EVALUATION OF RELIABILITY CHARACTERISTICS OF A SYSTEM WITH PREVENTIVE MAINTENANCE

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Abstract

This study deals with Mean time to system failure, Steady state availability, Steady state busy period, Steady state, the expected frequency of preventive maintenance per unit time and Cost analysis of a two-unit cold standby redundant system with preventive maintenance are calculated by using Kolmogorov's forward equations method. The random failure occurs at random times which follow an exponential distribution and also the repair time are assumed to be exponentially distributed.

Keywords: Cost analysis, Steady state availability, Mean time to system failure (MTSF), Preventive maintenance (PM) and the Kolmogorov's forward equations method.

I. Introduction

Reliability is a vital for proper utilization and maintenance of any system. It involves technique for increasing system effectiveness through reducing failure frequency and maintenance cost minimization. We analyzed the system by using linear first order differential equations. Repairable systems receive maintenance actions that restore/renew system components when they fail. These actions change the overall makeup of the system.

II. Review of literature

Kuo-Hsiung wang, Ching- Chang Kuo have studied cost and probabilistic analysis of series systems with mixed standby components [1]. Khaled. M. El-said and Mohamed Salah El-Sherbeny have studied Evaluation of reliability and Availability characteristics of two different systems by using Linear first order differential equations [2]. El-said have studied Cost analysis of a system with preventive maintenance by using the Kolmogorov's forward equations method [3]. M.Y. Haggag have studied Cost analysis of two dissimilar unit cold stand by system with three states and preventive maintenance using linear first order differential equations [4]. Ibrahim Yusuf, Nafiu Hussaini have studied Evaluation of reliability and availability characteristics of 2 out of 3 standby system under a perfect repair condition [5]. Uba Ahmad Ali, Naziru Idris Bala and Ibrahim Yusuf have studied Reliability Analysis of a two dissimilar unit cold standby system with three modes using Kolmogorov forward equation method [6]. Ibrahim Yusuf have studied Comparison of some reliability characteristics between redundant systems requiring supporting units for their operations [7].

M.Y. Haggag and Ahmed Khayar have studied Cost analysis of a two dissimilar-unit cold stand by system with preventive maintenance by Kolmogorov's forward method [8]. U.A. Ali, Saminu I.Bala, Ibrahim Yusuf have studied Evaluation of mean time to system failure of a repairable 3 out of 4 system with online preventive maintenance [9]. Ibrhim Yusuf, Nafiu Hussaini, Bashir M. Yakasai have studied Some reliability measures of a Deteriorating system [10]. Ibrahim Yusuf have studied Comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation [11]. Ibrahim Yusuf, Bashir Yusuf and Saminu I Bala have studied Mean time to system failure analysis of a linear consecutive 3 out of 5 warm standby system in the presence of common cause failure [12]. Ibrahim Yusuf, K. Suleiman and Yusuf Bashir have studied Stochastic modelling and analysis of a repairable 2 out of 4 system [13]. Pradeep. K. Joshi, Ravindrasen and Chitaranjan Sharma have studied Reliability and Availability characteristics of a two unit stand by redundant system by linear differential equation (LDE) solution Technique [14]. Ibrahim Yusuf have studied reliability modelling of a parallel system with a supporting device and two types of preventive maintenance [15]. K. Suleiman, U.A. Ali, Ibrahim Yusuf, A.D.Koko, S.I. Bala have studied comparison between four dissimilar solar panel configurations[16].

III. Objectives of the study

i. Mean time to system failure.
ii. Steady state availability.
iii. Steady state busy period.
iv. Steady state, the expected frequency of preventive maintenance per unit time.
v. Cost analysis.

IV. The following assumptions are adopted for the system

1. The system consists of two similar units. Initially one unit is operative and the other unit is kept as cold standby.
2. Standby is switched to operative state in negligible time.
3. A repaired unit works as a good as new.
4. The system is down when both units are non-operative.
5. Each unit has two types of failure.

