar{B}^*(2\lambda_1+2\lambda_2)(t)\};$$

$$p_{35}^{(7)}=(2\lambda_2)\{(2\lambda_1+2\lambda_2)^*(u)\}\{(2\lambda_1+2\lambda_2)\bar{B}^*(r_2+2\lambda_1+\lambda_2)(v/u)\}$$

$$p_{31,\vartheta7,8}=(2\lambda_2)\{(2\lambda_1+2\lambda_2)^*(u)\}\{r_2(r_2+2\lambda_1+2\lambda_2)^*(v/u)\}(2\lambda_1)\{\bar{B}^*(2\lambda_1+2\lambda_2)(y/x)\}$$

$$p_{31,\vartheta7}^{(8,7)}=(2\lambda_2)\{(2\lambda_1+2\lambda_2)^*(u)\}\{r_2(r_2+2\lambda_1+2\lambda_2)^*(v/u)[\sum_{n=1}^{\infty}\{\{2\lambda_2(2\lambda_1+2\lambda_2)^*(w/v)\}$$
$$\{r_2(r_2+2\lambda_1+2\lambda_2)^*(w/v)\}\}^n](2\lambda_1)\{\bar{B}^*(2\lambda_1+2\lambda_2)(y/x)\}$$

$$p_{30\vartheta7}^{(8,7)}=(2\lambda_2)\{(2\lambda_1+2\lambda_2)^*(u)\}\{r_2(r_2+2\lambda_1+2\lambda_2)^*(v/u)[\sum_{n=1}^{\infty}\{\{2\lambda_2(2\lambda_1+2\lambda_2)^*(w/v)\}$$
$$\{r_2(r_2+2\lambda_1+2\lambda_2)^*(w/v)\}\}^n](2\lambda_1)\{b^*(2\lambda_1+2\lambda_2)(y/x)\}$$

$$p_{35}^{(7,8)}=(2\lambda_2)\{(2\lambda_1+2\lambda_2)^*(u)\}[\sum_{n=1}^{\infty}\{\{r_2(r_2+2\lambda_1+2\lambda_2)^*(v/u)\}\{2\lambda_2(2\lambda_1+2\lambda_2)^*(w/v)\}\}^n]$$
$$\{(2\lambda_1+2\lambda_2)\bar{B}^*(r_2+2\lambda_1+2\lambda_2)(x/w)\}$$

$p_{41}=r_2/X_4$; $p_{45}=2(\lambda_1+\lambda_2)/X_4$; $p_{42}=r_1/X_4$; $p_{50}=1$

$p_{52}=2r_2/X_5$; $p_{64}=2(\lambda_1+\lambda_2)/X_5$

Also μ_i, the mean sojourn times in state S_i are:

$$\mu_0=\bar{A}^*(2\lambda_1+2\lambda_2)(t);\mu_1=\frac{1}{X_2};\mu_2=\frac{1}{X_7};\mu_3=\bar{B}^*(2\lambda_1+2\lambda_2)(t);$$
$$\mu_4=\frac{1}{X_4};\quad \mu_5=\int_0^{\infty}g_3(t)dt;\quad \mu_6=\frac{1}{X_5(25-31)}$$

(1-31)

$$where:$$
$$2\lambda_1+2\lambda_2+a=X_1;\ 2\lambda_1+2\lambda_2+r_1=X_2;\ 2\lambda_1+2\lambda_2+r_1+r_2=X_4$$
$$2\lambda_1+2\lambda_2+2r_2=X_5;p_1(\lambda_1+\lambda_2)+p_2\lambda_2+p_3\lambda_1+r_2=X_7$$
$$(2\lambda_1+2\lambda_2)^*(u)=\int_{u=0}^{\infty}e^{-(2\lambda_1+2\lambda_2)u}\,du$$
$$\bar{A}^*(2\lambda_1+2\lambda_2)(v,u)=\int_{v=u}^{\infty}\bar{A}(v)\{e^{-(2\lambda_1+2\lambda_2)(v-u)}\}\,dv$$
$$b^*(2\lambda_1+2\lambda_2)(v,u)=\int_{v=u}^{\infty}b(v)\{e^{-(2\lambda_1+2\lambda_2)(v-u)}\}\,dv$$
$$\bar{B}^*(2\lambda_1+2\lambda_2)(v,u)=\int_{v=u}^{\infty}\bar{B}(v)\{e^{-(2\lambda_1+2\lambda_2)(v-u)}\}\,dv$$

6. **Mean time to system failure**

Time to system failure can be regarded as the first passage time to the failed state. To obtain it, we regard the down states as absorbing states. Using arguments as for the regenerative process we obtain the following recursive relations for $\pi_i(t)$:

$$\pi_0(t)=\sum_{j=1,2,3}Q_{0j}(t)\circledS\pi_j(t)\qquad \pi_1(t)=\sum_{j=0,4,5}Q_{1j}(t)\circledS\pi_j(t)$$

$$\pi_2(t)=\sum_{j=0,1,4,6}Q_{2j}(t)\circledS\pi_j(t)$$

$$\pi_3(t)=Q_{30}(t)\circledS\pi_0(t)+Q_{31}(t)\circledS\pi_1(t)+Q_{32}^{(7,8)}(t)\circledS\pi_2(t)+Q_{32.97}(t)\circledS\pi_2(t)+$$
$$Q_{35}^{(7,8)}(t)+Q_{35.97}(t)+Q_{30.97}^{(8,7)}(t)\circledS\pi_0(t)+Q_{30.97,8}(t)\circledS\pi_0(t)+Q_{31.97,8}(t)\circledS\pi_1(t)+$$
$$Q_{31.97}^{(8,7)}(t)\circledS\pi_1(t)$$

$$\pi_4(t)=Q_{42}(t)\circledS\pi_2(t)+Q_{41}(t)\circledS\pi_1(t)+Q_{45}(t)$$

$$\pi_6(t)=Q_{62}(t)\circledS\pi_2(t)+Q_{64}(t)\circledS\pi_4(t)\qquad(32\text{-}37)$$

