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GENERALISED ESTIMATION OF YOUNG'S MODULUS OF IRON BY AN EXPANSION INTO DIRECTION COSINES α1,α2,α3 WITH RESPECT TO THE CRYSTAL AXES

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Abstract
In this present article, Young's Modulus is expressed by an expansion into Direction Cosines α1, α2, α3 with respect to the crystal axes. The General Equation of Young's Modulus can be used to determine **Y** values at <100>, <110>, <111> directions respectively. In the present article Iron, Copper, Aluminum Young's Modulus is determined at <100>, <110>, <111> directions respectively. The Equation can be generalized to include any element with anisotropic property.

Keywords:
isotropic, Young's Modulus, Direction Cosines

I. Introduction

Anisotropic Properties are those properties which vary with crystal direction Young's Modulus of iron is different at <100>,<110>,<111> directions viz. 125,210.5,272.5GPa respectively.
Young's Modulus can be expressed as an expansion into direction cosines α1,α2,α3 with respect to the crystal axes.In the present article, consideration is made up to three terms.

1.1 Standard Equation:

$$Y^* = K0 + K1\,(\sum\alpha^2 1\ \alpha^2 2) + K2\,(\prod\alpha^2 1) + K3\,(\sum\alpha^2 1\ \alpha^2 2)^2 + K4\,(\sum\alpha^2 1\ \alpha^2 2)(\prod\alpha^2 1) + K5\,(\sum\alpha^2 1\ \alpha^2 2)^3 + K6\,(\prod\alpha^2 1)^2$$

Considered Equation:

$$Y^* = K0 + K1\,(\sum\alpha^2 1\ \alpha^2 2) + K2\,(\prod\alpha^2 1)\,) + K3\,(\sum\alpha^2 1\ \alpha^2 2)^2 + K4\,(\sum\alpha^2 1\ \alpha^2 2)(\prod\alpha^2 1) + K5\,(\sum\alpha^2 1\ \alpha^2 2)^3 + K6\,(\prod\alpha^2 1)^2$$

[uvw]	a	b	c	α1	α2	α3	Y
<100>	0	90^0	90^0	1	0	0	K0
<110>	45^0	45^0	90^0	1/√2	1/√2	0	K0 + K1 /4
<111>	54.7^0	54.7^0	54.7^0	1/√3	1/√3	1/√3	K0 + K1 /3+ K2 / 27

II. Calculation Of Young's Modulus For Iron By An Expansion Into Direction Cosines α1,α2,α3 With Respect To The Crystal Axes

$$Y^* = K0 + K1\,(\sum\alpha^2 1\ \alpha^2 2) + K2\,(\prod\alpha^2 1)\,) + K3\,(\sum\alpha^2 1\ \alpha^2 2)^2 + K4\,(\sum\alpha^2 1\ \alpha^2\,)(\prod\alpha^2 1) + K5\,(\sum\alpha^2 1\ \alpha^2\,)^3 + K6\,(\prod\alpha^2 1)^2$$

For <100> directions, α1 =1, α2 =0, α3=0 ….[I]

For <110> directions, α1 =1/√2, α2 =1/√2, α3=0….[II] For <111> directions, α1 =1/√3, α2 =1/√3,

$\alpha_3 = 1/\sqrt{3}$….[III]

2.1 Calcution Of Young's Modulus For Iron By An Expansion Into Direction Cosines $\alpha_1, \alpha_2, \alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

	Modulus of Elasticity (GPa)		
Metal	*[100]*	*[110]*	*[111]*
Iron	125.0	210.5	272.7

For <100> directions, $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 0$ ….[I], in Standard Equation

$$Y^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1)) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2$$

We have

$$Y^*[100] = K_0 = 125;$$

For <110> directions, $\alpha_1 = 1/\sqrt{2}$, $\alpha_2 = 1/\sqrt{2}$, $\alpha_3 = 0$ Using [II] , in Standard Equation

$$Y^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1)$$

$$210.5 = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2$$

⇨ $210.5 = 125 + K_1/4 + K_3/16 + K_5/64$

⇨ $[16K_1 + 4K_3 + K_5] = 5472$........ **[IV];**

For <111> directions, $\alpha_1 = 1/\sqrt{3}$, $\alpha_2 = 1/\sqrt{3}$, $\alpha_3 = 1/\sqrt{3}$….[III];

Using [III] , in Standard Equation

$$Y^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2$$

$$272.7 = 125 + K_1/3 + K_2/27 + K_3/9 + K_4/81 + K_5/27 + K_6/729 = 125 + K_1/3 + K_5/27 + K_3/9 + K_4/81 + K_2/27 + K_6/729$$ **[re-arraging K_2, K_5**

⇨ $27[9K_1 + 3K_3 + K_5] + 9[K_4 + 3K_2 + K_6] = 147.7 * 729 = 107673.3$

⇨ $27 * 3980 + 9 * 23.7 = 107673.3$

⇨ $[9K_1 + 3K_3 + K_5] = 3980$ **….[V];**

⇨ $[K_4 + 3K_2 + K_6] = 23.7$

⇨ $3 + 6*3 + 2.7 = 23.7$

⇨ $K_4 = 3; K_2 = 6; K_6 = 2.7$

⇨ **From[IV] - [V];We have**

⇨ $16K_1 + 4K_3 + K_5 = 5472$ (-)

$9K_1 + 3K_3 + K_5 = 3980$

$7K_1 + K_3 = 1492$

⇨ **$7*210 + 22 = 1492$;**

⇨ **$K_1 = 210; K_3 = 22$;**

⇨ **$K_5 = 3980 - 3*22 - 9*210$**

⇨ **$K_5 = 2024$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$Y^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2$ $K_0=125$, $K_1=210$, $K_2=6$; $K_3=22$; $K_4= 3$; $K_5=2024$; $K_6 =2.7$**

$Y^*_{IRON} = 125 + 210 (\sum \alpha^2_1 \alpha^2_2) + 6 (\prod \alpha^2_1) + 22(\sum \alpha^2_1 \alpha^2_2)^2 + 3(\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + 2024 (\sum \alpha^2_1 \alpha^2_2)^3 + 2.7(\prod \alpha^2_1)^2$ .... [VI];

⇨ **[VI] ABOVE IS GENERALISED ESTIMATION OF YOUNG'S MODULUS OF IRON BY AN EXPANSION INTO DIRECTION COSINES α1,α2,α3 WITH RESPECT TO THE CRYSTAL AXES**

⇨ **FOR <100> Directions, $Y^*_{IRON} = 125$**

⇨ **FOR <110> Directions, $Y^*_{IRON} = 125 + 210/4 + 22/16 + 2024/64 = 210.5$**

⇨ **FOR <111> Directions, $Y^*_{IRON} = 125 + 210/3 + 6/27 + 22/9 + 3/81 + 2024/27 + 2.7/729 = 272.7$**

Metal	*Modulus of Elasticity (GPa)* [100]	[110]	[111]
Iron	125.0	210.5	272.7

⇨ **ABOVE VALUES, CONFORM TO THE STANDARD YOUNG'S MODULUS OF IRON.**

III Conclusion.

Youngs Modulus of an element can be expressed By an Expansion into Direction Cosines α1, α2, α3 With Respect To the Crystal Axes. Generalized equation of Young's Modulus can be utilized to obtain its value at any crystallographic direction with the provision of directional cosines α1, α2, α3 along that particular crystallographic direction.

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