V. **Description of the system**

In this paper, the system consists of nine units and the following notations are adopted for the system:

α_1 constant failure rate of type I.

α_2 constant failure rate of type II.

β_1 constant repair rate of type I.

β_2 constant repair rate of type II.

$p_i(t)$ probability of the system at time t, $(t \geq 0)$ at state S_i.

λ constant rate for taking a unit into preventive maintenance.

δ constant rate end of preventive maintenance.

O the unit is operative.

S the unit is standby.

F_{R1} the failed unit is under repair of type I.

F_{R2} the failed unit is under repair of type II.

F_{W1} the failed unit is waited for repair of type I.

F_{w2} the failed unit is waited for repair of type II.

O_p the operative unit is under preventive maintenance.

S_p the standby unit is under preventive maintenance.

The system can take one of the following states:

$S_0(O,S)$, $S_1(F_{R1},O)$, $S_2(F_{R2},O)$, $S_3(F_{R1},O)$, $S_4(F_{R2},O)$, $S_5(F_{R1},F_{W2})$, $S_6(F_{R2},F_{W1})$, $S_7(F_{R1},F_{W1})$, $S_8(F_{R2},F_{W2})$, $S_9(O_p,S_p)$.

VI. System configuration

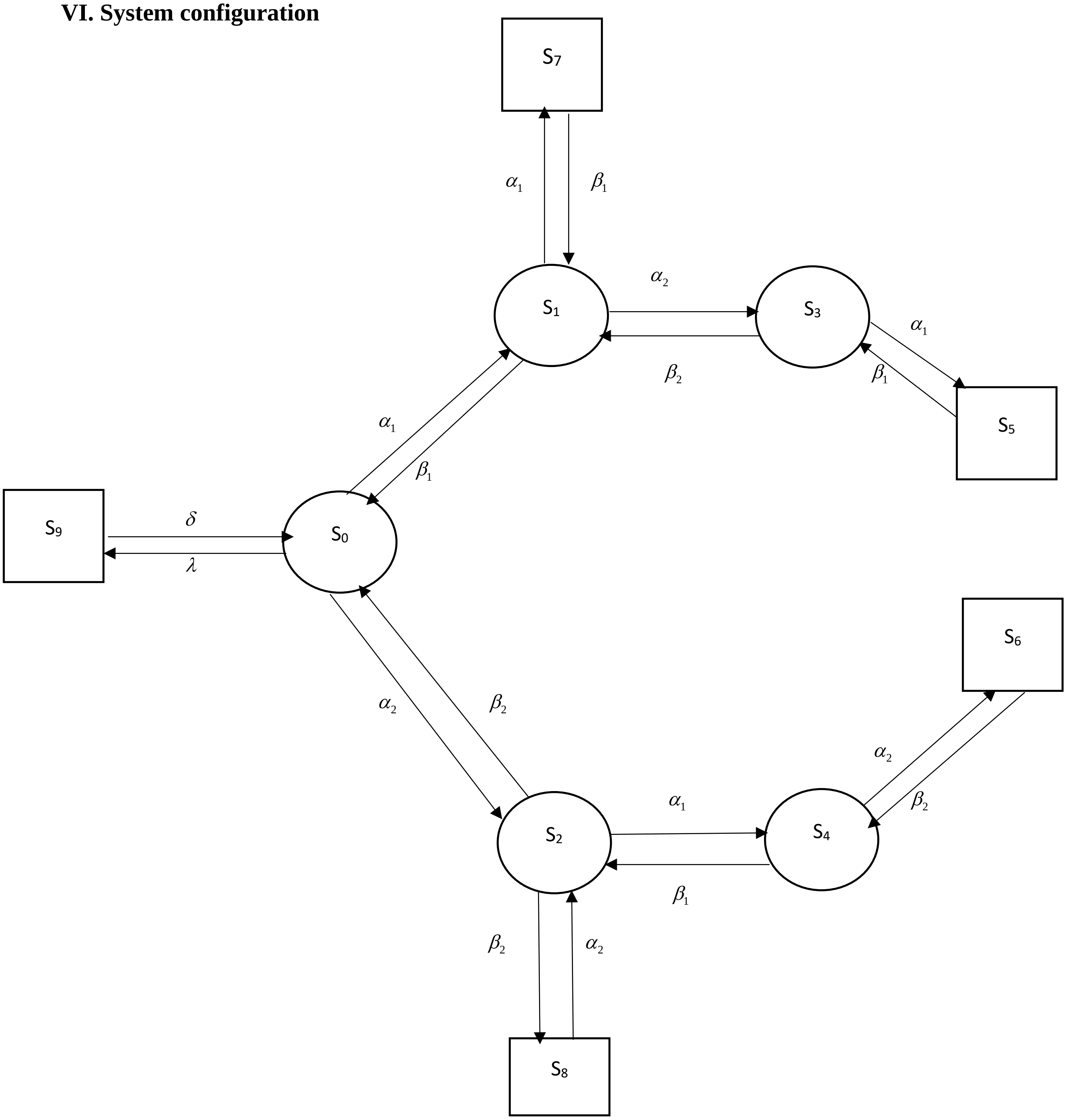

Fig.1: The states of the System

VII. Mean time to system failure

For Fig.1 let $P_i(t)$ probability of the system at time t, $(t \geq 0)$ at state S_i let $P(t)$ denote the probability row vector at time t, then the initial conditions for this problem are

$$P(0)=\left[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0),P_8(0),P_9(0)\right]=[1,0,0,0,0,0,0,0,0,0]$$

