



K₀,K₁,K₂,K₃, K₄, K₅, K₆ CONSTANTS EVALUATION OF MAGNETO- CRYSTALLINE ANISOTROPY ENERGY DENSITY EQUATION OF ELECTRICAL STEELS BASED ON TEXTURE FACTOR FOR IDEAL FIBRES

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1. ABSTRACT:

Texture Factor, A* and Magnetic Crystalline Anisotropy Energy Density* K₀,K₁,K₂,K₃, K₄, K₅, K₆ Constants are important parameters for Electrical Steels. While the former indicates volume density of crystals having preferred Orientation, latter indicates the easy and hard magnetization directions. Evaluation of these parameters for Pure Iron and Electrical Steel enables in reduction of core losses and improving the electrical energy efficiency in Transformers, Rotating Machines. In this research article, an attempt is made to compute Magneto- Crystalline Anisotropy Energy Density for pure iron based on Texture Factor for Ideal fibers.

Keywords:

Texture Factor, Magnetic Crystalline Anisotropy Energy Density, Core losses

2. INTRODUCTION:

The Magneto Crystalline Anisotropy constants K₀,K₁,K₂,K₃,K₄, K₅, K₆ values determine the extent to which a material is easily magnetizable. Their value depends on Chemical Composition, Crystal Structure, and Thermo-Mechanical Processing history of the given material. Texture factor constants K₀,K₁,K₂,K₃, K₄, K₅, K₆ values determines the preferred orientations of grains, the Overall Texture Factor is quantitative measurement of texture. The value signifies extent of presence of standard texture viz. Cube Texture (T.F = 22.5), Goss Texture (T.F = 35.6), Gamma Texture (T.F = 38.68) in the given material

3. ESTIMATION OF MAGNETIC ANISOTROPY CONSTANTS K₀,K₁,K₂,K₃, K₄, K₅, K₆ CONSTANTS EVALUATION OF FOR ELECTRICAL STEELS:

Magneto Crystalline Anisotropy Energy is generally expressed by an expansion into direction cosines $\alpha_1, \alpha_2, \alpha_3$ of the magnetization with respect to the crystal axes.

$$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 [1];$$

[uvw]	a	b	c	α_1	α_2	α_3	E
[100]	0	90 ⁰	90 ⁰	1	0	0	K ₀
[110]	45 ⁰	45 ⁰	90 ⁰	1/√2	1/√2	1/√2	K ₀ + K ₁ /4
[111]	54.7 ⁰	54.7 ⁰	54.7 ⁰	1/√3	1/√3	1/√3	K ₀ + K ₁ /3+ K ₂ / 27

From REF 1, we have

$$E^* = 0.355A^* + (0.163 - 0.013A^*)[\text{wt\%Si}] - 1.898$$

FOR A* for Ø fiber <100>//ND is 22.5 => E* = -0.5345 [wt%Si] + 6.0895



FOR A* for fiber <110>//ND is 35.6 => $E^* = -0.9406 [\text{wt}\% \text{Si}] + 10.74$

FOR A* for Y fibre <111>//ND is 38.68=> $E^* = -1.03608 [\text{wt}\% \text{Si}] + 11.8334$

$$E^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2) (\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2 \dots \dots \dots [I]$$

FOR [100] directions, $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0$

$$\Rightarrow E^* = K_0 = -0.5345 [\text{wt}\% \text{Si}] + 6.0895$$

FOR [110] directions, $\alpha_1 = 1/\sqrt{2}, \alpha_2 = 1/\sqrt{2}, \alpha_3 = 0$

$$\Rightarrow E^* = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + K_1/4 + K_3/16 + K_5/64$$

$$\Rightarrow -0.9406 [\text{wt}\% \text{Si}] + 10.74 = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + K_1/4 + K_3/16 + K_5/64$$

$$\Rightarrow (-0.4061[\text{wt}\% \text{Si}] + 4.6505) * 64 = 16K_1 + 4K_3 + K_5$$

$$\Rightarrow 16K_1 + 4K_3 + K_5 = 297.632 - 25.9904[\text{wt}\% \text{Si}] \dots \dots \dots [II]$$