Taking Laplace-Stieltjes transforms of equations [32-37] and after solving for $\tilde{\pi}_0(s)$, we have

$$\text{MTSF} = E(T) = -\frac{d}{ds}\tilde{\pi}_0(s)\Big|_{s=0} = \frac{D_1'(0)-N_1'(0)}{D_1(0)}\qquad(38)$$

$$D_1'(0)-N_1'(0)=\mu_0 b_0+\mu_1 b_1+\mu_2 b_2+n_3 b_3+\mu_4 b_4+\mu_6 b_6$$

$$D_1(0)=1-p_{01}p_{10}\bar{a}_{24}-p_{02}p_{10}p_{21}-p_{02}p_{20}\bar{a}_{14}-p_{42}p_{24}-p_{42}p_{14}p_{21}-p_{01}p_{14}p_{20}p_{42}$$
$$-p_{02}p_{20}\bar{a}_{14}-p_{41}p_{14}+p_{02}p_{10}p_{41}p_{24}-p_{03}p_{10}(B_{31}+p_{21}B_{32})-p_{03}\bar{a}_{14}(B_{30}+p_{20}B_{32})$$
$$+p_{03}p_{42}p_{24}B_{30}+p_{03}p_{42}p_{10}p_{24}B_{31}-p_{03}p_{42}p_{14}(p_{20}B_{31}-p_{21}B_{30})-p_{03}p_{41}p_{10}p_{24}B_{32}$$
$$-p_{26}p_{64}p_{42}+p_{01}p_{10}p_{42}p_{26}p_{64}-p_{02}p_{10}p_{41}p_{26}p_{64}-p_{26}p_{62}\bar{a}_{14}+p_{01}p_{10}p_{26}p_{62}+$$
$$p_{03}p_{10}p_{26}p_{64}(B_{31}p_{42}-p_{41}B_{32})+p_{03}p_{42}p_{26}p_{64}B_{30}+p_{03}p_{10}p_{26}p_{62}B_{31}+$$
$$p_{03}p_{26}p_{62}B_{30}\bar{a}_{14}$$

Where:

$$b_0=\bar{a}_{24}-p_{14}(p_{21}p_{42}+p_{41})-p_{26}(p_{42}p_{64}+p_{62})+p_{26}p_{14}p_{41}p_{62}$$
$$b_1=p_{01}\bar{a}_{24}+p_{02}(p_{21}+p_{24}p_{41})+p_{03}p_{21}p_{32}+p_{03}B_{31}\bar{a}_{26}-p_{03}p_{24}(B_{31}p_{42}-p_{41}B_{32})$$
$$-p_{01}p_{26}p_{42}p_{64}+p_{02}p_{26}p_{64}p_{41}-p_{01}p_{26}p_{62}-p_{03}p_{26}p_{64}(B_{31}p_{42}-p_{41}B_{32})$$
$$b_2=p_{02}\bar{a}_{14}+p_{03}B_{32}\bar{a}_{14}+p_{01}p_{14}p_{42}+p_{03}p_{14}p_{42}B_{31}$$
$$b_3=p_{03}\bar{a}_{24}-p_{03}p_{14}p_{41}-p_{03}p_{21}p_{14}p_{42}-p_{03}p_{42}p_{26}p_{64}-p_{03}p_{26}p_{62}\bar{a}_{14}$$
$$b_4=p_{01}p_{14}\bar{a}_{26}+p_{03}p_{14}B_{31}\bar{a}_{26}+(p_{14}p_{21}+p_{26}p_{64})(p_{03}B_{32}+p_{02})+p_{02}p_{24}+p_{03}p_{24}B_{32}$$
$$b_6=p_{02}p_{26}\bar{a}_{14}+p_{03}p_{26}B_{32}\bar{a}_{14}+p_{01}p_{42}p_{26}p_{14}+p_{03}p_{42}p_{26}p_{14}B_{31}$$

7. System Availability

$M_i(t)$ is the probability that the system, initially in regenerative state S_i, is up at time t without passing through any other regenerative state or returning to itself through one or more non-regenerative states, i.e. either it continues to remain in regenerative state S_i or a non-regenerative state including itself.

By probabilistic arguments, we have:

$$M_0(t)=e^{-(2\lambda_1+2\lambda_2)t}\bar{A}(t)$$

$$M_1(t)=e^{-(2\lambda_1+2\lambda_2+r_1)t}$$

$$M_2(t)=e^{-(r_2+p_1(\lambda_1+\lambda_2)+p_2\lambda_2+p_3\lambda_1)t}$$

$$M_3(t)=e^{-(2\lambda_1+2\lambda_2)t}\bar{B}(t)+\int_{u=0}^{t}(2\lambda_2)e^{-(2\lambda_1+2\lambda_2)u}du\,e^{-(2\lambda_1+2\lambda_2+r_2)(t-u)}\bar{B}(t)+\int_{u=0}^{t}(2\lambda_2)e^{-(2\lambda_1+2\lambda_2)u}du$$
$$+\int_{v=u}^{t}r_2e^{-(2\lambda_1+2\lambda_2+r_2)(v-u)}dv\,e^{-(2\lambda_1+2\lambda_2)(t-v)}\bar{B}(t)$$

$$M_4(t) = e^{-(2\lambda_1+2\lambda_2+r_1)t}$$
$$M_6(t) = e^{-(2\lambda_1+2\lambda_2+2r_2)t}$$

(39-44)