We obtain the following differential equation:

$$\frac{dP_0(t)}{dt}=-(\alpha_1+\alpha_2+\lambda)P_0(t)+\beta_1P_1(t)+\beta_2P_2(t)+\delta P_9(t) \tag{3.1}$$

$$\frac{dP_1(t)}{dt}=-(\alpha_1+\alpha_2+\beta_1)P_1(t)+\beta_1P_7(t)+\beta_2P_3(t)+\alpha_1P_0(t) \tag{3.2}$$

$$\frac{dP_2(t)}{dt}=-(\alpha_1+\alpha_2+\beta_2)P_2(t)+\beta_1P_4(t)+\beta_2P_8(t)+\alpha_2P_0(t) \tag{3.3}$$

$$\frac{dP_3(t)}{dt}=-(\alpha_1+\beta_2)P_3(t)+\beta_1P_5(t)+\alpha_2P_1(t) \tag{3.4}$$

$$\frac{dP_4(t)}{dt}=-(\alpha_2+\beta_1)P_4(t)+\beta_2P_6(t)+\alpha_1P_2(t) \tag{3.5}$$

$$\frac{dP_5(t)}{dt}=-\beta_1P_5(t)+\alpha_1P_3(t) \tag{3.6}$$

$$\frac{dP_6(t)}{dt}=-\beta_2P_6(t)+\alpha_2P_4(t) \tag{3.7}$$

$$\frac{dP_7(t)}{dt}=-\beta_1P_7(t)+\alpha_1P_1(t) \tag{3.8}$$

$$\frac{dP_8(t)}{dt}=-\beta_2P_8(t)+\alpha_2P_2(t) \tag{3.9}$$

$$\frac{dP_9(t)}{dt}=-\delta P_9(t)+\lambda P_0(t) \tag{3.10}$$

This can be written in the matrix form as

$$\dot{P}=QP \tag{3.11}$$

Where, $$Q=\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix}$$

To evaluate the transient solution is too complex therefore we will restrict ourselves in calculating the MTSF. To calculate the MTSF we take the transpose matrix of Q and delete the rows and columns for the absorbing state the new matrix is called A. The expected time to reach an absorbing state is calculated from

$$MTSF=E\left[T_{P(0)\to P(absorbing)}\right]=P(0)\left(-A^{-1}\right)\begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix} \tag{3.12}$$

