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ON FUZZY P-BAIRE SPACES

E. Poongothai, Department of Mathematics, Shanmuga Industries of Arts and Science college, Tiruvannamalai-606603, Tamilnadu, India. E-mail: epoongothai5@gmail.com
M. Nalini, Department of Mathematics, Shanmuga Industries of Arts and Science college, Tiruvannamalai-606603, Tamilnadu, India, mark270672@gmail.com.

Abstract:
In this paper a property which can be used to cy..and measure named fzy. P- Baire spaces in fzy.neutrosophictopolgl.spaces are introduced and studied. For this purpose fzy. P- den, fzy.nwe. P-den set, fzy. P- 1cy..set, fzy. P- Baire spaces are defined. Also, we discussed some results about these concepts are also obtained.

Keywords:
Fuzzy open P-set, Fuzzy nowhere P-den set, Fuzzy first category P-set and Fuzzy P-Baire spaces.

1.Introduction

In 1965 the concept of fzy.sets was introduced by **Zadeh[9].**since many authors have extensively developed the theory of fzy.setsand its application to several sectors of both pure and applied sciences, such as [2],[3] &[4].

Recently, in[1] authors extended min. structures to fzy.min. spaces. This paper deals with fzy. P-Baire spaces in fzy.min. spaces and a new generalization of the notion of fzy.topolgl. spaces. Also we discuss several characterizations fzy.P-Baire spaces in fzy.min. spaces.

2.Preliminaries

Definition 2.1 [4]

For a set X, we define a fzy.set in X to be a function μ:X→[0,1], where $\mu(x)$ represents the degree of membership of x in the fzy. set μ.

That is $\mu=\{(x, \mu(x)/x \in x\}$. the family of all fzy. sets on x is denoted by I_x. consisting of all the mappings from X to [0,1]. Any subset A of a set X can be identified with its characteristics function ψ_A: x →{0,1} defined by

$$\psi(x) = \begin{cases} 1, \text{if } x \in A \\ 1, \text{if } x \notin A \end{cases}$$

Definition 2.2 [3]

Let (X, T) be a fzy.topolgl. space and λ be any fzy. set in (X, T). we define

$cl(\lambda) = \wedge\{\mu/\lambda \le \mu, 1-\mu \in T\}$ and$int(\lambda) = \vee\{\mu/\mu \le \lambda, \mu \in T\}$

For any fzy.setin a fzy.topolgl.space (X,T), it is easy to see that

$1 - cl(\lambda) = int(1-\lambda)$ and $1 - int(\lambda) = cl(1-\lambda)$ [1]

Definition 2.3[1]

A min. structure m_X on a non-empty set X is denoted by (X, M_X) .A family M of fzy.sets in X is said to be a fzy.min. structure on X, if $0 \in M$ and $1 \in M$. In this case (X, M) is called a fzy.min. spaces.

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Definition 2.4 [1]

A family of μ fzy. sets in X is said to be a

a) Fzy.min. structure in Lowen sense on X if $\lambda 1_X \in \mu$ for any $\lambda \in I$, where I = [0, 1] ([4]).
b) Fzy.min. structure in Chang sense on X if $\lambda 1_X \in \mu$ for any $\lambda \in \{0, 1\}$ ([3]).

In these cases, (X, μ) is called afzy.min. space in Lowen sense (respectively Change sense).In the rest of this paper, fzy.min. structure is used for fzy.min. structure in Lowen sense.

Definition 2.5 [1]

For a fzy.set on λ on X, M-cl(λ) and M-Int(λ) represents the closure and interior respectively with respect to the fzy.min. spaces are defined as follows:

$$M - Cl(\lambda) = \wedge\{\mu : \lambda \leq \mu;\ 1\text{-}\mu \in M\},$$

$$M - Int(\lambda) = \vee\{\mu : \mu \leq \lambda;\ \mu \in M\}$$

where FMC(X) (respectively, FMO(X)) represents the collection of all fzy.M-closed (respectively, fzy. M-open) sets in X

Definition 2.6 [1]

[4], [6] For any two fzy. sets A and B

(a) m-Int(A) ≤ A and m-Int(A) = A if A is a fzy. m-open set.

