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FUZZY STRONGLY (GSP)*-CLOSED SETS IN FUZZY TOPOLOGICAL SPACE

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ABSTRACT
In this paper we have introduced a new class of fuzzy sets called fuzzy strongly (gsp)*-closed sets, and fuzzy strongly (gsp)*-continuous mappings are investigated its characterization in Fuzzy topological spaces.

Keywords:
fuzzy Strongly (gsp)*-closed sets , fuzzy Strongly (gsp)*-continuous maps , fuzzy Ts(gsp)*-space, fuzzy gTs(gsp)*- space, fuzzy g*Ts(gsp)*-space and fuzzy g**Ts(gsp)*-space.

I PRELIMINARIES:
Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family {Aα: ∈αΛ} of fuzzy sets of X is defined by to be the mapping sup Aα (resp. inf Aα) . A fuzzy set A of X is contained in a fuzzy set B of X if A(x) ≤ B(x) for each x∈X. A fuzzy point xβ in X is a fuzzy set defined by xβ (y) = β for y=x and x(y) =0 for y ≠□x, β∈[0,1] and y ∈□X .A
fuzzy point xβ is said to be quasi-coincident with the fuzzy set A denoted by xβqA if and only if β + A(x) > 1. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by AqB if and only if there exists a point x∈X such that A(x) + B(x) > 1 .A ≤ B if and only if ˥(AqBc).A family τ□of fuzzy sets of X is called a fuzzy topology on X if 0,1 belongs to τ□and τ□is closed with respect to arbitrary union and finite intersection .The members of τ□are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed sets of A and the interior of A (denoted by int(A))is the union of all fuzzy open subsets of A.
Throughout this paper (X, τ), (Y, σ) represent non-empty fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned.For a subset A of a fuzzy space (X,τ), cl(A) and int(A) denote the fuzzy closure and the fuzzy interior of A respectively.

Definition 1.1: A fuzzy Subset A of fuzzy topological space (X, τ) is called
1. fuzzy semi-open set if A⊆cl(int(A)) and a fuzzy semi-closed set if int(cl(A))⊆A.
2. fuzzy semi-pre open set if A⊆cl(int(cl(A)) and a fuzzy semi-pre closed set if int(cl(int(A)))⊆A
3. fuzzy regular -open set if int(cl(A))=A and a fuzzy regular -closed.

Definition 1.2:A fuzzy Subset A of fuzzy topological space (X ,τ) is called
1. fuzzy generalized closed set (briefly fuzzy g-closed) if cl(A)⊆U whenever A ⊆U and U is fuzzy open in (X, τ)
2. fuzzy g*-closed set if cl(A)⊆U whenever A⊆U and U is fuzzy g open in (X ,τ)
3. fuzzy g**-closed set if cl(A)⊆U whenever A⊆U and U is fuzzy g* open in (X ,τ)
4. fuzzy wg - closed set if cl(int(A))⊆U whenever A⊆U and U is fuzzy open in (X,τ)
5. fuzzy regular generalized closed set (briefly fuzzy rg-closed) if cl(A) ⊆U whenever A⊆U and U is fuzzy regular open in (X,τ)
6. fuzzy sg**-closed set if scl(A)⊆U whenever A⊆U and U is fuzzy g** open in (X,τ)
7. fuzzy sg*-closed set if scl(A)⊆U whenever A⊆U and U is fuzzy g* open in (X,τ)

8. fuzzy generalized semi-closed set (briefly fuzzy gs-closed) if scl(A)⊆U whenever A⊆U and U is fuzzy open in (X,τ)
9. fuzzy gsp - closed set if spcl(A)⊆U whenever A⊆U and U is fuzzy open in (X,τ)
10. fuzzy (gsp)*- closed set if cl(A)⊆U whenever A⊆U and U gsp is fuzzy open in (X,τ)