Where

$$A=\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \alpha_1 & \alpha_2 & 0 & 0 & \lambda \\ \beta_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \alpha_2 & 0 & 0 \\ \beta_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \alpha_1 & 0 \\ 0 & \beta_2 & 0 & -(\alpha_1+\beta_2) & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & -(\alpha_2+\beta_1) & 0 \\ \delta & 0 & 0 & 0 & 0 & -\delta \end{bmatrix}$$

$$MTSF=\frac{N_1}{D_1} \tag{3.13}$$

Where

$$\begin{aligned} N_1 = {} & 3\alpha_1\alpha_2^3\delta+3\alpha_1^3\alpha_2\delta+\alpha_1\alpha_2^3\lambda+\alpha_1^3\alpha_2\lambda+6\alpha_1^2\alpha_2^2\delta+\beta_1^1\beta_2^2\delta+2\alpha_1^2\alpha_2^2\lambda+\beta_1^2\beta_2^2\lambda+2\alpha_1\alpha_2\beta_1^2\delta \\ & +5\alpha_1\alpha_2^2\beta_1\delta+5\alpha_1^2\alpha_2\beta_1\delta+2\alpha_1\alpha_2\beta_2^2\delta+5\alpha_1\alpha_2^2\beta_2\delta+5\alpha_1^2\alpha_2\beta_2\delta+2\alpha_1\beta_1\beta_2^2\delta+\alpha_1\beta_1^2\beta_2\delta+2\alpha_1^2\beta_1\beta_2\delta \\ & +\alpha_2\beta_1\beta_2^2\delta+2\alpha_2\beta_1^2\beta_2\delta+2\alpha_2^2\beta_1\beta_2\delta+\alpha_1\alpha_2\beta_1^2\lambda+2\alpha_1\alpha_2^2\beta_1\lambda+2\alpha_1^2\alpha_2\beta_1\lambda+\alpha_1\alpha_2\beta_2^2\lambda+2\alpha_1\alpha_2^2\beta_2\lambda \\ & +2\alpha_1^2\alpha_2\beta_2\lambda+\alpha_1\beta_1\beta_2^2\lambda+\alpha_1\beta_1^2\beta_2\lambda+\alpha_1^2\beta_1\beta_2\lambda+\alpha_2\beta_1\beta_2^2\lambda+\alpha_2\beta_1^2\beta_2\lambda+\alpha_2^2\beta_1\beta_2\lambda+8\alpha_1\alpha_2\beta_1\beta_2\delta \\ & +4\alpha_1\alpha_2\beta_1\beta_2\lambda \end{aligned}$$

$$D_1 = \delta \begin{bmatrix} \alpha_1^4\alpha_2 + 3\alpha_1^3\alpha_2^2 + \alpha_1^3\alpha_2\beta_1 + 2\alpha_1^3\alpha_2\beta_2 + \alpha_1^3\beta_1\beta_2 + 3\alpha_1^2\alpha_2^3 + 3\alpha_1^2\alpha_2^2\beta_1 + 3\alpha_1^2\alpha_2^2\beta_2 + 2\alpha_1^2\alpha_2\beta_1\beta_2 \\ +\alpha_1^2\alpha_2\beta_2^2 + \alpha_1^2\beta_1\beta_2^2 + \alpha_1\alpha_2^4 + 2\alpha_1\alpha_2^3\beta_1 + \alpha_1\alpha_2^3\beta_2 + \alpha_1\alpha_2^2\beta_1^2 + 2\alpha_1\alpha_2^2\beta_1\beta_2 + \alpha_2^3\beta_1\beta_2 + \alpha_2^2\beta_1^2\beta_2 \end{bmatrix}$$