Recursive relations giving the pointwise availability $A_i(t)$ are:

$$A_0(t) = M_0(t) + \sum_{j=1,2,3} q_{0j}(t) \quad A_j(t \boxed{c}$$
$$A_1(t) = M_1(t) + \sum_{j=0,4,5} q_{1j}(t) \quad A_j(t \boxed{c}$$
$$A_2(t) = M_2(t) + \sum_{j=0,1,4,6} q_{2j}(t) \quad A_j(t \boxed{c}$$
$$A_3(t) = M_3(t) + q_{30}(t) \quad A_0(t \boxed{c} q_{31}(t) \quad A_1(t) \boxed{c} q_{32,9,7}(t) \quad A_2(t) \boxed{c} q_{32}^{(7,8)}(t) \quad A_2(t \boxed{c}$$
$$q_{35,9,7}(t) \quad A_5(t) \boxed{c} q_{35}^{(7,8)}(t) \quad A_5(t \boxed{c} q_{30,9,7,8}(t) \quad A_0(t \boxed{c} q_{30,9,7}^{(8,7)}(t) \quad A_0(t \boxed{c}$$
$$q_{31,9,7,8}(t) \quad A_1(t) \boxed{c} q_{31,9,7}^{(8,7)}(t) \quad A_1(t) \boxed{c}$$
$$A_4(t) = M_4(t) + q_{42}(t) \quad A_2(t \boxed{c} q_{41}(t) \quad A_1(t \boxed{c} q_{45}(t) \quad A_5(t \boxed{c}$$
$$A_5(t) = q_{50}(t) \quad A_0(t \boxed{c}$$
$$A_6(t) = M_6(t) + q_{62}(t) \quad A_2(t \boxed{c} q_{64}(t) \quad A_4(t \boxed{c}$$

(45-51)

Taking laplace transforms of [45-51] and after solving for $A_0^*(s)$, the steady state availability A_0 is:

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} A_0^*(s) = N_2(0)/D_2'(0) \qquad (52)$$

$$N_2(0) = \mu_0 b_0 + \mu_1 b_1 + \mu_2 b_2 + M_3^*(0) b_3 + \mu_4 b_4 + \mu_6 b_6$$
$$D_2'(0) = \mu_0 b_0 + \mu_1 b_1 + \mu_2 b_2 + n_3 b_3 + \mu_4 b_4 + \mu_5 b_5 + \mu_6 b_6$$

where $M_i^*(0)$ is the steady state probability that the system is initially up in regenerative state S_i without passing through any other regenerative state & n_3 is unconditional sojourn times in respective state S_3 and $b_0, b_1, b_2, b_3, b_4, b_6$ are same as given for MTSF.

$$\begin{aligned} b_5 = {} & p_{01}p_{14}p_{45}\bar{a}_{37} + p_{01}p_{15}\bar{a}_{24}\bar{a}_{37} + p_{02}p_{24}p_{45}\bar{a}_{37} + p_{02}p_{14}p_{21}p_{45}\bar{a}_{37} + p_{02}p_{15}(p_{21} + p_{24}p_{41})\bar{a}_{37} \\ & - p_{01}p_{15}p_{26}(p_{42}p_{64} + p_{62})\bar{a}_{37} + p_{02}p_{26}p_{64}p_{45}\bar{a}_{37} + p_{02}p_{26}p_{64}p_{15}p_{41}\bar{a}_{37} + p_{03}p_{31}p_{14}p_{45}\bar{a}_{26} \\ & - p_{03}p_{26}(p_{62} + p_{42}p_{64})(p_{31}p_{15} + p_{37}p_{75}) - p_{01}p_{26}p_{62}p_{14}p_{45}\bar{a}_{37} + p_{03}p_{37}p_{75}\bar{a}_{24} \\ & - p_{03}p_{14}p_{37}p_{75}(p_{41} + p_{21}p_{42}) + p_{03}p_{24}p_{37}p_{45}p_{72} + p_{03}p_{14}p_{37}p_{45}p_{72}p_{21} + p_{03}p_{37}p_{15}p_{72}(p_{21} + p_{24}p_{41}) \\ & + p_{03}p_{26}p_{37}p_{41}p_{75}p_{14}p_{62} + p_{03}p_{26}p_{37}p_{45}p_{72}p_{64} + p_{03}p_{26}p_{37}p_{41}p_{72}p_{15}p_{64} \end{aligned}$$

8. Expected busy period analysis of the repairman in simple repair (electrical, mechanical, on-line) (0,t] in (0,t]

$B_i^1(t)$ is defined as the probability that at time t, the repairman is busy in repair, given that the system entered state S_i at t=0. Developing similar recursive relations as in [45-51] and after solving the resulting recurrence equations of the Laplace transforms for $B_0^{1*}(s)$, in the long run , the fraction of time for which , repairman is busy with only repair is given by :

$$B_0^1 = \lim_{t\to\infty} B_0^1(t) = \lim_{s\to 0} sB_0^{1*}(s) = \frac{N_3(0)}{D_2'(0)} \qquad (53)$$

$$N_3(0) = \mu_1 b_1 + \mu_2 b_2 + \mu_4 b_4 + \mu_6 b_6$$

where;

$W_1(t)=e^{-(2\lambda_1+2\lambda_2+r_1)t}$; $W_2(t)=e^{-(p_1(\lambda_1+\lambda_2)+p_2\lambda_2+p_3\lambda_1+r_2)t}$

$W_4(t)=e^{-(2\lambda_1+2\lambda_2+r_1+r_2)t}$; $W_6(t)=e^{-(2\lambda_1+2\lambda_2+2r_2)t}$

Where $W_i(t)$ (i=1,2,4,6) denotes the probability that the repairman is busy initially in regenerative state S_i and remains busy at epoch t without transiting through any other regenerative state.

