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EXPERIMENTAL INVESTIGATION OF CYLINDRICAL ROLLER BEARING INNER AND OUTER RACE WITH DISTRIBUTED DEFECTS

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Abstract
Many rotating machines ‘performance is directly depends upon the bearing rotating system. One of the most vulnerable parts of mechanical systems is the bearing, and they are essential to its effective operation. In order to study bearing failure, different components of the bearing are typically given fabricated flaws to study, and vibration signature tools are used to analyses the flaws and track the bearing's state. Thus, vibration responses are used to monitor the condition of rolling bearings, which is a major concern. This research work presents an experimental examination of the vibration behaviour of healthy and faulty cylindrical roller bearings with waviness as a distributed defect under high speed and dynamic radial load. In the presence of defects with different waviness orders, the effects on peak amplitude, defect frequency amplitude, Root Mean Square (RMS) value, peak values, and peak to peak values were examined under operative conditions. A variable frequency drive is connected to the motor to control the speed. A pulley belt mechanism is used for the application of radial load on the shaft-bearing mechanism. This research will assist practicing engineers in determining the severity of vibrations caused by malfunctioning rolling element bearings.

Keywords: Rotating Machinery, Distributed Defects, Waviness, Wave Passage Frequency, Roller Bearing

I. Introduction

To accomplish the operational goals, numerous mechanisms and machines employ rolling elements (such as) bearings (REBs). It is imperative for REBs to keep track of rolling element bearings' health as a result of vibration analysis method because they are used in mechanical components where they must operate dependably and efficiently. The most frequent reason for bearing faults is surface damage to the raceways or rolling components of the bearings. These surface flaws can be distinguished into two categories: localized flaws and distributed flaws.[1] Cracks, pits and spalls, dents, scratches, bump flanking, and fault size prediction are examples of localized defects. Distributed defects include surface waviness, misaligned races, and off-sized rolling elements.[2]–[9] Localized flaws are caused by poor design, poor installation, lack of lubrication, and excessive fatigue loading, whereas distributed faults are brought about by poor design and manufacturing inaccuracies.[9].When a system is not stationary; the impact of local faults grows more severe and expands to create distributed defects. Every time a rolling element interacts with a fault on the surface of a bearing element, a series of impacts are created, which cause the bearing system to be excited. As a result, the level of vibration has been significantly raised. The rise in vibration level is caused by non-uniform forces acting on raceways and the rolling element and due to distributed faults. The study of dispersed flaws in REBs has therefore been highly helpful for bearing quality checks and health monitoring .[10]

II. Literature

Numerous researches have examined the study of vibration caused by bearings with dispersed defects. Patel et al. provide a summary of the research on bearing fault diagnostics for bearings with rolling elements having localized and spread flaws and various non-linear factors..[11]An experimental examination was conducted by Tallian and Gustafsson to forecast the REB's vibrations with the surface waviness of the bearing elements. The experimental findings showed that specific waviness orders controlled the vibration amplitude and generation caused by raceway compliance, variable contact, and bearing geometrical flaws.[12] Wardle suggested using a theoretical vibration theory with linear ball-race interface conformity to predict the dynamic response and harmonics of a ball bearing with thrust pressure. The surface waviness is created using a sinusoidal function. The author concluded that the outer race waviness is responsible for vibrations at the harmonics of the outer race ball passage frequency. [13], [14] Akturk has investigated how the waviness of the bearing surface affects the rotor's response to vibration. He observed that the frequency spectrum includes distinct frequency components for the outer raceway, inner raceway, and ball waviness orders of surface waviness.[15] Tandon and Chaudhary's theoretical model has been used to estimate the vibration characteristics of bearings with rolling elements in rotor bearing assemblies to distributed defects under radial load. The model suggests a continuous spectrum with specific frequency components for each level of waviness. [16] Ono and Okada investigated how the frequency response of the shaft bearing system is affected by the bearing outer race waviness in conjunction with shaft imbalance and radial clearance. [17]