VIII. Availability analysis

The initial conditions for this problem are the same as for the reliability case:

$$P(0) = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

The differential equations form can be expressed as:

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \end{bmatrix} = \begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix}$$

The steady state availability can be obtained using the following procedure. In the steady state, the derivatives of the state probabilities become zero. That allows us to calculate the steady state probabilities with.

$$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_9(\infty) \quad (4.1)$$

$$QP(\infty) = 0 \quad (4.2)$$

Or in the matrix form

$$\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

To obtain $P_0(\infty), P_1(\infty), P_2(\infty), P_3(\infty), P_4(\infty), P_9(\infty)$ we solve the equation (4.2) and the following normalizing condition

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$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)+P_8(\infty)+P_9(\infty)=1 \tag{4.3}$$

We substitute the equation (4.3) in any one of the redundant rows in equation (4.2)

$$\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The steady state availability $A(\infty)$ is given by

$$A(\infty)=\frac{N_2}{D_2} \tag{4.4}$$

Where

$$N_2=\beta_1\beta_2\left(2\alpha_1\alpha_2\delta+\alpha_1\beta_2\delta+\alpha_2\beta_1\delta+\beta_1\beta_2\delta+\beta_1\beta_2\lambda\right)$$

$$D_2=\alpha_1^2\beta_2^2\delta+\alpha_2^2\beta_1^2\delta+\beta_1^2\beta_2^2\delta+\beta_1^2\beta_2^2\lambda+\alpha_1\alpha_2^2\beta_1\delta+\alpha_1^2\alpha_2\beta_2\delta+\alpha_1\beta_1\beta_2^2\delta+\alpha_2\beta_1^2\beta_2\delta+2\alpha_1\alpha_2\beta_1\beta_2\delta$$

IX. Busy period analysis

The initial conditions for this problem are the same as for the reliability case

$$P(0)=\left[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0),P_8(0),P_9(0)\right]$$

$$=[1,0,0,0,0,0,0,0,0,0]$$

The differential equations form can be expressed as

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \end{bmatrix} = \begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix}$$

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The steady state busy period can be obtained using the following procedure. In the steady state, the derivatives of the state probabilities become zero. That allows us to calculate the steady state probabilities with

$$B(\infty)=1-\left[P_0(\infty)+P_9(\infty)\right] \tag{5.1}$$

$$QP(\infty)=0 \tag{5.2}$$

Or, in the matrix form

$$\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

to obtain $P_0(\infty), P_9(\infty)$ we solve the equation (5.2) and the following normalising condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)+P_8(\infty)+P_9(\infty)=1 \tag{5.3}$$

We substitute the equation (5.3) in any one of the redundant rows in equation (5.2) to yield

$$\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The steady state busy period $B(\infty)$ is given by

$$B(\infty)=1-\left[\frac{N_3}{D_2}\right] \tag{5.4}$$

Where $N_3=\beta_1\beta_2(\delta+\lambda)$

X. The expected frequency of preventive maintenance

The initial conditions for this problem are the same as for the reliability case:

$$P(0)=[1,0,0,0,0,0,0,0,0,0]$$

The differential equations form can be expressed as:

$$\begin{bmatrix}\dot{P}_0\\ \dot{P}_1\\ \dot{P}_2\\ \dot{P}_3\\ \dot{P}_4\\ \dot{P}_5\\ \dot{P}_6\\ \dot{P}_7\\ \dot{P}_8\\ \dot{P}_9\end{bmatrix}=\begin{bmatrix}-(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta\\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0\\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0\\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0\\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0\\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0\\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta\end{bmatrix}\begin{bmatrix}P_0\\ P_1\\ P_2\\ P_3\\ P_4\\ P_5\\ P_6\\ P_7\\ P_8\\ P_9\end{bmatrix}$$