(b) A ≤ m-Cl(A) and A = m-Cl(A) if A is a fzy. m-closed set.

(c) m-Int(A) ≤ m-Int(B) and m-Cl(A) ≤ m-Cl(B) if A ≤ B.

(d) m-Int(A ∧ B) = (m-Int(A)) ∧ (m-Int(B)) and (m-Int(A)) ∨ (m-Int(B)) ≤ m-Int(A ∨ B).

(e) m-Cl(A ∨ B) = (m-Cl(A)) ∨ (m-Cl(B)) and m-Cl(A ∧ B) ≤ (m-Cl(A)) ∧ (m-Cl(B)).

(f) m-Int(m-Int(A)) = m-Int(A) and m-Cl(m-Cl(B)) = m-Cl(B).

(g) $(m\text{-}Cl(A))^c = m\text{-}Int(A^c)$ and $(m\text{-}Int(A))^c = m\text{-}Cl(A^c)$

3. On fuzzy nowhere P-dense set

In this section we will introduce and study the concept of fzy.open P set (fzy. closed –P set) and fzy.nwe. P-den set.

Definition 3.1:

Let (X, M) be a fzy.min. spaces on fzy.topolgl. space. A fzy.set λ on X is said to be fzy.open-p (respectively fzy. closed-P) set, P-int(λ)= λ(respectively, P-cl(λ)= λ)

Example 3.2:

Let X = {a, b, c}, M = {0, 1, λ, μ, γ}, where λ = {(a, 0.3), (b, 0.2), (c, 0)}, M = {(a, 0.2),(b, 0.3),(c, 0.1)}, γ = {(a, 0.6),(b, 0.4),(c, 0.5)}.

$P\text{-}Int(\gamma) = \vee\{\delta : \delta \leq \gamma \text{ and } \delta \in M\} = \lambda \vee \mu \vee \gamma = \gamma$. This ⇒$\gamma$ is fzy.open-P set similarly 1- γ is a fzy. closed P-set.

Definition 3.3:

A fzy.set λ for a fzy.min. space (X, M) is called fzy.denP-set if there exists no fzy. closed set μ in (X, M) such that λ<μ<1. That is , P-cl(λ)=1.

Definition 3.4:

Let (X,M) be a fzy.min. space. A fzy.set λ in (X,M) is called a fzy.nwe. P-den set if there exists no non-zero fzy. open P-set μ in (X, M).
Such that, μ <P-cl(λ) that is , P-int P-cl(λ)=0.

Example 3.5:

Let X = {a, b, c}, M = {$\bar{0}$, $\bar{1}$, α, β, γ, λ, μ}, where

α = {(a, 0.9),(b, 0.3), (c, 0.6)},

β = {(a, 0.3),(b, 1), (c, 0.2)},

γ = {(a, 0.7),(b, 0.4), (c, 1)},

λ = {(a, 0.8),(b, 0.4), (c, 0.3)},

μ = {(a, 0.7),(b, 0.4),(c, 0.9)},

The non-zero fzy.nwe. P-den sets are α, β, γ, α ∨ β, β ∨ γ, γ ∨ λ,γ$V(\lambda \Lambda$μ), βΛ(α∨ γ),λV $(\beta \Lambda\, \gamma)$ are the fzy. open P-sets in (X,M).

Now, the fzy.sets$(1-\beta \vee \gamma), (1-\lambda \vee \mu), (1-\gamma) and\ (1-\alpha \vee \beta)$ are fzy.nwe. P-den sets in (X,M)

Theorem 3.6:

Let λ be a fzy.set of a fzy.min. space (X, M).Then

(i) P-cl(λ) ≥ λVcl(intλ);

(ii) P-int(λ)≤ λΛint (clλ).

Proof:

We will prove (i) statement.
P-cl(λ) is a fzy. closed P-set, we have cl(intλ) ≤ cl(int(P-clλ)) ≤ P-clλ. Thus λ Vcl(intλ) ≤P-clλ.
We have the relation P-clλ = λ∨cl(intλ), so int(P-clλ) = int(cl(intλ)).
The above theorem gives the motivation to introduce the class which will be discussed.