Harsha and Kankar proposed a theoretical model to predict the resilience of a ball bearing system with surface waviness on the raceway. It was also investigated how ball counts affected the system’s stability. [18]Harsha et al. represent the analytical model for the impact of surface waviness on the nonlinear vibrations of a rotor bearing system. The surface waviness on the bearing raceway was also modeled in their model using the sinusoidal displacement excitation model. The authors concluded that when the number of balls and waves is equal, the strongest vibrations occur in the outer ring of waviness.[19] A nonlinear analytical model was given by Jang and Jeong to examine the vibration response of a ball bearing with waviness on the rotor. They have noticed the fascinating frequencies and their harmonics that are produced by the different types of waviness in rolling elements.[20] In order to anticipate the primary Jang et al. created a nonlinear dynamic model to analyse the vibration frequencies and their harmonics of a robust rotor mechanical system with surface waviness on the raceway and rolling parts.. They formulated the rotational moment and inertial forces in their model. Surface waviness is created using a sinusoidal function.[21]Kulkarni et al. studied the vibration spectrum on the outer race of the bearing caused by a single roughness fault under pure radial load. Investigations have been conducted into how roughness size, speed, and load affect vibration response.[22]Wang et al. studied the 4-DOF dynamics equations for a rotor roller bearing system and found the nonlinear bearing forces of a roller bearing under four-dimensional loads. The findings demonstrate that as the rotational speed increases, the system is susceptible to instability brought on by quasi-periodic bifurcation, periodic-doubling bifurcation, and chaotic paths.[6]Babu et al. constructed a vibration model in order to predict the nonlinear vibration of an oiled stiff rotor bearing system with surface waviness on the raceway and rolling parts, taking the effect of friction moment into consideration.[23] Goverthan et al. investigate the roller bearing with distributed defects, considering the combined static and dynamic loading. The author came to the conclusion that the

order and amplitude of waviness had an impact on the frequency and amplitude spectra.[24] Shah et al. built a dynamic model of a deep groove ball bearing taking lubrication into account in order to investigate the effects of controlling parameters such as static loading, shaft speed of rotation, and waviness order on magnitude of waviness defect frequency. [25]
A review of the literature reveals that detailed work has been done on both defective and healthy bearings. However, the authors of this research discovered that there is a lack of vibration studies for bearings with distributed faults on bearing elements at high speed experimentally. There is also less work found on analysis o statical parameters of time domain frequency analysis. As a result, the present study contains a thorough analysis of the vibrations produced by rolling element bearings under various loading circumstances and high speed in the presence of distributed faults on the inner and outer races of the bearing. Under operational conditions, the impacts on peak amplitude, defect frequency amplitude, Root Mean Square (RMS) value, peak values, and peak to peak values were investigated in the presence of defects with various waviness orders.

III. Experimental / Computational details:

The line diagram of the test rig developed for the experiment is shown in Fig. 1.The motor is assemble to the shaft by a flexible jaw coupling. The experiment set up is equipped through a single-phase AC motor. Shaft with rotor and loading mechanism supported on two fixed specially designed bearing housings. The bearing assembly is made such that it is simple to replace the bearing for different test options. During the experiments, a healthy bearing was fixed in the left-side bearing housing, and it remained there throughout the experiment. Assembly of bearing with various selected defects mounted on the right side bearing housing. The rotational speed of an electric motor is adjusted and controlled using a variable frequency drive (VFD). A laser tachometer was used to measure the shaft's rotational speed. The mechanism's radial load is applied using a pulley belt system. Through an associated hook and hanger configuration, the weight is suspended at the base of the machine. On an M.S. plate with four vibration-absorbing pads, the entire test setup was mounted. The experimental setup's modular construction makes quick bearing removal and replacement possible.