9. Expected busy period analysis of the repairman in scheduled maintenance in (0,t]

$B_i^2(t)$ is defined as the probability that at time t, the repairman is busy in schedule maintenance, given that the system entered state S_i at t=0. Developing similar recursive relations as in [45-51] and after solving the resulting recurrence equations of the Laplace transforms for $B_0^{2*}(s)$, in the long run, the fraction of time for which, repairman is busy with schedule maintenance is given by:

$$B_0^2=\lim_{t\to\infty}B_0^2(t)=\lim_{s\to 0}sB_0^{2*}(s)=\frac{N_4(0)}{D_2'(0)} \quad (54)$$

$$N_4(0)=W_3^*(0)b_3$$

$$W_3(t)=e^{-(2\lambda_1+2\lambda_2)t}\overline{B}(t)+\int_{u=0}^{t}2\lambda_2e^{-(2\lambda_1+2\lambda_2)u}du\{e^{-(2\lambda_1+2\lambda_2+r_2)t}/e^{-(2\lambda_1+2\lambda_2+r_2)u}\}\overline{B}(t)+$$
$$\int_{u=0}^{t}2\lambda_2e^{-(2\lambda_1+2\lambda_2)u}du\int_{v=u}^{t}r_2\{e^{-(2\lambda_1+2\lambda_2+r_2)v}/e^{-(2\lambda_1+2\lambda_2+r_2)u}\}dv\{e^{-(2\lambda_1+2\lambda_2+r_2)t}/e^{-(2\lambda_1+2\lambda_2+r_2)v}\}\overline{B}(t)$$

10. Expected busy period analysis of the repairman in emergency repair (0,t]

$B_i^3(t)$ is defined as the probability that at time t, the repairman is busy in emergent repair, given that the system entered state S_i at t = 0. Developing similar recursive relations as in [45-51] and after solving the resulting recurrence equations of the Laplace transforms for $B_0^{3*}(s)$, in the long run, the fraction of time for which the repairman is busy with emergent repair is given by:

$$B_0^3=\lim_{t\to\infty}B_0^3(t)=\lim_{s\to 0}sB_0^{3*}(s)=\frac{N_5(0)}{D_2'(0)} \quad (55)$$

$$N_5(0)=\mu_5b_5$$

$$W_5(t)=\overline{G}_3(t)$$

11. Particular Cases

(1) When all the maintenance and repair time distributions are n-phase Erlang distributed i.e.

$$g_i(t)=nr_i(nr_it)^{n-1}e^{-nr}i^t/(n-1)!,i=3; \quad f(r_i)=nr_i(nr_it)^{n-1}e^{-nr}i^t/(n-1)!, \quad i=1,2$$

$$b(t)=(nb)(nbt)^{n-1}e^{-nbt}/(n-1)!; \quad \overline{G}_i(t)=\sum_{j=0}^{n-1}\{(nr_it)^je^{-nr_it}\}/j!; \quad i=1,2,3$$

and the rest of the time distributions are considered negative exponential i.e.
$a(t)=ae^{-at}$ and so on, then steady state equations become:

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$MTSF = K_0 / K_1, \quad AV = K_{01}/K_2, B_0^1 = K_{02}^1 / K_2, \quad B_0^2 = K_{02}^2 / K_2, \quad B_0^3 = K_{02}^3 / K_2$

$K_0 = e_0 + e_1 + e_2 + e_3 + e_4 + e_6 + e_7$

$K_1 = e_{01}$

$K_{01} = K_0$

$K_{02}^1 = e_1 + e_2 + e_4 + e_6 \; ; K_{02}^2 = e_3 + e_7 \; ; K_{02}^3 = e_5$

$K_2 = e_0 + e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7$

$e_0 = (1/X_1)[a_{01}a_{04} - a_{04}\{2\lambda_2 F_2 a_{07} - p_2\lambda_2 J_2 a_{10} + 2\lambda_2^2 F_2 F_5 p_2 F_8 J_2\}]$

$e_{01} = 1 - (2\lambda_1 / X_1)a_{04}\{F_1 a_{01} + (2\lambda_2 F_2)J_1 F_6 - p_2\lambda_2 J_2 F_1 F_8 + F_1 F_6 F_9 (p_2\lambda_2 J_2)\} + (2\lambda_1 / X_1)$
$F_1 F_6 F_8 (p_2\lambda_2 J_2) - (2\lambda_2 / X_1)\{F_1 p_1(\lambda_1 + \lambda_2) + J_1 a_{04} a_{14} + (p_2\lambda_2 J_2)F_1 F_5 F_9 a_{04} + (p_3\lambda_1 J_2)$
$F_1 F_5 a_{04} + (2\lambda_2 F_4)F_1 F_{10} p_1(\lambda_1 + \lambda_2)J_2\} - F_6 a_{04}\{(p_3\lambda_1 J_2) + (2\lambda_2 F_2)p_1(\lambda_1 + \lambda_2)J_2 -$
$F_5 a_{15}(2\lambda_2 F_2) - (a/X_1)\{F_1(2\lambda_1 F_4)a_{01} + F_3 a_{14} a_{15} - F_3 F_6 (p_3\lambda_1 J_2) + F_6(2\lambda_2 F_2)(2\lambda_1 F_4 J_1 -$
$F_3 p_1(\lambda_1 + \lambda_2)J_2) + (2\lambda_2 F_4)F_{11} J_1 a_{14} - (p_2\lambda_2 J_2)F_6 F_9 (2\lambda_1 F_1 F_4 - F_3) - (p_2\lambda_2 J_2)F_1 F_8 (2\lambda_1 F_4)$
$+ F_1 p_1(\lambda_1 + \lambda_2)J_2 (2\lambda_2 F_4)F_{11} + (2\lambda_2 F_4)F_1 F_5 (p_3\lambda_1 J_2)F_{11}\} - (p_2\lambda_2 J_2)a_{04}(F_6 F_9 + F_8) -$
$(2\lambda_2 F_4)F_{10} a_{14} - (2\lambda_2 F_4)(p_2\lambda_2 J_2)F_5\{F_8 F_{10}(2\lambda_2 F_2) + F_1 F_9 F_{11}(a/X_1)\}$

$e_1 = F_2[(2\lambda_1 / X_1)a_{01}a_{04} + a_{04}\{(2\lambda_2 / X_2)a_{13} - (2\lambda_1 / X_1)(p_2\lambda_2 J_2)a_{10} + (2\lambda_2 / X_2)(p_2\lambda_2 J_2)F_5 F_9\}$
$+ (a/X_1)(2\lambda_1 F_4)\{a_{01} - (p_2\lambda_2 J_2)a_{10}\} + (a/X_1)(2\lambda_2 F_4)F_{11}\{a_{13} + (p_2\lambda_2 J_2)F_5 F_9\}]$