FOR [111] directions, $\alpha_1 = 1/\sqrt{3}, \alpha_2 = 1/\sqrt{3}, \alpha_3 = 1/\sqrt{3}$

$$\Rightarrow -1.03608 [\text{wt}\% \text{Si}] + 11.8334 = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + K_1/3 + K_2/27 + K_3/9 + K_4/81 + K_5/27 + K_6/729$$

$$\Rightarrow 27(9K_1 + K_5 + 3K_3) + (9K_4 + 27K_2 + K_6) = 4187.3031 - 0.50158[\text{wt}\% \text{Si}] * 729$$

$$\Rightarrow 27(150) + 137.3031 = 4187.3031 - 365.65182[\text{wt}\% \text{Si}]$$

$$\Rightarrow (9K_1 + K_5 + 3K_3) = 150; \dots [III]$$

$$\Rightarrow (9K_4 + 3K_2 + K_6) = 137.3031 - 365.65182[\text{wt}\% \text{Si}] \dots [IV]$$

⇒ SUBTRACTING [III] and [II], we have

$$\Rightarrow 16K_1 + 4K_3 + K_5 = 297.632 - 25.9904[\text{wt}\% \text{Si}]$$

$$\Rightarrow 9K_1 + 3K_3 + K_5 = 150$$

$$\Rightarrow 7K_1 + K_3 = 147.632 - 25.9904[\text{wt}\% \text{Si}]$$

$$\Rightarrow 7*(21) + (0.632 - 25.9904[\text{wt}\% \text{Si}]) = 147.632 - 25.9904[\text{wt}\% \text{Si}]$$

$$\Rightarrow K_1 = 21; K_3 = 0.632 - 25.9904[\text{wt}\% \text{Si}]; K_5 = -40.896 + 77.9712[\text{wt}\% \text{Si}]$$

⇒ Next Equation....[IV], we have $9K_4 + 27K_2 + K_6 = 137.3031 - 365.65182[\text{wt}\% \text{Si}]$

$$\Rightarrow 9*(3) + 27*(4) + (2.3031 - 365.65182[\text{wt}\% \text{Si}]) = 137.3031 - 365.65182[\text{wt}\% \text{Si}]$$

$$\Rightarrow K_4 = 3; K_2 = 4; K_6 = 2.3031 - 365.65182[\text{wt}\% \text{Si}]$$

$$\Rightarrow K_0 = 6.0895; K_1 = 21; K_2 = 4; K_3 = 0.632 - 25.9904[\text{wt}\% \text{Si}]; K_4 = 3; K_5 = -40.896; K_6 = 2.3031 - 365.65182[\text{wt}\% \text{Si}]$$

⇒ Generalized Equation for Magneto-Anisotropic Energy Density is

$$\Rightarrow E^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2) (\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2$$



$$E^* = -0.5345 [\text{wt\%Si}] + 6.0895 + 21(\sum \alpha^2_1 \alpha^2_2) + 4(\prod \alpha^2_1) + (0.632 - 25.9904[\text{wt\%Si}]) (\sum \alpha^2_1 \alpha^2_2)^2 + 3(\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + (-40.896 + 77.9712[\text{wt\%Si}]) (\sum \alpha^2_1 \alpha^2_2)^3 + (2.3031 - 365.65182[\text{wt\%Si}]) (\prod \alpha^2_1)^2 \dots [IV]$$

$$\Rightarrow E^* = 6.0895 + 21(\sum \alpha^2_1 \alpha^2_2) + 4(\prod \alpha^2_1) + 0.632(\sum \alpha^2_1 \alpha^2_2)^2 + 3(\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) - 40.896(\sum \alpha^2_1 \alpha^2_2)^3 + (2.3031)(\prod \alpha^2_1)^2 + [-0.5345 [\text{wt\%Si}] - 25.9904[\text{wt\%Si}]) (\sum \alpha^2_1 \alpha^2_2)^2 + 77.9712[\text{wt\%Si}](\sum \alpha^2_1 \alpha^2_2)^3 - 365.65182[\text{wt\%Si}](\prod \alpha^2_1)^2]$$

$$\Rightarrow E^* = E^*_{\text{IRON}} + E^{**}$$

$$\Rightarrow \text{Where } E^*_{\text{IRON}} = 6.0895 + 21(\sum \alpha^2_1 \alpha^2_2) + 4(\prod \alpha^2_1) + 0.632(\sum \alpha^2_1 \alpha^2_2)^2 + 3(\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) - 40.896(\sum \alpha^2_1 \alpha^2_2)^3 + (2.3031)(\prod \alpha^2_1)^2$$

$$\Rightarrow E^{**} = [-0.5345 [\text{wt\%Si}] - 25.9904[\text{wt\%Si}]) (\sum \alpha^2_1 \alpha^2_2)^2 + 77.9712[\text{wt\%Si}](\sum \alpha^2_1 \alpha^2_2)^3 - 365.65182[\text{wt\%Si}](\prod \alpha^2_1)^2]$$

⇒ Above is the Standard Magnetic –Crystalline Anisotropy Energy Density Equation for Electrical Steel

⇒ Magneto-Crystalline Energy Density Equation of Electrical Steel in terms of 7 constants.

