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# NEW STRONGLY HOMEOMORPHISM IN INTUITIONISTIC TOPOLOGICAL SPACES

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**Abstract**

The purpose of this paper is to introduce the notion of $\mathcal{I}i$-homeomorphism and intuitionistic strongly $i$-homeomorphism in Intuitionistic Topological Spaces. Further, some of their basic properties of $\mathcal{I}i$-homeomorphism and intuitionistic strongly $i$-homeomorphism are investigated. Besides, we proved intuitionistic strongly $i$-homeomorphism is an equivalence relation.

**Keywords:** $\mathcal{I}i$-homeomorphism, intuitionistic strongly $i$-homeomorphism, $\mathcal{I}Si$-$h(A)$

## I. Introduction

Homeomorphism play a vital role in topology. The notion of intuitionistic sets and intuitionistic points was introduced by Coker[6]. Later he developed and introduced the Intuitionistic topological spaces[5] and explained some fundamental properties. Selvanayaki etal[3] introduced homeomorphism and discussed some basic properties. Suganya[1] et al introduced and derived some properties of $\mathcal{I}i$-open sets in Intuitionistic topological spaces. In this paper we explained a new class of functions on Intuitionistic topological space called $\mathcal{I}i$-homeomorphism and analyse their characterizations. Additionally, we also define intuitionistic strongly $i$-homeomorphism in intuitionistic topological space and we proved the family of all intuitionistic strongly $i$-homeomorphism satisfies the group properties.

## II. $\mathcal{I}i$-homeomorphism and Intuitionistic strongly $i$-homeomorphism

### 2.1 Preliminaries

We recall some definitions and results which are useful for this sequel. Throughout the present study, $\mathcal{I}$ means intuitionistic, a space $A$ means intuitionistic topological space $(A, \tau_{I_1})$ and $B$ means an intuitionistic topological space $(B, \tau_{I_2})$ unless otherwise mentioned.

**Definition 2.1.1. [6]** Let $A$ be a non-empty set. An intuitionistic set(IS for short) $H$ is an object having the form $H = <A, H_1, H_2>$ where $H_1, H_2$ are subsets of $A$ satisfying $H_1 \cap H_2 = \emptyset$. The set $H_1$ is called the set of members of $H$, while $H_2$ is called set of non members of $H$.

**Definition 2.1.2. [5]** An intuitionistic topology (for short IT) on a non-empty set $A$ is a family $\tau_I$ of intutionistic sets in $A$ satisfying following axioms.

1) $\tilde{\emptyset}, \tilde{A} \in \tau_I$
2) $G_1 \cap G_2 \in \tau_I$, for any $G_1, G_2 \in \tau_I$
3) $\cup G_\alpha \in \tau_I$ for any arbitrary family $\{ G_\alpha / \alpha \in J\}$ where $(A, \tau_I)$ is called an intuitionistic topological space and any intuitionistic set $H$ is called an intuitionistic open set (for short $\mathcal{I}OS$) in $A$. The complement $H^c$ of an $\mathcal{I}OS$ $H$ is called an intuitionistic closed set (for short $\mathcal{I}CS$) in $A$.

**Definition 2.1.3[1]** An intuitionistic set $D$ of an Intutionistic topological space $(A, \tau_I)$ is said to be an intutionistic $i$-open set (shortly $\mathcal{I}i$-open set) if there exist an intutionistic open set $H \neq \tilde{\emptyset}$ and $\tilde{A}$ such that $D \subseteq \mathcal{I}cl(D \cap H)$.

**Definition 2.1.4.[3]** A function $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ is $\mathcal{I}$-open map if the image of every $\mathcal{I}$-open set in $A$ is $\mathcal{I}$-open in $B$.

**Definition 2.1.5.[3]** A function $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ is $\mathcal{J}i$-closed map if the image of every $\mathcal{J}$-closed set in $A$ is $\mathcal{J}i$-closed in $B$ .
**Definition 2.1.6.[2]**A mapping $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ is $\mathcal{J}i$- continuous function if the inverse image of every intuitionistic open set in $(B,\tau_{I_2})$ is $\mathcal{J}i$-open in $(A,\tau_{I_1})$.
**Definition 2.1.7.[2]** A function $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ is said to be $\mathcal{J}$i-irresolute if $s^{-1}(G)$ is a $\mathcal{J}i$-open in $(A,\tau_{I_1})$ for every $\mathcal{J}i$-open set $G$ in $(B,\tau_{I_2})$.
**Definition 2.1.8.[4]** A bijective function $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ is said to be $\mathcal{J}$-homeomorphism if $s$ is $\mathcal{J}i$-continuous and $\mathcal{J}i$-open map.
**Definition 2.1.9.[1]** Let $(A,\tau_{I_1})$ be an Intuitionistic topological space and let $H\subseteq A$. The intuitionistic $i$-interior of $H$ is defined as the union of all $\mathcal{J}i$-open sets contained in $A$ and is denoted by $\mathcal{J}int_i(H)$. It is clear that $\mathcal{J}int_i(H)$ is the largest $\mathcal{J}i$-open set, for any subset $H$ of $A$.
**Definition 2.1.10.[1]** Let $(A,\tau_{I_1})$ be an intuitionistic topological space and let $H\subseteq A$. The $\mathcal{J}i$-closure of $H$ is defined as the intersection of all $\mathcal{J}i$-closed sets in $A$ containing $H$, and is denoted by $\mathcal{J}cl_i(H)$. It is clear that $\mathcal{J}cl_i(H)$ is the smallest $\mathcal{J}i$-closed set for any subset $H$ of $A$.