$e_2 = J_2[(2\lambda_2 / X_1)a_{04}a_{14} + (2\lambda_1 / X_1)(2\lambda_2 F_2)F_6 a_{04} + (a/X_1)(2\lambda_2 F_4)F_{11} a_{14} + (a/X_1)(2\lambda_2 F_2)(2\lambda_1 F_4)F_6]$

$e_3 = F_4[(a/X_1)\{a_{01} - (2\lambda_2 F_2)(p_1\lambda_1 + p_2\lambda_2)J_2 F_6 - (2\lambda_2 F_2)F_5 a_{15} - (p_2\lambda_2 J_2)a_{10}\}]$

$e_4 = F_7[(2\lambda_2 / X_1)a_{04}\{p_3\lambda_1 J_2 + (2\lambda_2 F_2)(p_1\lambda_1 + p_2\lambda_2)J_2 + (p_2\lambda_2 J_2)F_9\} + (2\lambda_2 / X_1)(2\lambda_2 F_2)$
$a_{15}(2\lambda_2 F_4)F_{10} + (a/X_1)(2\lambda_2 F_2)\{(p_1\lambda_1 + p_2\lambda_2)J_2 a_{15} + F_4 a_{11}(p_3\lambda_1 J_2) + F_4 F_{11}(2\lambda_2 F_2)$
$p_1(\lambda_1 + \lambda_2)J_2 a_{15} + F_4 F_{11}(2\lambda_2 F_2)p_1(\lambda_1 + \lambda_2)J_2 + F_4 F_{11} F_9 p_2\lambda_2 J_2\}]$

$e_5 = J_3[(2\lambda_1 / X_1)a_{04}\{2\lambda_2 F_2 F_7 + 2\lambda_1 F_2 a_{01} - 2\lambda_1 F_2 p_2\lambda_2 J_2 a_{10} - 2\lambda_2 F_2 p_2\lambda_2 J_2 F_7 F_8\} +$
$(2\lambda_2 / X_2)a_{04}\{p_3\lambda_1 J_2 F_7 + 2\lambda_1 F_2 a_{13} + p_2\lambda_2 J_2 F_7 F_9 2\lambda_1 F_2 p_2\lambda_2 J_2 F_5 F_9\} + (a/X_1)$
$\{4\lambda_1\lambda_2 F_4 F_7 a_{15} + p_2\lambda_2 J_2 a_{10}(4\lambda_1\lambda_2 F_2 F_4 + 2\lambda_2 F_4 F_{12}) + 2\lambda_2 F_4 F_{12} a_{01} -$
$4\lambda_2^2 F_2 F_4 F_{12} a_{07} 2\lambda_1\lambda_2 p_3 J_2 F_4 F_7 F_{11} + 4\lambda_2^2 F_2 F_4 F_{11} p_1(\lambda_1 + \lambda_2)J_2 F_7 + 4\lambda_1\lambda_2 F_2 F_4 F_{11} a_{13}$
$+ 4\lambda_2^3 p_2 J_2 F_4 F_{12} F_2 F_5 F_8 + 2\lambda_2^2 p_2 J_2 F_4 F_{11} F_9 (F_7 + 2\lambda_1 F_2 F_5)\}]$

$e_6 = F_9[(2\lambda_2 / X_1)(p_2\lambda_2 J_2)a_{04}a_{14} + (2\lambda_1 / X_1)(2\lambda_2 F_2)(p_2\lambda_2 J_2)F_6 a_{04} + (a/X_1)(p_2\lambda_2 J_2)$
$F_4\{(2\lambda_2)F_4 a_{14} + F_2 4\lambda_1\lambda_2 F_6 F_2\}]$

$e_7 = F_{12}[(a/X_1)(2\lambda_2 F_4)\{a_{01} - p_2\lambda_2 J_2 a_{10} - 2\lambda_2 F_2 (p_1(\lambda_1 + \lambda_2)J_2 + F_5 a_{15})\}]$

(i) When n=1, all the repair times and maintenance time follow exponential distribution.

(ii) When n=2, all the repair times and maintenance time follow 2-phase erlang distribution.

In case of Erlang distribution:

$p_{01} = 2\lambda_1 / X_1; \quad p_{02} = 2\lambda_2 / X_1; \quad p_{03} = a / X_1; \quad p_{10} = (nr_1 / 2\lambda_1 + 2\lambda_2 + nr_1)^n = F_1$

$p_{14} = (2\lambda_2)\sum_{j=0}^{n-1}(nr_1)^j / (2\lambda_1 + 2\lambda_2 + nr_1)^{j+1} = (2\lambda_2)F_2$

$$p_{15} = (2\lambda_1)\sum_{j=0}^{n-1}(nr_1)^j/(2\lambda_1 + 2\lambda_2 + nr_1)^{j+1} = (2\lambda_1)F_2$$

$$p_{20} = (nr_2/(p_1(\lambda_1+\lambda_2) + p_2\lambda_2 + p_3\lambda_1 + nr_2))^n = J_1$$

$$p_{21} = p_1(\lambda_1+\lambda_2)\sum_{j=0}^{n-1}(nr_2)^j/(p_1(\lambda_1+\lambda_2) + p_1\lambda_2 + p_3\lambda_1 + nr_2)^{j+1} = p_1(\lambda_1+\lambda_2)J_2$$

$$p_{24} = p_3\lambda_1\sum_{j=0}^{n-1}(nr_2)^j/(p_1(\lambda_1+\lambda_2) + p_2\lambda_2 + p_3\lambda_1 + nr_2)^{j+1} = p_3\lambda_1 J_2$$

$$p_{26} = p_2\lambda_2\sum_{j=0}^{n-1}(nr_2)^j/(p_1(\lambda_1+\lambda_2) + p_2\lambda_2 + p_3\lambda_1 + nr_2)^{j+1} = p_2\lambda_2 J_2$$