Definition 1.3: A fuzzy function f :(X,τ)→(Y,σ) is called;
1. fuzzy g– continuous if f^{-1}(V) is a fuzzy g-closed set of (X ,τ) for every fuzzy closed set V of (Y ,σ)
2. fuzzy g*–continuous if f^{-1}(V) is a fuzzy g*-closed set of (X ,τ) for every fuzzy closed set V of (Y,σ)
3. fuzzy g**–continuous if f^{-1}(V) is a fuzzy g**-closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
4. fuzzy rg–continuous if f^{-1}(V) is a fuzzy rg -closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
5. fuzzy wg–continuous if f^{-1}(V) is a fuzzy wg -closed set of (X,τ) for every fuzzy closed set V of (Y,σ)
6. fuzzy (gsp)*–continuous if f^{-1}(V) is a fuzzy (gsp)* -closed set of (X,τ) for every fuzzy closed set V of (Y,σ)

Definition 1.4: A fuzzy topological space (X,τ) is said to be
1. fuzzy $T_{1/2}$* space if every fuzzy g*-closed set in it is fuzzy closed.
2. fuzzy T_d space if every fuzzy gs -closed set in it is fuzzy g- closed.

2. Basic properties of fuzzy **strongly (gsp)* - closed sets in** fuzzy **topological Space**

We introduce the following definition

Definition 2.1: A subset A of a fuzzy Topological space (X,τ) is said to be a fuzzy strongly (gsp)*-closed set if cl(int(A))⊆U whenever A⊆U and U is fuzzy gsp-open.

Proposition 2.1: Every fuzzy closed set is fuzzy strongly (gsp)*-closed.

Proof: Let A be a fuzzy closed. Then cl(A)=A. Let us prove that A is fuzzy strongly (gsp)* - closed. Let A⊆U and U be fuzzy gsp-open. Then cl(A)⊆U .Since A is fuzzy closed . cl(int(A)) ⊆□cl(A) ⊆U. Then cl(int(A))⊆U whenever A⊆□U and U is fuzzy gsp - open. so□□A is fuzzy strongly (gsp)* - closed. The converse of the above proposition need not be true in general .**Proposition 2.2:** Every fuzzy g-closed set is fuzzy strongly (gsp)*-closed.

Proof: Let A be fuzzy g-closed. Then cl(A)⊆U Whenever A⊆□U and U is fuzzy open in (X, τ).To prove A is fuzzy strongly (gsp)*-closed. Then A⊆U and U be fuzzy (gsp) open .We have cl(A)⊆U Whenever A⊆U and U is fuzzy open in (X, τ).Since every fuzzy open set is (gsp) –open .We have cl(A)⊆U Whenever A ⊆□U and U is fuzzy (gsp)-open in (X, τ).But cl(int(A)) ⊆cl(A)⊆□U whenever A⊆U and U is fuzzy (gsp)-open in (X, τ).so□□cl(int(A))⊆U whenever A ⊆U and U is fuzzy (gsp) -open in (X,τ). So A is fuzzy strongly (gsp)*-closed. The converse of the above proposition need not be true in general as seen in the following example.

Proposition 2.3: Every fuzzy g*-closed set is fuzzy strongly (gsp)* - closed

The converse of the above proposition need not be true in general as seen in the following example.

Proposition 2.4: Every fuzzy rg–closed set is fuzzy strongly (gsp)*-closed

Proof: Let A be fuzzy rg-closed set. Then cl(A) ⊆□U Whenever A⊆U and U is fuzzy regular-open in (X,τ)**.**To prove A is fuzzy strongly (gsp)*-closed. Let A⊆U and U be fuzzy (gsp) open. Since every fuzzy regular-open set is fuzzy (gsp)-open .We have cl(A)⊆U Whenever A ⊆□U and U is fuzzy (gsp)-open in (X, τ)**.**But cl(int(A))⊆cl(A)⊆□U whenever A⊆U and U is fuzzy (gsp)-open in (X, τ)**.**∴□cl(int(A))⊆U whenever A⊆U and U is fuzzy (gsp)-open in (X,τ).i.e. A is fuzzy strongly (gsp)*-closed.