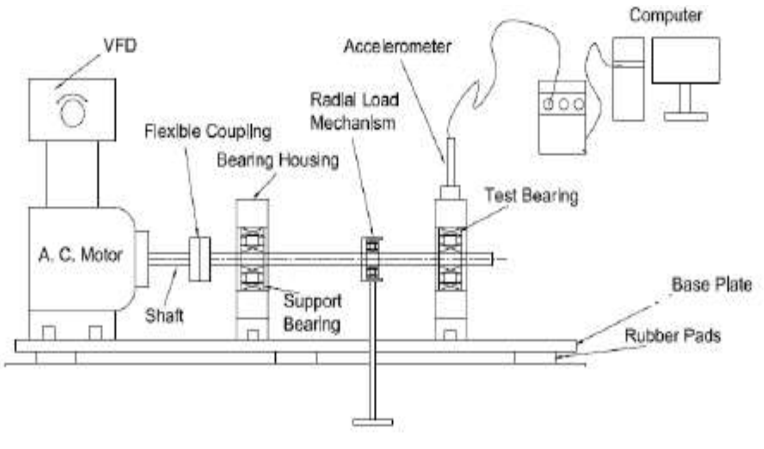


Figure 1. Line Diagram of the experimental set up

In this study, polymer cage test bearings (FAG-NJ205ECP) were used since they were simple to assemble and disassemble and allowed for the easy creation of fictitious faults in the bearing components. The test bearing operational conditions employed in this investigation are listed in Table I. The waviness of different orders 12, 13, and 14 developed on inner and outer races of the bearing. Here, the acceleration spectra of vibration caused by defective test bearings with distributed faults on the inner race and outer raceway are analyzed at high speed with and without loading conditions. The images of the waviness order of the inner and outer races are shown in fig. 2

(a) (b)

Figure 2. Distributed Defect On (a) inner race (b) outer race

Table 1 Operating Parameters

Inner Race Waviness,(N_w^{ir})	12,13,14
Outer Race Waviness, (N_w^{out})	12,13,14
Speed of shaft, (N)	2000 rpm (For all set ups)
Radial load applied on test bearing,(W)	No load and with 11 kg

The different characteristic defect frequency equations are listed in table II. The different defect frequencies and their side band frequencies are shown in table III with high speed shaft.

Table 2 Characteristic frequency equations [11][26]

Shaft Frequency	$f_s = \frac{2\Pi N}{60}$
Varying compliance frequency (VC)	$VC = f_c \times N$
Cage Frequency (FTF)	$f_c = \frac{f_s}{2}[1-(\frac{d}{D})\cos\alpha]$
Outer Race Defect Frequency (BPFO)	$f_{or} = \frac{Nf_s}{2}[1-(\frac{d}{D})\cos\alpha]$
Inner Race Defect Frequency (BPFI)	$f_{ir} = \frac{Nf_s}{2}[1+(\frac{d}{D})\cos\alpha]$
Rolling Element Defect Frequency (BSF)	$f_r = \frac{Df_s}{d}[1-(\frac{d^2}{D^2})\cos\alpha^2]$