### 2.2. $\mathcal{J}i$-homeomorphism

**Definition 2.2.1:** A function $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ is a $\mathcal{J}i$-homeomorphism if
1. $s$ is 1-1 and onto
2. $s$ is $\mathcal{J}i$-continuous
3. $s$ is $\mathcal{J}i$-open map

**Example 2.2.2:** Let $A=\{17,19,21\}$ with a family $\tau_{I_1}=\{\tilde{A},\widetilde{\emptyset},\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3,\mathcal{V}_4\}$ where $\mathcal{V}_1=<A,\{17\},\{19\}>$ , $\mathcal{V}_2=<A,\emptyset,19>$ , $\mathcal{V}_3=<A,\{17,21\},\emptyset>$ and $\mathcal{V}_4=<A,\{17\},\emptyset>$. Let $B=\{85,90,95\}$ with a family $\tau_{I_2}=\{\tilde{B},\widetilde{\emptyset},\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3\}$ where $\mathcal{H}_1=<B,\{85\},\{95\}>$ , $\mathcal{H}_2=<B,\{85,90\},\emptyset>$ and $\mathcal{H}_3=<B,\emptyset,\{85,95\}>$. Define $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ as $s(17)=85$, $s(19)=90$, $s(21)=95$ and $s(A)=B$. Then, $s(<A,\{17\},\{19\}>)=<B,\{85\},\{90\}>$, $s(<A,\{\emptyset\},\{19\}>)=<B,\{\emptyset\},\{90\}>$, $s(<A,\{17,21\},\{\emptyset\}>)=<B,\{85,95\},\{\emptyset\}>$ and $s(<A,\{17\},\{\emptyset\}>)=<B,\{85\},\{\emptyset\}>$. Therefore, $s$ is $\mathcal{J}i$-open. Also, $s^{-1}(<B,\{85\},\{95\}>=<A,\{17\},\{21\}>$, $s^{-1}(<B,\{85,90\},\{\emptyset\}>=<A,\{17,19\},\{\emptyset\}>$ and $s^{-1}(<B,\{\emptyset\},\{85,95\}>=<A,\{\emptyset\},\{17,21\}>$. Therefore, $s$ is $\mathcal{J}i$-continuous. Hence, $s$ is $\mathcal{J}i$-homeomorphism.
**Theorem 2.2.3:** Let $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ be a one-one onto mapping. Then, $s$ is a $\mathcal{J}i$-homeomorphism if and only if $s$ is $\mathcal{J}i$-closed and $\mathcal{J}i$-continuous.
**Proof:** Let $s$ be a $\mathcal{J}i$-homeomorphism. Then, $s$ is $\mathcal{J}i$-continuous. Let $K$ be a $\mathcal{J}$-closed set in $A$. Then $A-K$ is $\mathcal{J}$-open. Since $s$ is $\mathcal{J}i$-open, $s(A-K)$ is $\mathcal{J}i$-open in $B$. That is, $B-s(K)$ is $\mathcal{J}i$-open in $B$. Therefore, $s(K)$ is $\mathcal{J}i$-closed in $B$. Hence, the image of every $\mathcal{J}$-closed set in $A$ is $\mathcal{J}i$-closed in $B$. That is, $s$ is $\mathcal{J}i$-closed. Conversely, let $s$ be a $\mathcal{J}i$-closed and $\mathcal{J}i$-continuous. Let $R$ be $\mathcal{J}$-open in $A$. Then $A-R$ is $\mathcal{J}$-closed in $A$. Since $s$ is $\mathcal{J}i$-closed, $s(A-R)=B-s(R)$ is $\mathcal{J}i$-closed in $B$. Therefore, $s(R)$ is $\mathcal{J}i$-open in $B$. Thus, $s$ is $\mathcal{J}i$-open and hence, $s$ is a $\mathcal{J}i$-homeomorphism.
**Theorem 2.2.4:** Let $s:(A,\tau_{I_1})\rightarrow(B,\tau_{I_2})$ be an one-one, onto and $\mathcal{J}i$-continuous map. Then the following statements are equivalent
(i) $s$ is an $\mathcal{J}i$-open.
(ii) $s$ is an $\mathcal{J}i$-homeomorphism.
(iii) $s$ is an $\mathcal{J}i$-closed.
**Proof:**(i) ⇔ (ii) Obvious from the definition.
(ii) ⇔ (iii) Let $Y$ be a $\mathcal{J}$-closed set in $A$. Then $Y^c$ is $\mathcal{J}$-open in $A$. By hypothesis, $s(Y^c)=(s(Y))^c$ is an $\mathcal{J}i$-open in $B$. That is, $s(Y)$ is $\mathcal{J}i$-closed in $B$. Therefore, $s$ is an $\mathcal{J}i$-closed.
(iii) ⇔ (i) Let $C$ be a $\mathcal{J}$-open set in $A$. Then $C^c$ is $\mathcal{J}$-closed in $A$. By hypothesis, $s(C^c)=(s(C))^c$