$$p_{30} = (nb/2\lambda_1 + 2\lambda_2 + nb)^n = F_3\,;\, p_{37} = (2\lambda_2)\sum_{j=0}^{n-1}(nb)^j/(2\lambda_1 + 2\lambda_2 + nb)^{j+1} = (2\lambda_2)F_4$$

$$p_{31} = (2\lambda_1)\sum_{j=0}^{n-1}(nb)^j/(2\lambda_1 + 2\lambda_2 + nb)^{j+1} = (2\lambda_1)F_4$$

$$p_{41} = \sum_{j=0}^{n-1}(nr_2)^n(nr_1)^j(n+j-1)!/(n-1)!(j!)(2\lambda_1 + 2\lambda_2 + nr_1 + nr_2)^{n+j} = F_5$$

$$p_{42} = \sum_{j=0}^{n-1}(nr_1)^n(nr_2)^j(n+j-1)!/(n-1)!(j!)(2\lambda_1 + 2\lambda_2 + nr_1 + nr_2)^{n+i} = F_6$$

$$p_{45} = 2(\lambda_1+\lambda_2)\sum_{j=0}^{n-1}(nr_1)^j(nr_2)^j(2j)!/(j!)^2(2\lambda_1 + 2\lambda_2 + nr_1 + nr_2)^{2j+1} = F_7$$

$$p_{50} = 1\,;\quad p_{62} = (2nr_2/2\lambda_1 + 2\lambda_2 + 2nr_2)^n = F_8$$

$$p_{64} = 2(\lambda_1+\lambda_2)\sum_{j=0}^{n-1}(2nr_2)^j/(2\lambda_1 + 2\lambda_2 + 2nr_2)^{j+1} = F_9$$

$$p_{73} = \sum_{j=0}^{n-1}(nr_2)^n(nb)^j(n+j-1)!/(n-1)!(j!)(2\lambda_1 + 2\lambda_2 + nr_2 + nb)^{n+j} = F_{10}$$

$$p_{72} = \sum_{j=0}^{n-1}(nr_2)^j(nb)^n(n+j-1)!/(n-1)!(j!)(2\lambda_1 + 2\lambda_2 + nr_2 + nb)^{n+j} = F_{11}$$

$$p_{75} = 2(\lambda_1+\lambda_2)\sum_{j=0}^{n-1}(nr_2)^j(nb)^j(2j)!/(j!)^2(2\lambda_1 + 2\lambda_2 + nr_2 + nb)^{2j+1} = F_{12}$$

$$\mu_0 = 1/X_1;\mu_1 = F_2;\mu_2 = J_2;\mu_3 = F_4;\mu_4 = F_7;\mu_5 = \sum_{j=0}^{n-1}(1/nr_3) = J_3$$

$$\mu_6 = F_9;\mu_7 = F_{12}$$

$$\bar{a}_{24} = 1 - p_{24}p_{42} = \{1-(p_3\lambda_1 J_2)F_6\} = a_{01};\bar{a}_{37} = 1 - p_{37}p_{73} = \{1-(2\lambda_2 F_4)F_{10}\} = a_{04}$$

$$\bar{a}_{26} = 1 - p_{26}p_{62} = \{1-(p_2\lambda_2 J_2)F_8\} = a_{15};\bar{a}_{14} = 1 - p_{14}p_{41} = \{1-(2\lambda_2 F_2)F_5\} = a_{14}$$

$$p_{21}p_{42} + p_{41} = (p_1(\lambda_1+\lambda_2)J_2)F_6 = a_{07};p_{42}p_{64} + p_{62} = F_6F_9 + F_8 = a_{10}$$

$$p_{21} + p_{41}p_{24} = p_1(\lambda_1+\lambda_2)J_2 + F_5(p_3\lambda_1 J_2) = a_{13}$$

In case of exponential distribution:

$p_{10} = r_1 / X_2;\quad p_{14} = 2\lambda_2 / X_2;\quad p_{15} = 2\lambda_1 / X_2;\quad p_{20} = r_2 / X_7;\quad p_{21} = p_1(\lambda_1 + \lambda_2) / X_7$

$p_{24} = p_3\lambda_1 / X_7;\quad p_{26} = p_2\lambda_2 / X_7;\quad p_{30} = b / X_3;\quad p_{31} = 2\lambda_1 / X_3;\quad p_{37} = 2\lambda_2 / X_3$

$p_{41} = r_2 / X_4;\quad p_{45} = 2(\lambda_1 + \lambda_2) / X_4;\quad p_{42} = r_1 / X_4;\quad p_{50} = 1;\quad p_{62} = 2r_2 / X_5$

$p_{64} = 2(\lambda_1 + \lambda_2) / X_5;\quad p_{73} = r_2 / X_6;\quad p_{72} = b / X_6;\quad p_{75} = 2(\lambda_1 + \lambda_2) / X_6$

$\mu_0 = 1/X_1;\quad \mu_1 = 1/X_2;\quad \mu_3 = 1/X_3;\quad \mu_4 = 1/X_4;\quad \mu_5 = 1/r_3;\quad \mu_6 = 1/X_5;\quad \mu_7 = 1/X_6$

$\bar{a}_{24} = 1 - p_{24}p_{42} = \{1 - (p_3\lambda_1 / X_7)(r_1 / X_4)\} = a_{02}$

$\bar{a}_{37} = 1 - p_{37}p_{73} = \{1 - (2\lambda_2 / X_3)(r_2 / X_6)\} = a_{05}$

$\bar{a}_{26} = 1 - p_{26}p_{62} = \{1 - (p_2\lambda_2 / X_7)(2r_2 / X_5)\} = a_{20}$

$\bar{a}_{14} = 1 - p_{14}p_{41} = \{1 - (2\lambda_2 / X_2)(r_2 / X_4)\} = a_{18}$

$p_{21}p_{42} + p_{41} = \{\{p_1(\lambda_1 + \lambda_2) / X_7\}(r_1 / X_4) + (r_2 / X_4)\} = a_{08}$