Remark 2.1: fuzzy Strongly (gsp)* - closedness is independent of fuzzy semi-closedness **Proposition 2.5:** Every fuzzy (gsp)* - closed set is fuzzy strongly (gsp)*- closed set

Proof: Let A be fuzzy (gsp)* -closed set. Then cl(A)⊆U Whenever A⊆U and U is fuzzy gsp-open in (X, τ).To prove A is fuzzy strongly (gsp)* - closed. Let A⊆U and U be fuzzy (gsp) open. Since every fuzzy (gsp)* - open set is fuzzy (gsp)-open .We have cl(A)⊆U Whenever A ⊆□U and U is fuzzy (gsp)-open in (X, τ).But cl(int(A))⊆cl(A)⊆U whenever A ⊆□U and U is fuzzy (gsp)-open in (X, τ) i.e.□□cl(int(A))⊆U whenever A⊆U and U is fuzzy (gsp)-open in (X, τ).∴A is fuzzy strongly (gsp)* -closed. The converse of the above proposition need not be true in general.

Theorem 2.1: Every fuzzy g** -closed set is fuzzy strongly (gsp)* -closed.The converse of the above proposition need not be true in general.

Remark 2.2: fuzzy Strongly (gsp)*-closedness is independent of fuzzy sg**closedness

Remark 2.3: fuzzy Strongly (gsp)*-closedness is independent of fuzzy sg* -closedness

proposition 2.6: Every fuzzy wg-closed set is fuzzy strongly (gsp)* - closed.

Proof: Let A be fuzzy wg-closed **.**Then cl(int(A))⊆□U Whenever A⊆U and U is fuzzy open in (X, τ)**.**To prove A is fuzzy strongly (gsp)* -closed. Let A⊆U and U be fuzzy (gsp) open. Since every fuzzy wg-open set is fuzzy (gsp)-open **.**We have cl(int(A))⊆U whenever A⊆U and U is fuzzy (gsp)-open in (X, τ)**.** ∴□cl(int(A))⊆U whenever A⊆U and U is fuzzy (gsp)-open in (X, τ)**.**∴A is fuzzy strongly (gsp)* -closed.

Proposition 2.7: If A and B are fuzzy strongly (gsp)* -closed sets, then AUB is also fuzzy strongly (gsp)* -closed.

Proof: Let A and B be fuzzy strongly (gsp)* -closed. Let AUB⊆□U and U be fuzzy (gsp)-open. Then A⊆U and B⊆U where U is fuzzy (gsp)-open , cl(int(A))⊆□U, whenever A⊆U and U is fuzzy (gsp)-open and cl(int(B)) ⊆□U, whenever B ⊆□U and U is fuzzy (gsp)-open Since A and B are fuzzy strongly (gsp)* -closed**.** cl(int(A)Uint(B))=cl(int(A))Ucl(int(B))⊆□U whenever AUB⊆□U and U is fuzzy (gsp)-open. Therefore AUB is also fuzzy strongly (gsp)* -closed.

proposition 2.8: If A is a fuzzy strongly (gsp)* -closed set of (X,τ) such that A⊆□B⊆□cl(int(A)) , then B is also fuzzy strongly (gsp)* -closed set of (X,τ)

Proof: Let U be a fuzzy (gsp) –open set in (X,τ) such that B⊆U.Then A⊆U, Since A is fuzzy strongly (gsp)* -closed ,cl(int(A))⊆□U**.** Now cl(int(B)) ⊆□cl(int(A)), since B⊆□cl(int(A)) .Therefore cl(int(B))⊆cl(int(A))⊆□U cl(int(B))⊆□U whenever B⊆U and U is fuzzy (gsp)-open.=> B is fuzzy strongly (gsp)* -closed

The above results can be represented in the following figure:

Where A B represents A implies B and B need not imply A

A B represents A and B are independent.