is $\mathcal{J}i$-closed in $B$. That is, $s(C)$ is $\mathcal{J}i$-open in $B$. Therefore, $s$ is an $\mathcal{J}i$-open map.
**Theorem 2.2.5:** If $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ is bijective and $s(\mathcal{J}cl_i(N)) = \mathcal{J}cl(s(N))$ then $s$ is $\mathcal{J}i$-homeomorphism for every subset $N$ of $A$.
**Proof:** If $s(\mathcal{J}cl_i(N)) = \mathcal{J}cl(s(N))$ for every subset $N$ of $A$, then $s$ is $\mathcal{J}i$-continuous. If $N$ is $\mathcal{J}$-closed in $A$ then $N$ is $\mathcal{J}i$-closed in $A$.Then $\mathcal{J}cl_i(A) = A \Rightarrow s(\mathcal{J}cl_i(A)) = s(A)$. Hence by the given hypothesis, it follows that $\mathcal{J}cl(s(A)) = s(A)$. Thus $s(A)$ is $\mathcal{J}$-closed in $B$ and hence $\mathcal{J}i$-closed in $B$ for every $\mathcal{J}$-closed set $N$ in $A$. That is, $s$ is $\mathcal{J}i$-closed. Hence $s$ is $\mathcal{J}i$-homeomorphism.
**Remark 2.2.6:** The reverse implication is not true as shown in the following example.
**Example 2.2.7:** Let $A = \{p, q\}$ with a family $\tau_{I_1} = \{\tilde{A}, \tilde{\emptyset}, \mathcal{V}_1, \mathcal{V}_2\}$ where $\mathcal{V}_1 = < A, \{\emptyset\}, \{q\} >$ and $\mathcal{V}_2 =< A, \{p\}, \emptyset >$. Let $B = \{x, y\}$ with a family $\tau_{I_2} = \{\tilde{B}, \tilde{\emptyset}, \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3\}$ where $\mathcal{H}_1 =< B, \{\emptyset\}, \{\emptyset\} >$, $\mathcal{H}_2 = < B, \{x\}, \{\emptyset\} >$ and $\mathcal{H}_3 =< B, \{x\}, \{y\} >$. Define $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ as $s(p) = x$, $s(q) = y$ and $s(A) = B$. Then, $s(< A, \{\emptyset\}, \{q\} >) =< B, \{\emptyset\}, \{y\} >$, $s(A, \{p\}, \{\emptyset\} >) = < B, \{x\}, \{\emptyset\} >$. Therefore, $s$ is $\mathcal{J}i$-open. Also, $s^{-1}(< B, \{x\}, \{\emptyset\} > = < A, \{p\}, \{\emptyset\} >$, $s^{-1}(< B, \{\emptyset\}, \{\emptyset\} > = < A, \{\emptyset\}, \{\emptyset\} >$ and $s^{-1}(< B, \{x\}, \{y\} > = < A, \{p\}, \{q\} >$. Therefore, $s$ is $\mathcal{J}i$-continuous. Hence, $s$ is $\mathcal{J}i$-homeomorphism. Now, $s(\mathcal{J}cl_i(< A, \{\emptyset\}, \{q\} >)) = s(< A, \{\emptyset\}, \{q\} >) =< B, \{\emptyset\}, \{y\} >$ and $\mathcal{J}cl(s\,(< A, \{\emptyset\}, \{q\} >)) = < B, \{\emptyset\}, \{\emptyset\} >$. Hence, $s(\mathcal{J}cl_i(< A, \{\emptyset\}, \{q\} >)) \neq \mathcal{J}cl(s\,(< A, \{\emptyset\}, \{q\} >))$.
**Theorem 2.2.8:** Every intuitionistic homeomorphism is $\mathcal{J}i$-homeomorphism but not conversely.
**Proof:** Let $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ be intuitionistic homeomorphism then $s$ is intuitionistic continuous and intuitionistic open. Let $L$ be intuitionistic open set in $A$. Since every intuitionistic open set is $\mathcal{J}i$-open and $s$ is $\mathcal{J}$-open map, then $s(L)$ is $\mathcal{J}i$-open in $B$. Hence, $s$ is $\mathcal{J}i$-open. Let $K$ be a intuitionistic open set in $B$. Since, $s$ is intuitionistic continuous, $s^{-1}(K)$ is intuitionistic open in $A$. Since every intuitionistic open is $\mathcal{J}i$-open, $s^{-1}(K)$ is $\mathcal{J}i$-open in $A$ which implies $s$ is $\mathcal{J}i$-continuous. Hence $s$ is $\mathcal{J}i$-homeomorphism.