The steady state, the expected frequency of preventive maintenance per unit time can be obtained using the following procedure. In the steady state, the derivatives of the state probabilities become zero. That allows us to calculate the steady state probabilities with

$$K(\infty)=P_9(\infty) \qquad (6.1)$$

$$QP(\infty)=0 \qquad (6.2)$$

Or, in the matrix form

$$\begin{bmatrix}-(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta\\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0\\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0\\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0\\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0\\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}P_0\\ P_1\\ P_2\\ P_3\\ P_4\\ P_5\\ P_6\\ P_7\\ P_8\\ P_9\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

To obtain $P_9(\infty)$ we solve the equation (6.2) and the following normalizing condition:

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)+P_8(\infty)+P_9(\infty)=1 \qquad (6.3)$$

We substitute the equation (6.3) in any one of the redundant rows in equation (6.2) to yield

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$$\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 & 0 & 0 \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_1+\beta_2) & 0 & \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_2+\beta_1) & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady state, the expected frequency of preventive maintenance per unit time $K(\infty)$ is given by

$$K(\infty) = \frac{\beta_1^2\beta_2^2\lambda}{D_2} = \frac{N_4}{D_2} \tag{6.4}$$

Where $N_4 = \beta_1^2\beta_2^2\lambda$

XI. Cost analysis

The expected total profit per unit time incurred to the system in the steady state is given by:

$$PF(\infty) = C_0A(\infty) - C_1B(\infty) - C_2K(\infty) \tag{7.1}$$

Where,

PF is the profit incurred to the system,

C_0 is the revenue per unit up-time of the system,

C_1 is the cost per unit time which the system is under repair

C_2 is the cost per preventive maintenance.

XII. Special case

After study the system when the preventive maintenance is not allowed, we get the mean time to system failure is given by

$$MTSF = \frac{\hat{N}_1}{\hat{D}_1} \tag{8.1}$$

Where

$$\hat{N}_1 = 3\alpha_1\alpha_2^3\delta + 3\alpha_1^3\alpha_2\delta + 6\alpha_1^2\alpha_2^2\delta + \beta_1^1\beta_2^2\delta + 2\alpha_1\alpha_2\beta_1^2\delta + 5\alpha_1\alpha_2^2\beta_1\delta + 5\alpha_1^2\alpha_2\beta_1\delta + 2\alpha_1\alpha_2\beta_2^2\delta + 5\alpha_1\alpha_2^2\beta_2\delta$$
$$+5\alpha_1^2\alpha_2\beta_2\delta + 2\alpha_1\beta_1\beta_2^2\delta + \alpha_1\beta_1^2\beta_2\delta + 2\alpha_1^2\beta_1\beta_2\delta + \alpha_2\beta_1\beta_2^2\delta + 2\alpha_2\beta_1^2\beta_2\delta + 2\alpha_2^2\beta_1\beta_2\delta + 8\alpha_1\alpha_2\beta_1\beta_2\delta$$

$$\hat{D}_1 = \delta\begin{bmatrix} \alpha_1^4\alpha_2 + 3\alpha_1^3\alpha_2^2 + \alpha_1^3\alpha_2\beta_1 + 2\alpha_1^3\alpha_2\beta_2 + \alpha_1^3\beta_1\beta_2 + 3\alpha_1^2\alpha_2^3 + 3\alpha_1^2\alpha_2^2\beta_1 + 3\alpha_1^2\alpha_2^2\beta_2 + 2\alpha_1^2\alpha_2\beta_1\beta_2 \\ +\alpha_1^2\alpha_2\beta_2^2 + \alpha_1^2\beta_1\beta_2^2 + \alpha_1\alpha_2^4 + 2\alpha_1\alpha_2^3\beta_1 + \alpha_1\alpha_2^3\beta_2 + \alpha_1\alpha_2^2\beta_1^2 + 2\alpha_1\alpha_2^2\beta_1\beta_2 + \alpha_2^3\beta_1\beta_2 + \alpha_2^2\beta_1^2\beta_2 \end{bmatrix}$$