Table 3 Theoretical calculated frequency

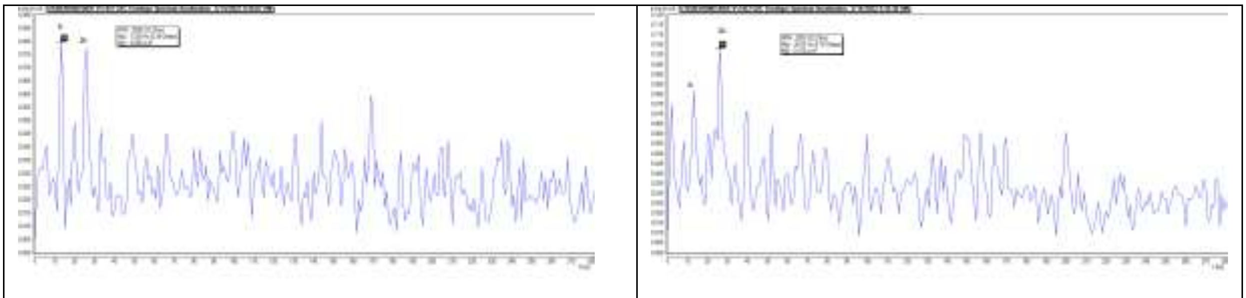
Sr. No	Frequency	@ Speed 2000 (rpm)
1	Shaft Frequency (rad/s)	209.33
2	Shaft Frequency (Hz)	33.33
3	Cage Frequency (Hz)	13.42
4	Inner Race Defect Frequency (Hz)	258.87
5	Outer Race Defect Frequency (Hz)	174.46
6	Roller Element Frequency (Hz)	164.62
7	Inner Race Defect Frequency + Shaft Frequency (Hz)	292.21
8	Inner Race Defect Frequency – Shaft Frequency (Hz)	225.54
9	Outer Race Defect Frequency + Shaft Frequency (Hz)	207.79
10	Outer Race Defect Frequency – Shaft Frequency (Hz)	141.13
11	Roller Defect Frequency + Shaft Frequency (Hz)	197.95
12	Roller Defect Frequency - Shaft Frequency (Hz)	131.28

3.1 Experimental Methodology:

Bearing manufacturing flaws are what produce the surface waviness on the bearing raceway. Races typically exhibit waviness in the form of peaks and valleys with variable heights. To express the periodic waviness, a sinusoidal wave is used to simulate the bearing race's waviness.. The wave number refers to the number of waves per race's perimeter, also called the waviness order. Because the rollers move with cage speed and the inner raceway surface waviness fault moves constantly with shaft rotational speed, this interaction occurs in loaded zone and unloaded zone. While the rolling parts are in motion at the cage's rotational speed, the outer raceway's waviness remains stationary. The vibration frequency spectra are shown below with different waviness orders for the bearing inner race and outer race. The experiments are run initially with a healthy bearing, and then the inner and outer races with surface waviness and dispersed flaws are tested on a design test rig. Different waviness orders, 12, 13, and 14, are tested on the inner and outer races at high speeds of 2000 rpm, both with and without an 11 kg load.

3.2 Vibration Response of Healthy Bearing Elements

The vibration response of a healthy bearing with load and without load conditions is shown in fig. 3.The peak amplitude is visible at cage frequency or harmonics of cage frequency. Due to the length of the research paper, it is challenging to depict each dynamic behavior of a bearing with faults under various operating situations.

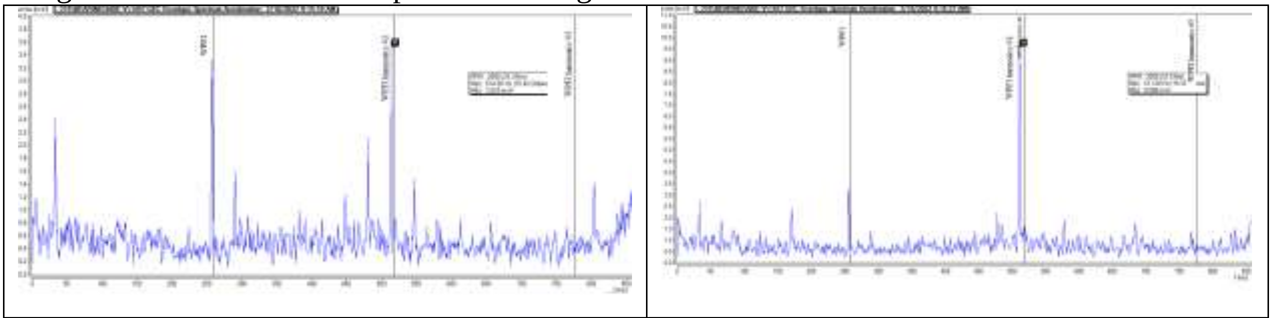


(a) (b)