**Remark 2.2.9:** The reverse implication need not be true as seen from the following example.
**Example 2.2.10:** Let $A = \{i, j\}$ with $\tau_{I_1} = \{\tilde{A}, \tilde{\emptyset}, \mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$ where $\mathcal{V}_1 = < A, \{\emptyset\}, \{j\} >, \mathcal{V}_2 = < A, \{i\}, \{j\} >$ and $\mathcal{V}_3 = < A, \{i\}, \{\emptyset\} >$. Let $B = \{u, v\}$ with a family $\tau_{I_2} = \{\tilde{B}, \tilde{\emptyset}, \mathcal{H}_1, \mathcal{H}_2\}$ where $\mathcal{H}_1 = < B, \{u\}, \{\emptyset\} >$ and $\mathcal{H}_2 = < B, \{\emptyset\}, \{v\} >$ . Define $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ as $s(i) = u$, $s(j) = v$ and $s(A) = B$. Then, $s^{-1}(< B, \{\emptyset\}, \{v\} > = < A, \{\emptyset\}, \{j\} >$ , $s^{-1}(< B, \{u\}, \{\emptyset\} >) = < A, \{i\}, \{\emptyset\} >$ which are $\mathcal{J}i$-open set in $A$. Hence, $s$ is $\mathcal{J}i$-continuous. Also, $s(< A, \{\emptyset\}, \{j\} >) = < B, \{\emptyset\}, \{v\} >$, $s(< A, \{i\}, \{\emptyset\} >) = < B, \{u\}, \{\emptyset\} >$ and $s(< A, \{i\}, \{j\} >) = < B, \{u\}, \{v\} >$ which are $\mathcal{J}i$-open sets in $B$. Hence, $s$ is $\mathcal{J}i$-open map. Therefore, $s$ is $\mathcal{J}i$-homeomorphism. But $s(< A, \{i\}, \{j\} >) = < B, \{u\}, \{v\} >$ which is not intuitionistic open set in $B$. Hence $s$ is not intuitionistic homeomorphism.
**Theorem 2.2.11:** Every intuitionistic $\alpha$-homeomorphism is $\mathcal{J}i$-homeomorphism.
**Proof:** Let $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ be an intuitionistic $\alpha$-homeomorphism, then $s$ is bijective, intuitionistic $\alpha$-continuous and intuitionistic $\alpha$-open. Let $E$ be an intuitionistic-open set in $B$. Since, $s$ is intuitionistic $\alpha$-continuous, $s^{-1}(E)$is intuitionistic $\alpha$-open in X. Since, all the intuitionistic $\alpha$-open set is $\mathcal{J}i$-open, $s^{-1}(E)$ is $\mathcal{J}i$-open in $A$ which implies $s$ is $\mathcal{J}i$-continuous. Let $H$ be a intuitionistic-open set in $A$. Since, $s$ is intuitionistic $\alpha$-open, $s(H)$ is intuitionistic $\alpha$-open in $B$. Since, every intuitionistic $\alpha$-open set is $\mathcal{J}i$-open, $s(H)$ is $\mathcal{J}i$-open in $B$ which implies $s$ is $\mathcal{J}i$-open. Thus, $s$ is $\mathcal{J}i$-homeomorphism.
**Remark 2.2.12:**The reverse implication need not be true as seen from the following example.
**Example 2.2.13:** Let $A = \{g, h\}$ with a family $\tau_{I_1} = \{\tilde{A}, \tilde{\emptyset}, \mathcal{V}_1, \mathcal{V}_2\}$ where $\mathcal{V}_1 = < A, \{\emptyset\}, \{\emptyset\} >$ and $\mathcal{V}_2 =< A, \{h\}, \{\emptyset\} >$. Let $B = \{u, v\}$ with a family $\tau_{I_2} = \{\tilde{B}, \tilde{\emptyset}, \mathcal{H}_1, \mathcal{H}_2\}$ where $\mathcal{H}_1 = < B, \{v\}, \{u\} >$ and $\mathcal{H}_2 = < B, \{\emptyset\}, \{u\} >$ . Define $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ as $s(g) = u$, $s(h) = v$ and $s(A) = B$. Then, $s^{-1}(< B, \{\emptyset\}, \{u\} > = < A, \{\emptyset\}, \{g\} >$ , $s^{-1}(< B, \{v\}, \{u\} >) = < A, \{h\}, \{g\} >$ which are $\mathcal{J}i$-open set in $A$. Hence, $s$ is $\mathcal{J}i$-continuous. Also, $s(< A, \{\emptyset\}, \{\emptyset\} >) = < B, \{\emptyset\}, \{\emptyset\} >$, $s(<$