Example 3.7:

Let X = {a, b, c}, and α, β, γ are fzy. sets of X defined as it follows.

α = {(a, 0.4),(b, 0.2),(c, 0.7)},

β = {(a, 0.8),(b, 0.8),(c, 0.4)},

γ = {(a, 0.8),(b, 0.9), (c, 0.6)}

Let M={$\bar{0}, \bar{1}$, α, β, α∧ β, α ∨ β}. We see that P-cl(γ) $> \gamma \vee cl(int\, \gamma)$ and P-int(1-γ) $< (1-\gamma) \wedge int(cl\, 1-\gamma)$.

The above example shows that in theorem 3.6 inequality may not be satisfied.

Proposition 3.8:

If λ is a fzy.nwe.den set and in a fzy.min. space (X,M), then P-int(λ)=0.

Proof:

Let λ be a fzy.nwe. P-den set in (X, M). Then, we have P-intP-cl(λ)=0. By theorem 3.6, we have P-int(λ)≤ λ ΛP-intcl(λ). Then, P-int(λ)≤ λΛ0=0.That is, P-int(λ)=0.

Proposition 3.9:

If λ is a fzy.nwe.P-den set and μ is a fzy.set in a fzy.min. space (X, M), then (λΛ μ) is a fzy.nwe. P-den set in (X, M).

Proof:

Let λ be a fzy.nwe.P-den set in (X, M). Then,P-intcl(λ)=0. Now P-int(P-cl(λΛμ)≤P-int(P-cl(λ)ΛP-int(P-cl(μ)) ≤0 Λ P-int(P-cl(μ)) =0. That is P-int(P-cl(λΛμ)) =0. Hence (λ Λ μ) is a fzy.nwe. P-den set in (X, M).

Proposition 3.10:

If λ is a fzy.den P-set and fzy.open P-set in fzy.min. space (X, M) and if μ≤1- λ, then μ is a fzy.nwe. P-den set in (X, M).

Proof:

Let λ be a fzy.den P-set and fzy. open P-set in (X, M). Then we have P-cl(λ)=1 and P-int(λ)= λ. Now μ≤1– λ, ⇒ P-cl(μ) ≤ P-cl (1- λ). Then P-cl(μ)≤ 1-P-int(λ) = 1 – λ. Hence P-cl(μ) ≤ (1– λ), which ⇒ P-int(P-cl(μ)) ≤ P-int (1– λ) = 1– P-cl(λ)=1 – 1 = 0. That is, P-int(P-cl(μ)) =0. Hense μ is afzy.nwe. P-den set in (X, M).

Proposition 3.11:

If λ is a fzy.nwe. P-den set in a fzy.min. space (X, M), then 1 – λ is a fzy.den P-set in (X, M).

Proof:

Let λ be a fzy.nwe. P-den set in (X, M). Then P-int P-cl (λ) =0. Now $\lambda \leq$ P-cl(λ) ⇒ P-int(λ) ≤ P-int(P-cl(λ)) =0. Then, P-int(λ)=0 and P-cl (1- λ) =1–P-int(λ)=1–0=1 and hence 1–λ is a fzy.den P-set in (X, M).

Definition 3.12:

Let (X,M) be a fzy.min.space in a fzy.topolgl. space. A fzy.set λ in (X,M) is called fzy.1 P-cy..if$\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fzy.nwe. P - densetsin(X,M). Any other fzy.set in (X, M) is said to be a fzy.2 P-cy...

Example 3.13:

Let X = {a, b, c}, and M= {$\bar{0}, \bar{1}$,α, β} where

α = {(a, 0.5), (b, 0.9), (c, 0.7)},

β = {(a, 0.8), (b, 0.9), (c, 0.6)},

now α, β, α∨β, αΛ β, are the non zerofzy. open P-sets in (X, M).