Figure 3. Vibration response of healthy bearing at 2000 rpm (a) without load (b) with load 11 kg

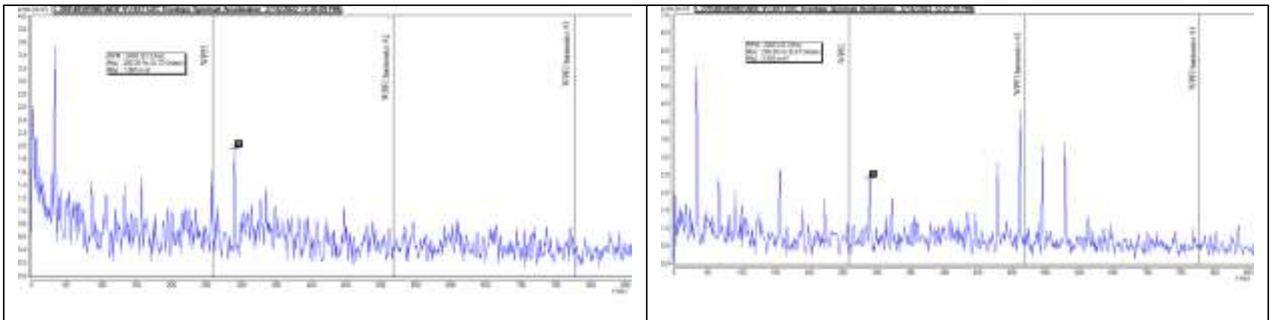
3.3 Vibration Response of Defective Bearing Elements

At 2000 rpm, the vibration spectra of the inner race having waviness order 13 and 14 are represented in fig. 4 and fig. 5, respectively. At high speed, 2000 rpm, the vibration spectra of the outer race having waviness order 13 are represented in fig. 6.



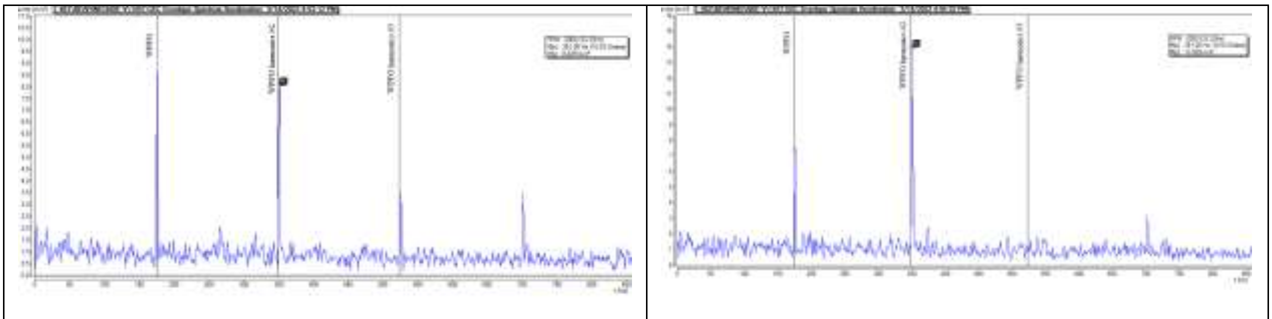
(a) (b)

Figure 4. Vibration response of defective bearing for Nw^{in} =13 at 2000 rpm (a) without load (b) with load



(a) (b)

Figure 5. Vibration response of defective bearing for Nw^{in} =14 at 2000 rpm (a) without load (b) with load



(a) (b)

Figure 6. Vibration response of defective bearing for Nw^{out} =13 at 2000 rpm (a) without load (b) with load

IV. Result and Discussion:

4.1 The effect of waviness orders on peak amplitude:

The peak amplitude of vibration is shown in fig. 4 and fig. 6 for different operating conditions. Table IV lists the pertinent waviness orders, respective peak amplitudes, and harmonies in the bearing spectrum. The value of peak amplitude is found higher with load then without load for both the inner and outer raceway of the bearing. Vibration amplitude change is severe in the outer raceway when the number of waves and rollers are the same. This is also noticed in literature done by various researchers.