$A,\{h\},\{\emptyset\}>) =< B,\{v\},\{\emptyset\}>$ which are $\mathcal{J}i$-open set in $B$. Hence, $s$ is $\mathcal{J}i$-open map. Therefore, $s$ is $\mathcal{J}i$-homeomorphism. But $s(< A,\{\emptyset\},\{\emptyset\}>) =< B,\{\emptyset\},\{\emptyset\}>$ which is not intuitionistic $\alpha$-open set in $B$. Hence $s$ is not intuitionistic $\alpha$-homeomorphism.

**Theorem 2.2.14:** Every intuitionistic semi homeomorphism is $\mathcal{J}i$-homeomorphism.

**Proof:** Let $s:(A,\tau_{I_1}) \to (B,\tau_{I_2})$ be an intuitionistic semi-homeomorphism, then $s$ is bijective, intuitionistic semi-continuous and intuitionistic semi-open. Since, every intuitionistic semi-continuous map is $\mathcal{J}i$-continuous and intuitionistic semi-open map is $\mathcal{J}i$-open which implies $s$ is both $\mathcal{J}i$-continuous and $\mathcal{J}i$-open. Therefore, $s$ is $\mathcal{J}i$-homeomorphism.

**Remark 2.2.15:** The reverse implication is not true as seen from the following example.

**Example 2.2.16:** Let $A=\{7,14,21\}$ with a family $\tau_{I_1}=\{\tilde{A},\tilde{\emptyset},\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3\}$ where $\mathcal{V}_1=< A,\{14\},\{7\}>$, $\mathcal{V}_2=< A,\{14\},\{7,21\}>$ , $\mathcal{V}_3=< A,\emptyset,\{7\}>$ and $\mathcal{V}_4=< A,\emptyset,\{7,21\}>$. Let $B=\{m,n,o\}$ with a family $\tau_{I_2}=\{\tilde{B},\tilde{\emptyset},\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3\}$ where $\mathcal{H}_1=< B,\{o\},\{k\}>$ , $\mathcal{H}_2=< B,\{m,n\},\emptyset>$ and $\mathcal{H}_3=< B,\emptyset,\{m,o\}>$. Define $s:(A,\tau_{I_1})\to(B,\tau_{I_2})$ as $s(7)=o$, $s(14)=m$ , $s(21)=n$ and $s(A)=B$. Now, $s^{-1}(< B,\{\emptyset\},\{m,o\}>=< A,\{\emptyset\},\{7,14\}>$, $s^{-1}(< B,\{m,n\},\{\emptyset\}>)=< A,\{14,21\},\{\emptyset\}>$, $s^{-1}(< B,\{m\},\{o\}>=< A,\{14\},\{7\}>$ which are $\mathcal{J}i$-open set in $A$. So, $s$ is $\mathcal{J}i$-continuous. Also, $s(< A,\{14\},\{7\}>) =< B,\{m\},\{o\}>$, $s(< A,\{14\},\{7,21\}>)=< B,\{m\},\{n,o\}>$, $s(< A,\{\emptyset\},\{7\}>) =< B,\{\emptyset\},\{o\}>$, $s(< A,\{\emptyset\},\{7,21\}>) =< B,\{\emptyset\},\{n,o\}>$, which are $\mathcal{J}i$-open set in $B$. Hence, $s$ is $\mathcal{J}i$-open map. Therefore, $s$ is $\mathcal{J}i$-homeomorphism. But , $s^{-1}(< B,\{\emptyset\},\{m,o\}>=< A,\{\emptyset\},\{7,14\}>$,which is not intuitionistic semi-open set in $A$. Therefore, $s$ is not intuitionistic semi-homeomorphism.

**Remark 2.2.17:** Composition of two $\mathcal{J}i$-homeomorphism is not $\mathcal{J}i$-homeomorphism.

**Example 2.2.18:** Let $A=\{17,19,21\}$ with $\tau_{I_1}=\{\tilde{A},\tilde{\emptyset},\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3\}$ where $\mathcal{V}_1=< A,\{17\},\{19\}>$ ,$\mathcal{V}_2=< A,\{\emptyset\},\{19\}>$ , $\mathcal{V}_3=< A,\{17,21\},\{\emptyset\}>$ and $\mathcal{V}_4=< A,\{17\},\{\emptyset\}>$. Let $B=\{i,j,k\}$ with a family $\tau_{I_2}=\{\tilde{B},\tilde{\emptyset},\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3\}$ where $\mathcal{H}_1=< B,\{i\},\{k\}>$ , $\mathcal{H}_2=< B,\{i,j\},\{\emptyset\}>$ and $\mathcal{H}_3=< B,\{\emptyset\},\{i,k\}>$. Let $C=\{2,3,5\}$ with $\tau_{I_3}=\{\tilde{C},\tilde{\emptyset},\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3\}$ where $\mathcal{V}_1=< C,\{3\},\{2\}>$, $\mathcal{V}_2=< C,\{3\},\{2,5\}>$ , $\mathcal{V}_3=< C,\{\emptyset\},\{2\}>$ and $\mathcal{V}_4=< C,\{\emptyset\},\{2,5\}>$. Define $s:(A,\tau_{I_1})\to(B,\tau_{I_2})$ and $t:(B,\tau_{I_2})\to(C,\tau_{I_3})$ as $s(17)=i$ , $s(19)=j$, $s(21)=k$ , $s(A)=B$ , $t(i)=2$, $t(j)=3$ , $t(k)=5$ and $t(B)=C$. Then $s$ and t are $\mathcal{J}i$-homeomorphism. But, $(t\circ s)< A,\{17,21\},\{\emptyset\}>= t(s(< A,\{17,21\},\{\emptyset\}>))=t(< B,\{i,k\},\{\emptyset\}>) =< C,\{2,5\},\{\emptyset\}>$ which is not $\mathcal{J}i$-open set in $C$. Hence, $(t\circ s)$ is not $\mathcal{J}i$-open map. Therefore, $(t\circ s)$ is not $\mathcal{J}i$-homeomorphism.

**Theorem 2.2.19:** If $s:(A,\tau_{I_1})\to(B,\tau_{I_2})$ and $t:(B,\tau_{I_2})\to(C,\tau_{I_3})$ are $\mathcal{J}i$-homeomorphism where $B$ is a $\mathcal{J}i$-$T_{1/2}$ space then $t\circ s:(A,\tau_{I_1})\to(C,\tau_{I_3})$ is also $\mathcal{J}i$-homeomorphism.

**Proof:** Let $M$ be intuitionistic open in $C$. Since $t$ is $\mathcal{J}i$-homeomorphism, $t$ is $\mathcal{J}i$-continuous. Therefore $t^{-1}(M)$ is $\mathcal{J}i$-open in $B$. Since $B$ is $\mathcal{J}i$-$\boldsymbol{T}_{\mathbf{1/2}}$ space , $t^{-1}(M)$ is intuitionistic open. Therefore $s^{-1}(t^{-1}(M))$ is $\mathcal{J}i$-open in $A$. Hence $(t\circ s)$ is $\mathcal{J}i$-continuous. Let $L$ be intuitionistic open in $A$. Then $s(L)$ is $\mathcal{J}i$-open in $B$. Since $B$ is $\mathcal{J}i$-$\boldsymbol{T}_{\mathbf{1/2}}$ space , $s(L)$ is intuitionistic open. Hence $t(s(L))$ is $\mathcal{J}i$-open in $C$. Hence $(t\circ s)$ is $\mathcal{J}i$-open map. Therefore $(t\circ s)$ is $\mathcal{J}i$-homeomorphism.