$p_{42}p_{64} + p_{62} = \{(r_1 / X_4)\{2(\lambda_1 + \lambda_2) / X_5\} + 2r_2 / X_5\} = a_{11}$

$p_{21} + p_{41}p_{24} = \{\{p_1(\lambda_1 + \lambda_2) / X_7\} + (r_2 / X_4)(p_3\lambda_1 / X_7)\} = a_{16}$

In case of 2-phase Erlang distribution:

$p_{10} = (2r_1 / Y_1)^2;\quad p_{14} = (2\lambda_2 / Y_1) + (4\lambda_2 r_1 / Y_1^2) = a_{23};\quad p_{15} = (2\lambda_1 / Y_1) + (4\lambda_1 r_1 / Y_1^2) = a_{24}$

$p_{20} = (2r_2 / Y_6^2);\quad p_{21} = \{p_1(\lambda_1 + \lambda_2) / Y_6\}\{1 + (2r_2 / Y_6)\} = a_{25};\quad p_{24} = (p_3\lambda_1 / Y_6)\{1 + (2r_2 / Y_6)\} = a_{26}$

$p_{26} = (p_2\lambda_2 / Y_6)\{1 + (2r_2 / Y_6)\} = a_{27};\quad p_{30} = (2b / Y_2)^2;\quad p_{31} = (2\lambda_1 / Y_2) + (4\lambda_1 b / Y_2^2) = a_{28}$

$p_{37} = (2\lambda_2 / Y_2) + (4\lambda_2 b / Y_2^2) = a_{29};\quad p_{41} = (2r_2^2 / Y_3^3) + (16r_1 r_2^2 / Y_3^3) = a_{30}$

$p_{42} = (2r_1^2 / Y_3^3) + (16r_1^2 r_2 / Y_3^3) = a_{31};\quad p_{45} = 2(\lambda_1 + \lambda_2)[(1/Y_3) + (8r_1 r_2 / Y_3^3)] = a_{32};\quad p_{50} = 1$

$p_{62} = (2r_2 / Y_4^2);\ p_{64} = 2(\lambda_1 + \lambda_2)[(1/Y_4) + (4r_2 / Y_4^2)] = a_{33}$

$p_{73} = (2r_2 / Y_5)^2 + (16r_2^2 b / Y_5^3) = a_{34};\quad p_{72} = (2b / Y_5)^2 + (16b^2 r_2 / Y_5^3) = a_{35}$

$p_{75} = 2(\lambda_1 + \lambda_2)[(1/Y_5) + (8r_2 b / Y_5^3)] = a_{36}$

$\bar{a}_{24} = 1 - p_{24}p_{42} = 1 - (p_3\lambda_1)\{(Y_6 + 2r_2) / Y_6^2\}\{(2r_1^2 / Y_3^2) + (16r_1 r_2 / Y_3^3)\} = a_{03}$

$\bar{a}_{37} = 1 - p_{37}p_{73} = 1 - \{(2\lambda_2 / Y_2) + (4\lambda_2 b / Y_2^2\}\{(2r_2^2 / Y_5^2) + 16r_2^2 b / Y_5^3)\} = a_{06}$

$\bar{a}_{26} = 1 - p_{26}p_{62} = 1 - (p_2\lambda_2 / Y_6)\{1 + (2r_2 / Y_6)\}\{4r_2^2 / Y_4^2\} = a_{21}$

$\bar{a}_{14} = 1 - p_{14}p_{41} = 1 - 2\lambda_2\{1/Y_1 + 2r_1 / Y_1^2\}\{(2r_2^2 / Y_3^2) + (16r_1 r_2 / Y_3^3)\} = a_{19}$

$p_{21}p_{42} + p_{41} = p_1(\lambda_1 + \lambda_2)\{1/Y_6 + 2r_2 Y_6^2\}\{(2r_1^2 / Y_3^2) + (16r_1 r_2 / Y_3^3)\} + \{(2r_2^2 / Y_3^2) + (16r_1 r_2 / Y_3^3)\} = a_{09}$

$p_{42}p_{64} + p_{62} = \{(2r_1^2 / Y_3^2) + (16r_1 r_2 / Y_3^3)\}\{(2\lambda_1 + 2\lambda_2) / Y_4) + (r_2(8\lambda_1 + 8\lambda_2) / Y_4^2)\} + 4r_2^2 / Y_4^2 = a_{12}$

$p_{21} + p_{41}p_{24} = p_1(\lambda_1 + \lambda_2)\{1/Y_6 + 2r_2 Y_6^2\} + \{(2r_2^2 / Y_3^2) + (16r_1 r_2 / Y_3^3)\}\{(p_3\lambda_1)(Y_6 + 2r_2) / Y_6^2\} = a_{17}$

$\mu_0 = 1/X_1;\ \mu_1 = (1/Y_1) + (2r_1 / Y_1^2);\qquad \mu_2 = (1/Y_6) + (2r_2 / Y_6^2)$

$\mu_3 = (1/Y_2) + (2b / Y_2^2);\qquad \mu_4 = (1/Y_3) + (8r_1 r_2 / Y_3^3);\qquad \mu_5 = (1/r_3)$

$\mu_6 = (1/Y_4) + (4r_2 / Y_4^2);\qquad \mu_7 = (1/Y_5) + (8r_2 b / Y_5^2)$

12. Cost Analysis

The cost benefit analysis of the system can be carried by considering the expected busy period of the repairman in repair in (0,t]. Therefore,

G(t) = expected revenue earned by the system in (0,t] - expected repair cost of the repair facility in (0,t]

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$$= C_1\mu_{up}(t) - C_2\mu_b^1(t) - C_3\mu_b^2(t) - C_4\mu_b^3(t)$$

Here:

$$\mu_{up}(t) = \int_0^t A_0(t)dt\,;\ \mu_b^1(t) = \int_0^t B_0^1(t)dt\,;\ \mu_b^2(t) = \int_0^t B_0^2(t)dt\,;\ \mu_b^3(t) = \int_0^t B_0^3(t)dt$$

The expected profit per unit of time in steady state is

$$G = \lim_{t\to\infty}\frac{G(t)}{t} = \lim_{s\to 0} s^2 G^*(s) = C_1\mu_{up}(t) - C_2\mu_b^1(t) - C_3\mu_b^2(t) - C_4\mu_b^3(t)$$

where C_1 is the revenue per unit up time and C_2, C3 and C4 are the repair cost, scheduled maintenance cost and emergent repair cost, respectively.