Table 4 Summary of Bearing Elements Waviness

Operating Condition	Bearing element	Waviness order (lobes/ circumference)	Peak Amplitude (m/s^2)	Bearing Spectrum Frequency
Without load @ 2000 rpm	Outer Race	12	4.611	$2f_{or}$
		13	8.029	$2f_{or}$
		14	1.948	$2f_{or}$
	Inner Race	12	10.22	$f_{ir}-f_s$
		13	3.523	$2f_{ir}$
		14	1.965	$f_{ir}+f_s$
With load at @ 2000 rpm	Outer Race	12	3.637	$2f_{or}$
		13	13.928	$2f_{or}$
		14	2.874	$2f_{or}$
	Inner Race	12	17.8	$f_{ir}-f_s$
		13	9.586	$2f_{ir}$
		14	2.403	$f_{ir}+f_s$

The outcomes are consistent with [13], [14], [19] discovery of amplitude maxima at sidebands in the vibration spectrum of bearings with waviness orders of (N_w^{ir}) = $N_b \pm 1$ in case of the inner race. When the waviness orders and the number of rollers are the same, the amplitude maxima at the harmonics of the wave passage frequency of the inner race are discovered. The amplitude peak is noticed at harmonics of the wave passage frequency of the outer race, irrespective of waviness orders. This is consistent with the findings of researchers [22]–[24] in the published literature.

4.2 The effect of waviness orders on defect frequency amplitude:

The defect frequency for inner race waviness is called wave passage frequency for inner race (WPFI), while the defect frequency for outer race waviness is called wave passage frequency for outer race (WPFO). For different waviness orders, vibration amplitudes at defect frequencies for outer and inner raceway are listed in table V. The defect frequency for the outer raceway is 174.46

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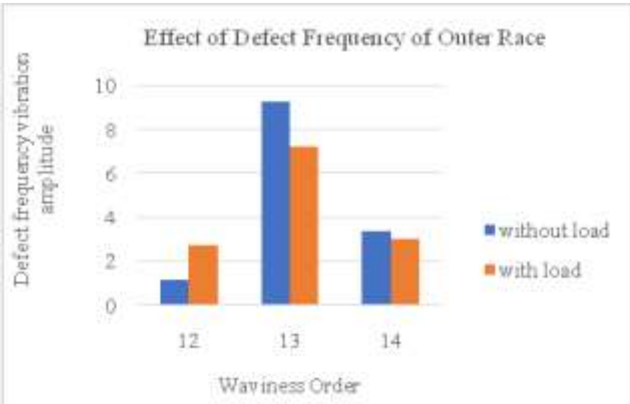
Hz, whereas that for the inner raceway is 258.87 Hz. Different fault frequencies and their harmonics are clearly visible in fig. 4-fig. 6 with their different operating conditions.

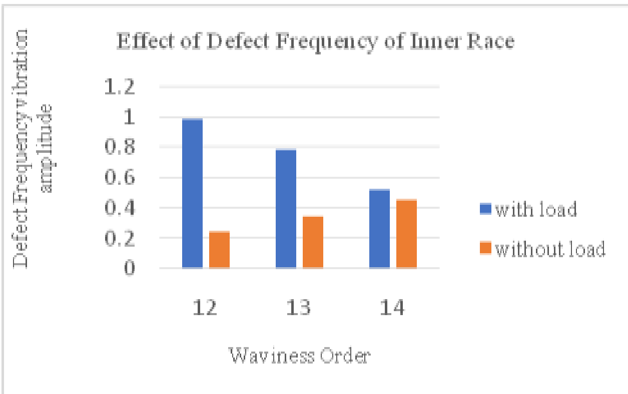
Figure 7 depicts the analysis of defect frequency for bearing elements with varying waviness orders, with and without load for bearing races. The maximum vibration amplitude is found for the outer race with an equal number of waviness orders and rollers. In loading conditions, the amplitude of the inner race defect frequency decreases as the number of waves increases, whereas in no load it is the opponent. This research output is in line with the published literature. [19]