### 2.3. Intuitionistic Strongly $\boldsymbol{i}$-homeomorphism

**Definition 2.3.1:** A bijection $s:(A,\tau_{I_1})\to(B,\tau_{I_2})$ is said to be intuitionistic strongly homeomorphism if both $s$ and $s^{-1}$ are intuitionistic irresolute.

**Definition 2.3.2:** A bijection $s:(A,\tau_{I_1})\to(B,\tau_{I_2})$ is said to be intuitionistic strongly i-homeomorphism if both $s$ and $s^{-1}$ are $\mathcal{J}i$-irresolute.

**Example 2.3.3:** Let $A=\{i,j\}$ with a family $\tau_{I_1}=\{\tilde{A},\tilde{\emptyset},\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3\}$ where $\mathcal{V}_1=< A,\{\emptyset\},\{j\}>$, $\mathcal{V}_2=< A,\{i\},\{j\}>$ and $\mathcal{V}_3=< A,\{i\},\emptyset>$. Let $B=\{x,y\}$ with a family $\tau_{I_2}=\{\tilde{B},\tilde{\emptyset},\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3\}$ where $\mathcal{H}_1=< B,\emptyset,\emptyset>$, $\mathcal{H}_2=< B,\{x\},\emptyset>$ and $\mathcal{H}_3=< B,\{x\},\{y\}>$. Define $s:(A,\tau_{I_1})\to(B,\tau_{I_2})$

as $s(i) = x$, $s(j) = y$ and $s(A) = B$. Now, $s^{-1}(< B,\{\emptyset\},\{\emptyset\} >) = < A,\{\emptyset\},\{\emptyset\} >$, $s^{-1}(< B,\{\emptyset\},\{x\} >) = < A,\{\emptyset\},\{i\} >$, $s^{-1}(< B,\{\emptyset\},\{y\} >) = < A,\{\emptyset\},\{j\} >$, $s^{-1}(< B,\{x\},\{\emptyset\} >) = < A,\{i\},\{\emptyset\} >$, $s^{-1}(< B,\{y\},\{\emptyset\} >) = < A, \{j\},\{\emptyset\} >$ and $s^{-1}(< B,\{x\},\{y\} > = < A,\{i\},\{j\} >$. Therefore, $s$ is $\mathcal{J}i$-irresolute. Also, $s(< A,\{\emptyset\},\{\emptyset\} > = < B,\{\emptyset\},\{\emptyset\} >$, $s(< A,\{\emptyset\},\{i\} > = < B,\{\emptyset\},\{x\} >$, $s(< A,\{\emptyset\},\{j\} > = < B,\{\emptyset\},\{y\} >$, $s(< A,\{i\},\{\emptyset\} > = < B,\{x\},\{\emptyset\} >$, $s(< A,\{j\}, \{\emptyset\} >) = < B,\{y\},\{\emptyset\} >$ and $s(< A,\{i\},\{j\} > = < B,\{x\},\{y\} >$. Therefore, $s^{-1}$ is $\mathcal{J}i$-irresolute. Hence $s$ is intuitionistic strongly $i$-homeomorphism

We denote the family of all intuitionistic strongly $i$-homeomorphism of an Intuitionistic topological space $(A,\tau_{I_1})$ into itself by $\mathcal{J}Si\text{-}h(A)$.

**Theorem 2.3.4:** Every intuitionistic strongly $i$-homeomorphism is a $\mathcal{J}i$-homeomorphism.

**Proof:** Let $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ be a bijective map which is intuitionistic strongly $i$-homeomorphism. Then $s$ and $s^{-1}$ are $\mathcal{J}i$-irresolute. Since, every $\mathcal{J}i$-irresolute are $\mathcal{J}i$-continuous, $s$ and $s^{-1}$ are $\mathcal{J}i$-continuous. Since, $s^{-1}$ is $\mathcal{J}i$-continuous, $s$ is $\mathcal{J}i$-open map. Thus, $s$ is both $\mathcal{J}i$-continuous and $\mathcal{J}i$-open. Therefore, $s$ is $\mathcal{J}i$-homeomorphism.