The steady state availability is given by

$$\hat{A}(\infty)=\frac{\hat{N}_2}{\hat{D}_2} \tag{8.2}$$

Where

$$\hat{N}_2=3\alpha_1^3\alpha_2+6\alpha_1^2\alpha_2^2+5\alpha_1^2\alpha_2\beta_1+5\alpha_1^2\alpha_2\beta_2+2\alpha_1^2\beta_1\beta_2+3\alpha_1\alpha_2^3+5\alpha_1\alpha_2^2\beta_1+5\alpha_1\alpha_2^2\beta_2+2\alpha_1\alpha_2\beta_1^2$$
$$+8\alpha_1\alpha_2\beta_1\beta_2+2\alpha_1\alpha_2\beta_2^2+\alpha_1\beta_1^2\beta_2+2\alpha_1\beta_1\beta_2^2+2\alpha_2^2\beta_1\beta_2+2\alpha_2\beta_1^2\beta_2+\alpha_2\beta_1\beta_2^2+\beta_1^2\beta_2^2$$

$$\hat{D}_2=\alpha_1^4\alpha_2+3\alpha_1^3\alpha_2^2+\alpha_1^3\alpha_2\beta_1+2\alpha_1^3\alpha_2\beta_2+\alpha_1^3\beta_1\beta_2+3\alpha_1^2\alpha_2^3+3\alpha_1^2\alpha_2^2\beta_1+3\alpha_1^2\alpha_2^2\beta_2+2\alpha_1^2\alpha_2\beta_1\beta_2$$
$$+\alpha_1^2\alpha_2\beta_2^2+\alpha_1^2\beta_1\beta_2^2+\alpha_1\alpha_2^4+2\alpha_1\alpha_2^3\beta_1+\alpha_1\alpha_2^3\beta_2+\alpha_1\alpha_2^2\beta_1^2+2\alpha_1\alpha_2^2\beta_1\beta_2+\alpha_2^3\beta_1\beta_2+\alpha_2^2\beta_1^2\beta_2$$

The steady state busy period is given by

$$\hat{B}(\infty)=1-\frac{\hat{N}_3}{\hat{D}_2} \tag{8.3}$$

Where,

$$\hat{N}_3=\beta_1\beta_2$$

The steady state, the expected frequency of preventive maintenance per unit time is given by

$$\hat{K}(\infty)=\frac{\hat{N}_4}{\hat{D}_2} \tag{8.4}$$

Where,

$$\hat{N}_4=\beta_1^2\beta_2^2$$

Table 1: Relation between failure rate of type I and both the MTSF and the profit of the system (with and without PM)

α_1	MTSF of the system with PM	MTSF of the system without PM 1.0e+03	The profit of the system with PM	The profit of the system without PM 1.0e+03
1	869.6870	1.8927	451.5158	1.9779
1.5	620.9395	1.8896	325.1392	1.9703
2	449.0964	1.8858	237.0482	1.9615
2.5	334.9267	1.8813	178.1200	1.9514
3	257.9897	1.8761	138.1779	1.9403
3.5	204.5985	1.8703	110.3155	1.9282
4	166.3617	1.8638	90.2662	1.9152
4.5	138.1608	1.8568	75.4131	1.9014
5	116.8097	1.8491	64.1203	1.8869