Table 5 Summary of bearing elements defect frequency vibration amplitude

Operating Condition	Bearing element	Waviness order (lobes/ circumference)	Defect frequency vibration amplitude (m/s^2)
Without load @ 2000 rpm	Outer Race	12	1.153
		13	9.217
		14	3.361
	Inner Race	12	0.245
		13	0.348
		14	0.455
With load at @ 2000 rpm	Outer Race	12	2.744
		13	7.21
		14	3.004
	Inner Race	12	0.978
		13	0.778
		14	0.519



(a)



(b)

Figure 7. Effect of defect frequency on (a) Outer Race (b) Inner Race

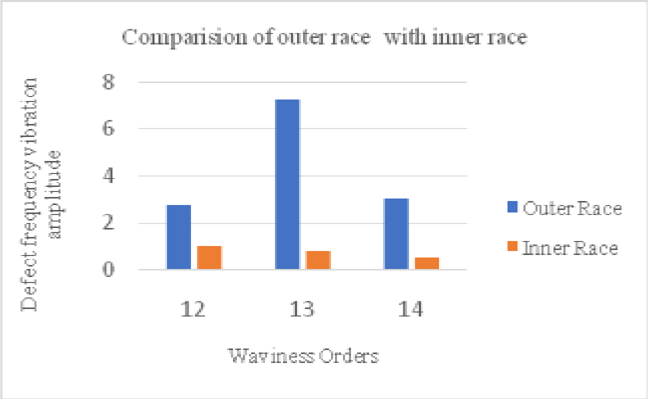


Figure 8. Comparison of outer race with inner race

The vibration amplitude of the inner and outer races for different waviness orders is compared in fig. 8.The vibration amplitude of the outer race is found to be higher at defect frequencies compared to the inner race. This is in line with the many published literature of research also.[16][27]

4.3 The effect of waviness orders on RMS, Peak, Peak to Peak Value:

The different aspects of the vibration signal, such as RMS, peak, & peak to peak, are analyzed to recognize bearing flaws. A root mean square value is defined as the square root of the sum of squares of all deviation values divided by the number of samples, where Xi is the i^{th} data point, for a dispersed dataset with N number of data points and X_m as an arithmetic mean

$$RMS = \sqrt{\frac{\sum_{i=1}^{n}(X_i - X_m)^2}{N}}$$

Since it directly correlates with the energy content of the vibration profile and, consequently, the vibration's capacity for causing damage, the RMS (root mean square) value is typically the most helpful.
The peak value is the maximum value in the signal. The Peak Level of the discrete time signal is:
Peak = maximum (X)
Peak to Peak is defined as the range between the maximum and minimum value in the signal.
Peak to Peak = maximum (x) – minmum (x)
For different waviness orders, the values of RMS, peak, and peak to peak for the outer race and inner race are listed in table VI.

Table 6 Summary of bearing elements vales of RMS, peak, and peak to peak

Operating Condition	Bearing element	Waviness order	RMS Value	Peak	Peak to Peak
Without load @ 2000 rpm	Outer Race	12	14.46	36.02	71.08
		13	4.91	21.27	39.04
		14	19.29	39.81	77.53
	Inner Race	12	9.94	31.31	60.02
		13	7.33	28.78	54.89
		14	5.62	23.24	43.19
With load at	Outer	12	20.01	40.24	79.5
		13	7.29	28.97	49.5

@ 2000 rpm	Race	14	22.79	46.19	90.98
	Inner Race	12	15.61	45.39	86.63
		13	9.66	35.16	66.97
		14	7.69	30.02	55.3