CRYSTALLOGRAPHIC DIRECTION	MAGNETO- CRYSTALLINE ANISOTROPY ENERGY DENSITY
[100] $\alpha_1=1, \alpha_2=0, \alpha_3=0$	$E^*_{[100]} = 6.0895 - 0.5345 [\text{wt\%Si}]$
[110] $\alpha_1=1/\sqrt{2}, \alpha_2=1/\sqrt{2}, \alpha_3=0$	$E^*_{[110]} = 10.74 - 0.9406 [\text{wt\%Si}]$
[111] $\alpha_1=1/\sqrt{3}, \alpha_2=1/\sqrt{3}, \alpha_3=1/\sqrt{3}$	$E^*_{[111]} = 11.8334 - 1.03608 [\text{wt\%Si}]$

S. N O.	Standard Crystallographic Directions	E* For Pure Iron	E*	E* for Fe-0.51%Si	E* for Fe-1.38 %Si	E* for Fe-2.8 %Si	E* for Fe-3.2 %Si
1	[100]	$E^*_{[100]} = 6.0895$	$E^*_{[100]} = -0.5345([\text{wt\%Si}]) + 6.0895$	5.81	5.35	4.59	4.37
2	[110]	$E^*_{[110]} = 10.74$	$E^*_{[110]} = -0.9406[\text{wt\%Si}] + 10.74$	10.26	9.44	8.10	7.73

3	[111]	$E^*_{[111]} = \frac{11.8}{634}$	$E^*_{[111]} = \frac{-1.0360}{8}$ [wt%Si] + 11.863 4	11.33	10.43	8.96	8.54
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⇒ **DISCUSSION:**

⇒ From REF¹, The <100>//ND fibre accounts for the lowest anisotropy energy since the flux lines, distributed homogenously in a plane of the rotating laminated sheet, have an easiest magnetization direction with the in-plane rotated cube texture components. On the contrary, the Y and the <011>//ND fiber orientations have relatively high anisotropy energy and as such, the occurrence of these components in electrical steels is undesirable.

3. ESTIMATION OF TEXTURE FACTOR CONSTANTS $K_0, K_1, K_2, K_3, K_4, K_5, K_6$ FOR PURE IRON

$$A^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2 \dots [V]$$

From REF 1, we have

$$E^* = 0.355A^* + (0.163 - 0.013A^*)[\text{wt\%Si}] - 1.898$$

⇒ We have, $E^* = (0.355 - 0.013[\text{wt\%Si}]) A^* + 0.163[\text{wt\%Si}] - 1.898$

⇒ $(0.355 - 0.013[\text{wt\%Si}]) A^* = -0.163[\text{wt\%Si}] + 1.898 - 0.5345 [\text{wt\%Si}] + 6.0895 + 21(\sum \alpha^2_1 \alpha^2_2) + 4(\prod \alpha^2_1) +$

$(0.632 - 25.9904[\text{wt\%Si}]) (\sum \alpha^2_1 \alpha^2_2)^2 + 3(\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + (-40.896 + 77.9712[\text{wt\%Si}]) (\sum \alpha^2_1 \alpha^2_2)^3 + (2.3031 - 365.65182[\text{wt\%Si}]) (\prod \alpha^2_1)^2$

⇒ $(0.355 - 0.013[\text{wt\%Si}]) A^* = [-0.6975[\text{wt\%Si}] + 7.9875] + 21(\sum \alpha^2_1 \alpha^2_2) + 4(\prod \alpha^2_1) + (0.632 - 25.9904[\text{wt\%Si}]) (\sum \alpha^2_1 \alpha^2_2)^2 + 3(\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + (-40.896 + 77.9712[\text{wt\%Si}])$

$(\sum \alpha^2_1 \alpha^2_2)^3 + (2.3031 - 365.65182[\text{wt\%Si}]) (\prod \alpha^2_1)^2$

⇒ Comparing with Standard Equation [V] we have,

⇒ $A^* = K_0 + K_1 (\sum \alpha^2_1 \alpha^2_2) + K_2 (\prod \alpha^2_1) + K_3 (\sum \alpha^2_1 \alpha^2_2)^2 + K_4 (\sum \alpha^2_1 \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha^2_1 \alpha^2_2)^3 + K_6 (\prod \alpha^2_1)^2$..[VI]