**Remark 2.3.5:** The reverse implication need not be true as seen from the following example.

**Example 2.3.6:** Let $A = \{u, v\}$ with a family $\tau_{I_2} = \{\tilde{A},\widetilde{\emptyset},\mathcal{H}_1,\mathcal{H}_2\}$ where $\mathcal{H}_1 = < A,\{v\},\{u\} >$ and $\mathcal{H}_2 = < A,\{\emptyset\},\{u\} >$ . Let $B = \{k, l\}$ with a family $\tau_{I_2} = \{\tilde{B},\widetilde{\emptyset},\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3\}$ where $\mathcal{H}_1 =< B,\{\emptyset\},\{\emptyset\} >$, $\mathcal{H}_2 = < B,\{k\},\{\emptyset\} >$ and $\mathcal{H}_3 =< B,\{k\},\{l\} >$. Define $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ as $s(u) = l$ , $s(v) = k$ and $s(A) = B$. Then, $s^{-1}(< B,\{k\},\{\emptyset\} > = < A,\{v\}, \{\emptyset\} >$, $s^{-1}(< B,\{k\},\{l\} > ) = < A,\{v\},\{u\} >$ , $s^{-1}(< B,\{\emptyset\},\{\emptyset\} >) = < A,\{\emptyset\},\{\emptyset\} >$ which are $\mathcal{J}i$-open set in $A$. So, $s$ is $\mathcal{J}i$-continuous. Also, $s(< A,\{v\},\{u\} >) = < B,\{k\},\{l\} >$, $s(< A,\{\emptyset\},\{u\} >) = < B,\{\emptyset\},\{l\} >$ which are $\mathcal{J}i$-open set in $B$. Hence, $s$ is $\mathcal{J}i$-open map. Therefore, $s$ is $\mathcal{J}i$-homeomorphism. But, $s^{-1}(< B,\{\emptyset\},\{k\} > = < A,\{\emptyset\}, \{v\} >$,which is not $\mathcal{J}i$-open set in $A$. Therefore, $s$ is not $\mathcal{J}i$-irresolute. Hence, $s$ is not intuitionistic strongly $i$-homeomorphism.

**Theorem 2.3.7:** If $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ and $t : (B,\tau_{I_2}) \to (C,\tau_{I_3})$ are intuitionistic strongly $i$-homeomorphism then $t \circ s : (A,\tau_{I_1}) \to (C,\tau_{I_3})$ is also intuitionistic strongly $i$-homeomorphism.

**Proof:**(i) $(t \circ s)$ is $\mathcal{J}i$-irresolute

Let $P$ be a $\mathcal{J}i$-open in $C$. Now, $(t \circ s)^{-1}(P)= s^{-1}(t^{-1}(P))=s^{-1}(Q)$ where $Q= t^{-1}(P)$. By hypothesis, $Q= t^{-1}(P)$ is $\mathcal{J}i$-open in $B$ and again, by hypothesis $s^{-1}(Q)$ is $\mathcal{J}i$-open in $A$.

(ii) $(t \circ s)^{-1}$is $\mathcal{J}i$-irresolute

Let $G$ be a $\mathcal{J}i$-open in $A$. By hypothesis, $s(G)$ is $\mathcal{J}i$-open in $B$. Again, by hypothesis $(t \circ s)(G) = t(s(G))$ is $\mathcal{J}i$-open in $C$. Thus, $(t \circ s)^{-1}$is $\mathcal{J}i$-irresolute.

From (i) and (ii), $t \circ s : (A,\tau_{I_1}) \to (C,\tau_{I_3})$ is also intuitionistic strongly $i$-homeomorphism.

**Theorem 2.3.8:** Every intuitionistic strongly $i$-homeomorphism is $\mathcal{J}i$-irresolute.

**Proof:** Obvious from the definition.

**Remark 2.3.9:** The reverse implication need not be true as shown in the following example.

**Example 2.3.10:** Let $A = \{w, e\}$ with a family $\tau_{I_1} = \{\tilde{A},\widetilde{\emptyset},\mathcal{V}_1,\mathcal{V}_2\}$ where $\mathcal{V}_1 = < A,\{\emptyset\},\{ e\} >$ and $\mathcal{V}_2 =< A,\{w\},\emptyset >$. Let $B = \{o, n\}$ with a family $\tau_{I_2} = \{\tilde{B},\widetilde{\emptyset},\mathcal{H}_1,\mathcal{H}_2\}$ where $\mathcal{H}_1 = < B, \{n\},\{o\} >$ and $\mathcal{H}_2 = < B,\{\emptyset\},\{o\} >$ . Define $s : (A,\tau_{I_1}) \to (B,\tau_{I_2})$ as $s(w) = o$, $s(e) = n$ and $s(A) = B$. Then, $s^{-1}(< B,\{\emptyset\},\{\emptyset\} > = < A,\{\emptyset\},\{\emptyset\} >$, $s^{-1}(< B,\{o\},\{\emptyset\} > ) = < A,\{e\},\{\emptyset\} >$ ,$s^{-1}(< B,\{\emptyset\},\{o\} >) = < A,\{\emptyset\},\{e\} >$,$s^{-1}(< B,\{n\},\{\emptyset\} >) = < A,\{w\}, \{\emptyset\} >$ and $s^{-1}(< B,\{n\},\{o\} > = < A,\{w\},\{e\} >$. Therefore, $s$ is $\mathcal{J}i$-irresolute. But $(s^{-1})^{-1}(< A,\{\emptyset\},\{w\} >) = < B,\{\emptyset\},\{n\} >$ which is not $\mathcal{J}i$-open in $B$. Hence $(s^{-1})$ is not $\mathcal{J}i$-irresolute. Therefore, $s$ is not intuitionistic strongly $i$-homeomorphism.