13. Graphical Representation

Figure 2 shows the behavior of the mean- time-to-system-failure of the Tippler-Conveyor system with respect to λ_1, which is the failure rate of Tippler, for varying values of λ_2 which is the failure rate of Conveyor belt. Here, repair time distribution of conveyor belt is taken to be exponentially distributed. From the graph it can be observed that MTSF of the Tippler-Conveyor system decreases as λ_2 increases. Same case arises in case of availability and expected cost or profit as shown in figure 3 & figure 4.

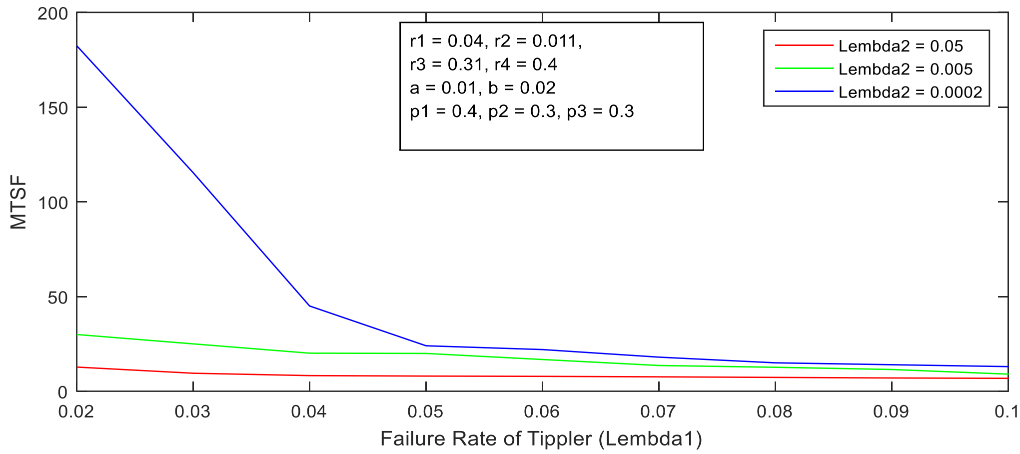

Figure-2. Effect of λ_1 on MTSF for varying values of λ_2

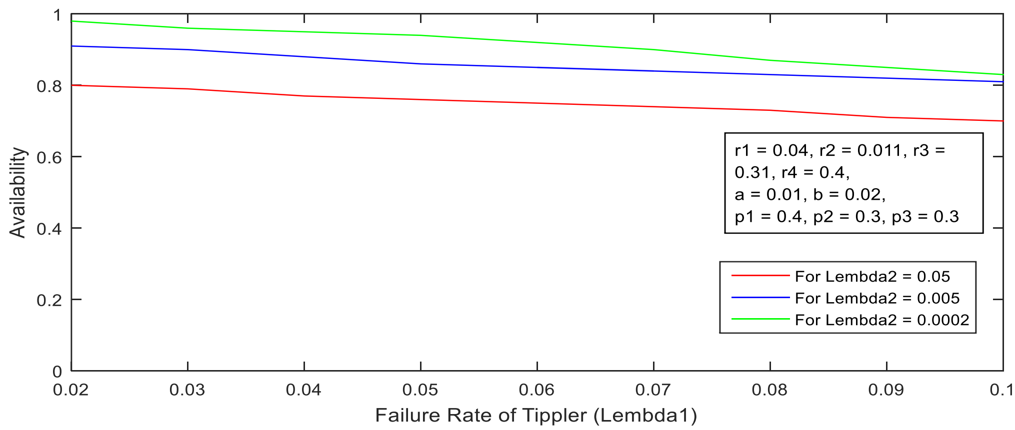

Figure-3. Effect of λ_1 on Availability for varying values of λ_2

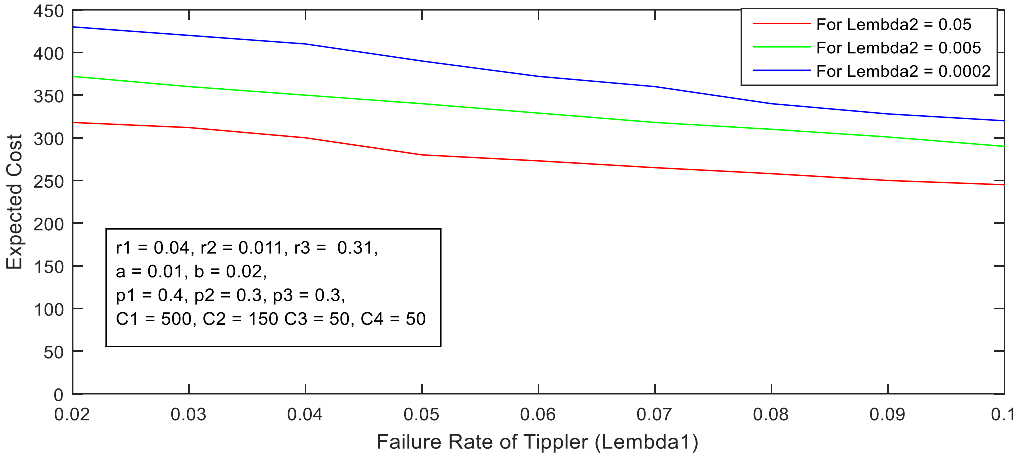

Figure-4. Effect of λ_1 on Expected Cost for varying values of λ_2

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