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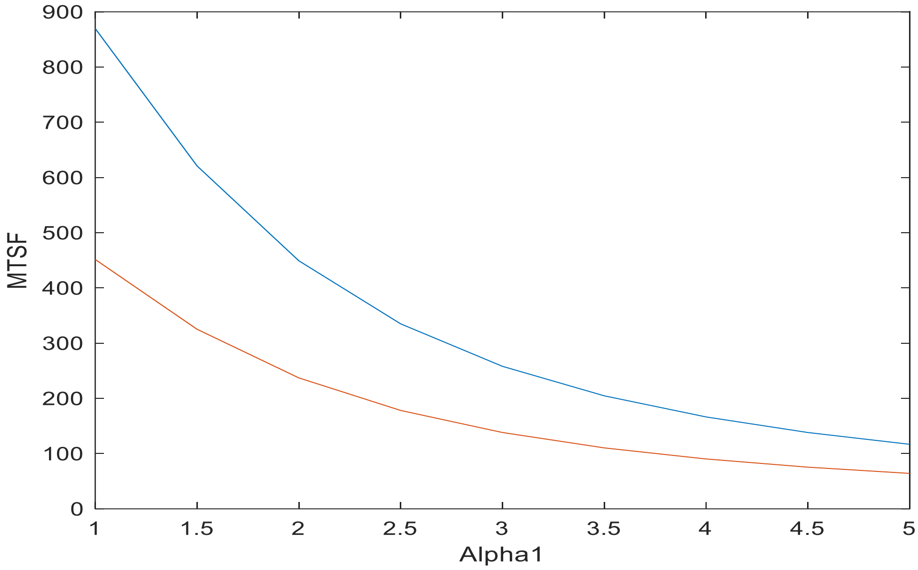

Fig. 2: Relation between the failure rate of type I and MTSF

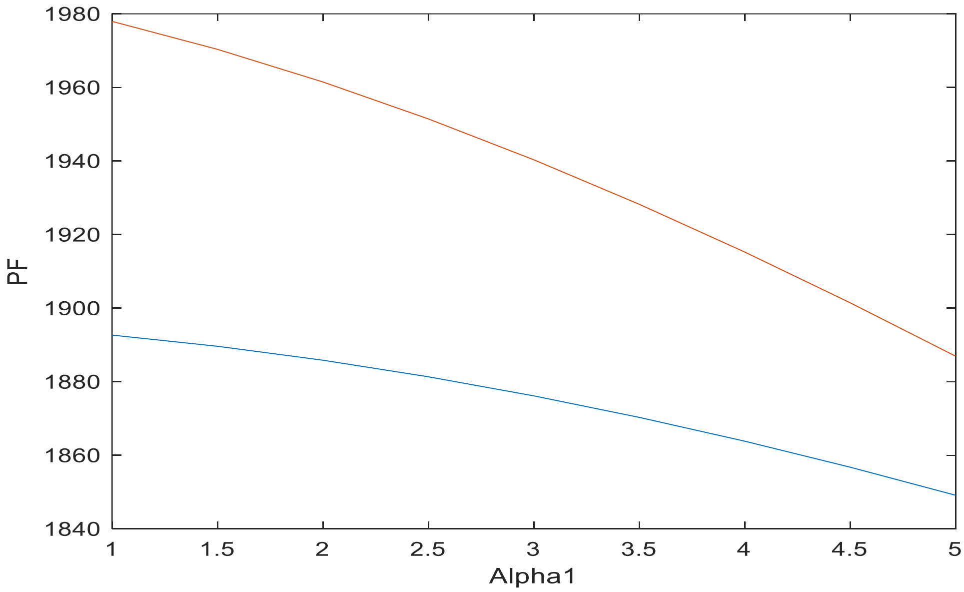

Fig. 3 Relation between the failure rate of type I and the Profit

XIII. Conclusion

MTSF and profit function with respect to α_1 for both systems with and without preventive maintenance graphically, it was observing that, the increase of failure rate α_1 at constant $\alpha_2 = 0.04$, $\beta_1 = 0.5$, $\beta_2 = 1$, $\lambda = 0.2$, $\delta = 0.2$, $C_0 = 2000$, $C_1 = 200$, $C_2 = 200$. The MTSF and the profit function of the system decreases for both systems with and without preventive maintenance and also a system with preventive maintenance is greater than the system without preventive maintenance with respect to the MTSF and the profit function incurred to the model.

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