**Theorem 2.3.11:** The set $\mathcal{J}Si\text{-}h(A)$ is a group under the composition of maps.

**Proof:** Define a binary operation ‘ * ’ from $\mathcal{J}Si\text{-}h(A)$ x $\mathcal{J}Si\text{-}h(A) \to \mathcal{J}Si\text{-}h(A)$ , by $s * t = s \circ t$ for all $s$ and $t$ in $\mathcal{J}Si\text{-} h(A)$ and $\circ$ is the usual operation of composition of maps. Then by theorem 2.3.7, $s \circ$

$t \in \mathcal{J}Si\text{-}h(A)$ . We know that the composition of maps are associative and the identity map $i : \mathcal{J}Si\text{-}h(A) \to \mathcal{J}Si\text{-}h(A)$ belonging to $\mathcal{J}Si\text{-}h(A)$ is the identity element. If $s \in \mathcal{J}Si\text{-}h(A)$ then $s^{-1} \in \mathcal{J}Si\text{-}h(A)$ such that $s \circ s^{-1} = s^{-1} \circ s = i$ and hence inverse exists for each element of $\mathcal{J}Si\text{-}h(A)$. Therefore, $\mathcal{J}Si\text{-}h(A)$ is a group under the composition of maps.

**Theorem 2.3.12:** Let $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ be an intuitionistic strongly $i$-homeomorphism. Then $s$ induces an isomorphism from the group $\mathcal{J}Si\text{-}h(A)$ onto the group $\mathcal{J}Si\text{-}h(A)$.

**Proof:** Using the map $s$, we define a map $\psi_s$: $\mathcal{J}Si\text{-}h(A) \to \mathcal{J}Si\text{-}h(B)$ by $\psi_s$(h) = $s \circ t \circ s^{-1}$ for each $t \in \mathcal{J}Si\text{-}h(A)$. By theorem 2.3.7, $\psi_s$ is well defined in general, because $s \circ t \circ s^{-1}$ is a intuitionistic strongly $i$-homeomorphism for every intuitionistic strongly $i$-homeomorphism $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$. Clearly, $\psi_s$ is bijective. Further for all $t_1, t_2 \in \mathcal{J}Si\text{-}h(A)$ , $\psi_s(t_1 \circ t_2) = s \circ (t_1 \circ t_2) \circ s^{-1} = (s \circ t_1 \circ s^{-1}) \circ (s \circ t_2 \circ s^{-1}) = \psi_s(t_1) \circ \psi_s(t_2)$. Therefore, $\psi_s$ is a homeomorphism and hence it induces an isomorphism induced by $s$.

**Theorem 2.3.13:** Intuitionistic strongly $i$-homeomorphism is an equivalence relation on the collection of all Intuitionistic topological spaces.

**Proof:** Reflexive and symmetry are obvious and transitivity follows from theorem 2.3.7.

**Theorem 2.3.14:** If $s : (A, \tau_{I_1}) \to (B, \tau_{I_2})$ is an intuitionistic strongly $i$-homeomorphism, where $B$ is $\mathcal{J}i\text{-}T_{1/2}$ space then $\mathcal{J}cl_i(s^{-1}(H)) = s^{-1}(\mathcal{J}cl(H))$ for every IS $H$ in $B$.

**Proof:** Let $H \subseteq B$. Then $\mathcal{J}cl(H)$ is an $\mathcal{J}$-closed set in $B$. Since $s$ is an $\mathcal{J}i$-irresolute mapping, $s^{-1}(\mathcal{J}cl(H))$ is an $\mathcal{J}i$-closed set in $A$. This implies $\mathcal{J}cl_i(s^{-1}(\mathcal{J}cl(H))) = s^{-1}(\mathcal{J}cl(H))$. Now $\mathcal{J}cl_i(s^{-1}(H)) \subseteq \mathcal{J}cl_i(s^{-1}(\mathcal{J}cl(H))) = s^{-1}(\mathcal{J}cl(H))$. Since $s^{-1}$ is $\mathcal{J}i$-irresolute mapping and $\mathcal{J}cl_i(s^{-1}(H))$ is an $\mathcal{J}i$-closed in $A$, $(s^{-1})^{-1}(\mathcal{J}cl_i(s^{-1}(H))) = s(\mathcal{J}cl_i(s^{-1}(H)))$ is an $\mathcal{J}i$-closed in $B$. Now $H \subseteq (s^{-1})^{-1}(s^{-1}(H)) \subseteq (s^{-1})^{-1}(\mathcal{J}cl_i(s^{-1}(H))) = s(\mathcal{J}cl_i(s^{-1}(H)))$. Therefore $\mathcal{J}cl(H) \subseteq \mathcal{J}cl(s(\mathcal{J}cl_i(s^{-1}(H)))) = s(\mathcal{J}cl_i(s^{-1}(H)))$ since $B$ is an $\mathcal{J}i\text{-}T_{1/2}$ space. Hence $s^{-1}$ $(\mathcal{J}cl(H)) \subseteq s^{-1}(s(\mathcal{J}cl_i(s^{-1}(H)))) \subseteq \mathcal{J}cl_i(s^{-1}(H))$. Hence, $s^{-1}(\mathcal{J}cl(H)) \subseteq \mathcal{J}cl_i(s^{-1}(H))$.Thus we get $\mathcal{J}cl_i(s^{-1}(H)) = s^{-1}(\mathcal{J}cl(H))$ and hence the proof.

## III. Conclusion

In this paper we have defined the $\mathcal{J}i$-homeomorphism and intuitionistic strongly $i$-homeomorphism and studied their properties. We conclude that the results of $\mathcal{J}i$-homeomorphism and intuitionistic strongly $i$-homeomorphism is very useful for future works in Intuitionistic Topological Spaces.

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