3. FUZZY **STRONGLY (gsp)* -CONTINUOUS MAPS**

We introduce the following definitions:

Definition 3.1: A fuzzy function f:(X,τ) → (Y,σ) is called fuzzy Strongly (gsp)* -continuous if f^{-1}(V) is a fuzzy strongly (gsp)* -closed set in (X,τ) for every fuzzy closed set V of (Y,σ).

Theorem 3.1: Every fuzzy continuous map is fuzzy strongly (gsp)* -continuous

Proof: Let f :(X,τ) → (Y,σ) be a fuzzy continuous map. Let us prove that f is fuzzy strongly (gsp)* -continuous. Let F be a fuzzy closed set in (Y,σ).Since f is fuzzy continuous f^{-1}(F) is fuzzy closed in (X,τ) and f^{-1}(F) is fuzzy strongly (gsp)* -closed . i.e., f is fuzzy strongly (gsp)* -continuous. The converse of the above Theorem is not true

Theorem 3.2 Every fuzzy g-continuous map is fuzzy strongly (gsp)* -continuous

Proof: Let f :(X,τ) → (Y,σ) be a fuzzy g-continuous**.** Let F be a fuzzy closed set in (Y,σ). Since f is fuzzy g-continuous f^{-1}(F) is fuzzy g-closed in (X,τ) .By theorem (3.1),f^{-1}(F) is fuzzy strongly (gsp)* -closed. i.e. f is fuzzy strongly (gsp)* -continuous closed fuzzy g**-closed fuzzy g-closed fuzzy wg-closed fuzzy sg**-closed fuzzy g*-closed, fuzzy Strongly (gsp)*-closed (gsp)*-closed

Semi-closed, sg*-closed, sg-closed

The converse of the above Theorem is not true

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Theorem 3.3: Every fuzzy g* -continuous map is fuzzy strongly (gsp)* -continuous
Proof: Let f :(X,τ) → (Y,σ) be a fuzzy g* -continuous**.** Let F be a fuzzy closed set in (Y,σ)**.** Since f is fuzzy g* -continuous $f^{-1}(F)$ is fuzzy g* -closed in (X,τ) **.**By Theorem (3.1),$f^{-1}(F)$ is fuzzy strongly (gsp)* -closed. i.e. f is fuzzy strongly (gsp)* -continuous The converse of the above Theorem is not true
Theorem 3.4: Every fuzzy g** -continuous map is fuzzy strongly (gsp)* -continuous
Proof: Let f :(X,τ) → (Y,σ) be a fuzzy g** -continuous**.** Let F be a fuzzy closed set in (Y,σ).Since f is fuzzy g** -continuous, $f^{-1}(F)$ is fuzzy g** -closed in (X ,τ)so $f^{-1}(F)$ is fuzzy strongly (gsp)* -closed∴f is fuzzy strongly (gsp)* -continuous
The converse of the above Theorem is not true
Theorem 3.5: Every fuzzy rg-continuous map is fuzzy strongly (gsp)* -continuous
Proof: Let f :(X,τ) → (Y,σ) be a fuzzy rg-continuous **.** Let F be a fuzzy closed set in (Y,σ) **.** Since f is fuzzy rg-continuous $f^{-1}(F)$ is fuzzy rg-closed in (X,τ). then,$f^{-1}(F)$ is fuzzy strongly (gsp)* -closed∴f is fuzzy strongly (gsp)* -continuous
Theorem 3.6: Every fuzzy (gsp)* -continuous map is fuzzy strongly (gsp)* -continuous
Proof: Let f :(X,τ) → (Y,σ) be a fuzzy (gsp)* -continuous **.** Let F be a fuzzy closed set in (Y,σ) **.** Since f is fuzzy (gsp)* -continuous $f^{-1}(F)$ is fuzzy (gsp)* -closed in (X,τ) then $f^{-1}(F)$ is fuzzy strongly (gsp)* -closed so f is fuzzy strongly (gsp)*-continuous The converse of the above Theorem is not true
Theorem 3.7: Every fuzzy wg-continuous map is fuzzy strongly (gsp)* -continuous
Proof: Let f :(X,τ) → (Y,σ) be a fuzzy wg-continuous **.** Let F be a fuzzy closed set in (Y,σ) **.** Since f is fuzzy wg-continuous $f^{-1}(F)$ is fuzzy wg-closed in (X,τ) then (3.22)$f^{-1}(F)$ is fuzzy strongly (gsp)* -closed∴f is fuzzy strongly (gsp)* -continuous The above results can be represented in the following figure :g Where AB represents A implies B and B need not imply A