$K_0 = \frac{-0.6975[\text{wt\%Si}] + 7.9875}{(0.355 - 0.013[\text{wt\%Si}])}$

$K_1 = \frac{21}{(0.355 - 0.013[\text{wt\%Si}])}$

⇒ $K_2 = \frac{4}{(0.355 - 0.013[\text{wt\%Si}])}$

⇒ $K_3 = \frac{(0.632 - 25.9904[\text{wt\%Si}])}{(0.355 - 0.013[\text{wt\%Si}])}$

⇒ $K_4 = \frac{3}{(0.355 - 0.013[\text{wt\%Si}])}$

⇒ $K_5 = \frac{(-40.896 + 77.9712[\text{wt\%Si}])}{(0.355 - 0.013[\text{wt\%Si}])}$

⇒ $K_6 = \frac{(2.3031 - 365.65182[\text{wt\%Si}])}{(0.355 - 0.013[\text{wt\%Si}])}$



S.N O.	CONSTANT	Fe + 0.51%Si	Fe + 1.38%Si	Fe + 2.8%Si	Fe + 3.2%Si
1.	K ₀	22.5	22.5	22.5	22.5
2.	K ₁	61.9122	67.2602 6	78.2997 7	82.0953 8
3.	K ₂	11.7928	12.8114 79	14.9142 43	15.6372 16
4.	K ₃	- 37.21543 6	- 112.852 32	- 268.982 55	- 322.663 33
5.	K ₄	8.8446	9.60860 9	11.1856 82	11.7279 12
6.	K ₅	-3.33349	213.645 0	661.533 78	815.527 13
7.	K ₆	- 542.9975 18	- 1608.78 99	- 3808.80 68	- 4565.21 78

⇒ ABOVE IS STANDARD EQUATION FOR TEXTURE FACTOR FOR ELECTRICAL STEELS IN TERMS OF 7 CONSTANTS

$$A^* = 22.5 + 61.9122 (\sum \alpha_1^2 \alpha_2^2) + 11.7928 (\prod \alpha_1^2) - 37.215436 (\sum \alpha_1^2 \alpha_2^2)^2 + 8.8446 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) - 3.33349 (\sum \alpha_1^2 \alpha_2^2)^3 - 542.997518 (\prod \alpha_1^2)^2 \dots [1]$$

$$A^* = 22.5 + 67.26026 (\sum \alpha_1^2 \alpha_2^2) + 12.811479 (\prod \alpha_1^2) - 112.85232 (\sum \alpha_1^2 \alpha_2^2)^2 + 9.608609 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) - 213.6450 (\sum \alpha_1^2 \alpha_2^2)^3 - 1608.7899 (\prod \alpha_1^2)^2 \dots [2]$$

$$A^* = 22.5 + 78.29977 (\sum \alpha_1^2 \alpha_2^2) + 14.914243 (\prod \alpha_1^2) - 268.98255 (\sum \alpha_1^2 \alpha_2^2)^2 + 11.185682 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) - 661.53378 (\sum \alpha_1^2 \alpha_2^2)^3 - 3808.8068 (\prod \alpha_1^2)^2 \dots [3]$$

$$A^* = 22.5 + 82.09538 (\sum \alpha_1^2 \alpha_2^2) + 15.637216 (\prod \alpha_1^2) - 322.66333 (\sum \alpha_1^2 \alpha_2^2)^2 + 11.727912 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) - 815.52713 (\sum \alpha_1^2 \alpha_2^2)^3 - 4565.2178 (\prod \alpha_1^2)^2 \dots [4]$$

S.N O	TEXTURE FACTOR, A*	ELECTRICAL STEELS Fe + 0.51% [wt %Si]	ELECTRICAL STEELS Fe + 1.38% [wt%Si]	ELECTRICAL STEELS Fe + 2.8% [wt%Si]	ELECTRICAL STEELS Fe + 3.2% [wt%Si]
1.	22.5 for θ <100> fibre	22.5	22.5	22.5	22.5
2.	35.6 for <110> fibre	35.6	35.6	35.6	35.6
3.	γ fibre <111> // ND is 38.68	38.68	38.68	38.68	38.68



5. CONCLUSIONS:

Magneto-Crystalline Anisotropy Energy Density value is least for [100] directions, and higher for [110] & [111] directions. Therefore [100] directions are easy directions of magnetization for pure iron and [111] hardest direction for magnetization of pure iron, [110] direction is harder direction for magnetization of pure iron. Texture Factor Equation results are consistent with the standard results and conforms to the value of ideal fibres.

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