(a) (b)

Figure 9. Effect of RMS on (a) Inner Race (b) Outer Race

Fig. 9 gives RMS versus waviness orders for both conditions of load at a speed of 2000 rpm. The RMS value is decreased with the increase of waviness with load and without load in the case of the inner race, while in the case of the outer race waviness, the RMS value is lower when the number of rollers and waviness orders are the same**.**

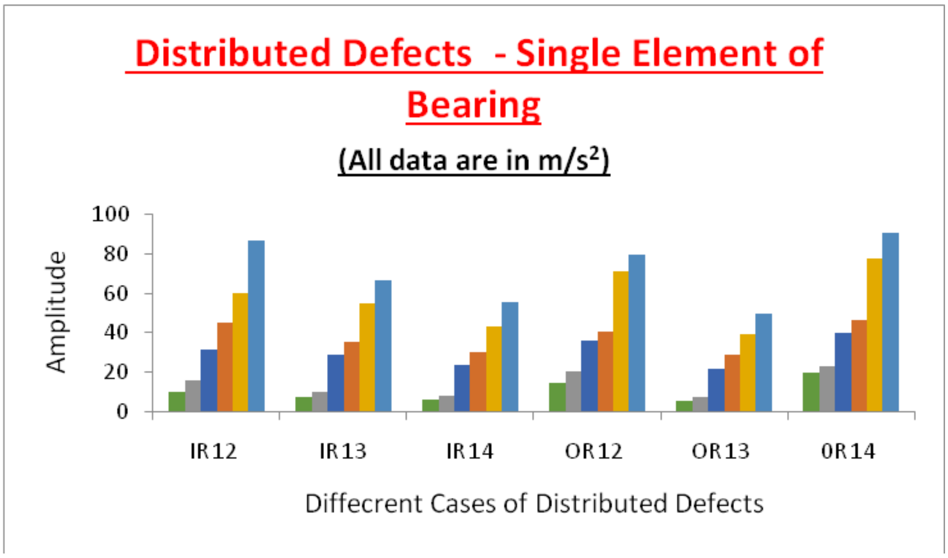


Figure 10. Effect of RMS, Peak, Peak to Peak on Inner Race and Outer Race

Fig. 10 depicts a comparison of the amplitudes of the RMS, Peak, and Peak to Peak for various waviness orders of a bearing's inner and outer races. The amplitude variation trend of peak value and peak to peak value is the same as the RMS value for the inner and outer races. The peak to peak value gives better detect ability if the defected bearings are compared to the RMS and peak values of the defected bearings. The result is also consistent with the work done in previous literature.[28][29]

V. Conclusion

The results of a thorough experimental vibration investigation led to the following conclusions:

1. Irrespective of the waviness order on the outer race, peak amplitude is noticed at the wave passage frequency of the outer race or its harmonics. (WPFO).
2. In the inner race, when the number of rollers and waviness order are the same, peak amplitude is noticed at the wave passage frequency of inner race or its harmonics (WPFI), while in other cases it is noticed at the side band frequency of the inner race or its harmonics.
3. The vibration amplitude change is severe in the outer race when the number of waves and rollers are the same.
4. In loading conditions, the amplitude of the inner race defect frequency decreases as the number of waves increases, whereas in no load it is the opponent.
5. The RMS value is decreased with the increase of waviness with and without load in the case of the inner race, while in the case of the outer race, the RMS value is lower when the number of rollers and waviness orders are the same.
6. The amplitude of RMS, peak, and peak to peak is found higher with load compare to without load.
7. The peak to peak value increases the delectability of defective bearings compared to the RMS and peak values.

VI. Future Scope

The measurement of vibration amplitudes at the outer race and inner race flaws constitutes the entire scope of this research work. However, by adjusting these parameters, it is possible to do analysis on several defective outer races, inner races, and rollers, where the impact of the roller on both the inner and outer rings may generate excitation forces in the roller.

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