4. APPLICATIONS OF fuzzy STRONGLY (gsp)* - CLOSED SETS

In this section application of fuzzy strongly (gsp)*-closed sets, new fuzzy spaces, called as fuzzy Ts(gsp)* space , fuzzy gT_s-(gsp)* space , fuzzy g^*T_s-(gsp)* , fuzzy $g^{**}T_s$(gsp)*space are introduced.
DEFINITION 4.1: A fuzzy space (X, τ) is called a fuzzy T_s(gsp)* -space if every fuzzy strongly (gsp)* - closed set is closed.
DEFINITION 4.2: A fuzzy space (X, τ) is called a fuzzy gT_s(gsp)* - space if every fuzzy strongly (gsp)* - closed set is fuzzy g-closed.
DEFINITION 4.3: A fuzzy space (X, τ) is called a fuzzy g^*T_s(gsp)* -space if every fuzzy strongly (gsp)* - closed set is fuzzy g*-closed
DEFINITION 4.4: A fuzzy space (X, τ) is called a fuzzy $g^{**}T_s$(gsp)* -space if every fuzzy strongly (gsp)* - closed set is fuzzy g**-closed.
THEOREM 4.1: Every fuzzy T_s(gsp)* -space is fuzzy $T_{1/2}$* -space.
PROOF: Let (X, τ) be a fuzzy T_s(gsp)* - space.Let us prove that (X, τ) is a fuzzy $T_{1/2}$* - space**.** Let A be a fuzzy g* -closed set. Since every fuzzy g* - closed set is fuzzy strongly (gsp)* -closed, A is fuzzy strongly (gsp)* -closed. Since (X, τ) is a fuzzy T_s(gsp)* - space , A is fuzzy closed.∴(X,τ) is a fuzzy $T_{1/2}$* - space. The converse is not true. Fuzzy Strongly (gsp)*- continuous g**-continuous continuous fuzzy (gsp)*-continuous fuzzy g*-continuous
fuzzy wg-continuous fuzzy g-continuous fuzzy rg-continuous
THEOREM 4.2: Every fuzzy T_s(gsp)* -space is fuzzy gT_s(gsp)*-space.
PROOF: Let A be a fuzzy strongly (gsp)*-closed set**.** Then A is fuzzy closed . Since the space is fuzzy T_s(gsp)* -space. And Every fuzzy closed set is fuzzy g-closed **.**Hence A is fuzzy g- closed**.** ∴(X,τ) is a fuzzy gT_s(gsp)*- space.
The converse is not true.
Theorem 4.3: Every fuzzy g^*T_s(gsp)* -space is fuzzy gT_s(gsp)* -space

Proof: Let A be a fuzzy strongly (gsp)* -closed.Then A is fuzzy g* -closed, since the fuzzy space is a fuzzy g*T_s(gsp)* -space. SinceEvery fuzzy g* -closed set is fuzzy g-closed. Hence A is fuzzy g-closed so (X,τ) is a fuzzy gT_s(gsp)* -space but the converse is not true.

Theorem 4.4: Every fuzzy g*T_s(gsp)* -space is fuzzy g**T_s(gsp)* -space

Proof: Let A be a fuzzy strongly (gsp)*-closed set **.** Then A is fuzzy g* -closed, since the space is a fuzzy g*T_s(gsp)* .Since Every fuzzy g** -closed set is fuzzy g* -closed **.**Hence A is fuzzy g** -closed **..** ∴ (X,τ) is a fuzzy g**T_s(gsp)* -space.

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