Then 1-α, 1– β, 1 –(α∨β), 1 –(αΛβ) are fzy.nwe. P-den sets in (X, M) and [(1–α)∨(1–β)∨(1–(α∨β))∨(1–(αΛβ)]=1- αΛβ is a fzy.1 P-cy.. sets in (X, M)

Definition 3.14:

If λ is a fzy.1 P-cy..set in a fzy.min. space (X,M). Then 1- is called a fzy.rel. P-set in (X, M),

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Proposition3.15:
If λ is a fzy.nwe. P-den(X, M) then P-int P-cl(λ)=0, now $\lambda \leq$ p-cl(λ) $\Rightarrow$p-int(λ) $\leq$ p-int(p-cl(λ)). Hence p-int(λ)=0.
Proof:
Let λ be a fzy.nwe. P-den set in (X, M).
Then, P-int P-cl(λ)=0. Now λ≤ P-cl(λ) ⇒ P-int(λ)≤P-in(P-cl(λ)). Hence P-int(λ)=0.

4. Fuzzy P-Baire Spaces

Definition4.1:
Let (X, M) be a fzy.min.space on a fzy.topolgl.space. Then (X, M) is called a fzy.P-Baire space if P $-int(\vee_{i=1}^{\infty}\lambda i)$)=0 where λ_i's are fzy.nwe. P-den set in (X, M).

Example4.2:

Let X= {a, b, c}. M= {$\bar{0}$, $\bar{1}$, α, β, γ} where

α = {(a, 0.5),(b, 0.7), (c, 0.6)},

β = {(a, 0.8),(b, 0.9), (c, 0.9)},

γ = {(a, 0.6),(b, 0.9), (c, 0.8)},

Now α, β, α∨β, αΛ β, γ, α∨γ, β∨γ are the non zerofzy.open P-sets in (X,M).

Then 1–α,1–β, 1 – γ, 1–α∨β, 1 –αΛβ are nwe. P-den sets in (X, M) and P-int [(1–α)∨(1–β)∨ (1–γ) ∨ (1–α∨β) ∨(1–αΛβ)]=0. Hence (X, M) is a fzy.P-Baire space.

Proposition4.3:
If P-$cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$,where λ_i's are fzy.den P-set and fzy. open P-sets in a fzy.min. space (X, M), then (X, M) is a fzy.P-Baire space.
Proof:

Now P–($\wedge_{i=1}^{\infty}(\lambda_i)$)=1⇒1–P-($\wedge_{i=1}^{\infty}(\lambda_i)$)=0. Then we have P-i(1−$\wedge_{i=1}^{\infty}(\lambda_i)$)=0, which ⇒P-$int(\vee_{i=1}^{\infty}(1-\lambda i)$)=0.Since ($\lambda_i$)'s are fzy.den P - sets in (X,M),P-cl(λ_i)=1and P-int (1–λ_i)=1–P-cl(λ_i)=1–1=0.

Hence we have P-i($\vee_{i=1}^{\infty} 1-\lambda i$))=0, where P-$int$(1–$\lambda_i$)=0 and (1–$\lambda_i$)'s are fzy.closed sets in (X, M). Then, by proposition4.3, (X, M) is a fzy.P-Baire space.

Proposition4.4:
If P-int (int($\vee_{i=1}^{\infty}(\lambda i)) = 0$, where P-int($\lambda_i$)=0 and λ_i's are fzy. closed P-set in a fzy.min.(X, M),then(X,M) is a fzy.P-Baire space.
Proof
Let λ_i's are fzy.closed P-sets in (X,M). since P-int(λ_i)=0, by prop.3.15, λ_i's are fzy.nwe. P-den sets in (X, M).
Therefore we have P-int($\vee_{i=1}^{\infty}(\lambda i)) = 0$, where λ_i's are fzy.nwe. P-den sets in (X,M). hence (X, M) is a fzy.P-Baire spaces.

Shortcut keywords:

S.NO	KEYWORDS	SHORTCUTS
1	Fuzzy	fzy.
2	Topological	topolgl.
3	Category	cy..
4	Nowhere	nwe.
5	First	1
6	Minimal	min.
7	Second	2
8	Implies that	$\Rightarrow$
9	Residual	Rel.
10	Dense	den.

Conclusion :

In this paper, the concept of fzy.P-den., fzy.nwe.P-den.set, fzy.P-Baire spaces and properties are discussed. Some of its characteristic and examples are established. This shall be extended in the future research studies.

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