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WIENER POLYNOMIAL OF SPLITTING GRAPH OF A GRAPH

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Abstract:

Let *G* be a connected graph on *n* vertices ,for each vertex *v* of a graph *G*, take a new vertex v'. Join v' to all the vertices of *G* adjacent to *v*. If *q* is the parameter, the Wiener polynomial of *G* is $W(G,q) = \sum_{\{u,v\}\subseteq V(G)} q^{d(u,v)}$. Here the sum is taken over all set of vertices. In this paper we obtain a relation between the Wiener polynomial of Splitting graph *S*(*G*) and the graph *G* when $G \cong P_n, C_n, K_n, W_{1,n}, P_n \circ K_2, C_n \circ K_2, T_{n,k}$.

Keywords: splitting Graph, Wiener Index, Wiener Polynomial.

1. ***Introduction***: A topological index (*TI*) is a number associated with a chemical structure presented by a connected graph where atoms are represented by vertices (points) and covalent bonds by edges (lines) connecting adjacent vertices [1] . Since then, many new *Topological Indices* have been added for quantitative structure-property relationship (QSPR). Apparently, the chemist Harry Wiener was the first to point out in 1947 that *W*(*G*) is well correlated with certain physico-chemical properties of organic compound from which *G* is derived. In 1976, Entringer, Jackson and Snyder published a paper [5] which is historically the first mathematics paper on *W*(*G*). The graphical invariant *W*(*G*) has been studied by many researchers [2,3,4] under different names such as distance, transmission, total status and sum of all distances. A quantity closely related to *W*(*G*) is the mean distance between the vertices or the average distance between the vertices of *G* when *G* represents a network.(*eg*. An interconnection network connecting many processors). The average distance of *G* between the nodes of the network is the measure of the average delay of messages for traversing from one node to another. Today , the *wiener index* is one of the most widely used topological index in chemical graph theory. Due to its strong connection to chemistry, it is related to boiling point , heat of evaporation, heat of formation, surface tension, total energy of polymers, ultra sonic sound velocity ,internal energy etc.[6]. For this reason wiener index is widely studied by chemists.

The Wiener index of a connected graph is defined as the sum of the distances between all unordered pairs of vertices of the graph, where the distance between two vertices is the length of the shortest path connecting them in the graph.

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$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v)$$

where $d(u, v)$ denote the distance between vertices u and v in a graph G. The Wiener polynomial is a related generating function which was first defined by HaruoHosoya [7] with this name, in honor of Harry Wiener but also known today as Hosoya polynomial, extends this concept to capture the complete distribution of distances in graph. If q is a parameter, then the Wiener polynomial of G is

$$W(G;q) = \sum_{\{u,v\} \subseteq V(G)} q^{d(u,v)}$$

This paper gives the expressions for Wiener polynomial of *Splitting graph* of path, cycle, wheel, tadpole, and sunlet graphs.

2. *Splitting graph of a graph* : The splitting graph is introduced by Prof . E. Sampath Kumar, Prof. H.B. Walikar in [1]. For a graph G , Splitting graph is denoted by $S(G)$. For each vertex v of a graph G, take a new vertex v' .Join v' to all the vertices of G adjacent to v. The graph $S(G)$ thus obtained is called Splitting graph or Duplicate graph of G. In [1], they have studied some properties of $S(G)$ and obtained the characterization of $S(G)$. Prof. Jan Mycielski, in his work in sets and logic used a graph called *shadow graph* $S(G)$. For a graph G , the *shadow graph* $S(G)$ is obtained from G by adding for each vertex v of G ,a new vertex v' called shadow vertex of v, and joining v' to the neighbour of v in G. The splitting graph of $P6$ is shown in the Figure 1.

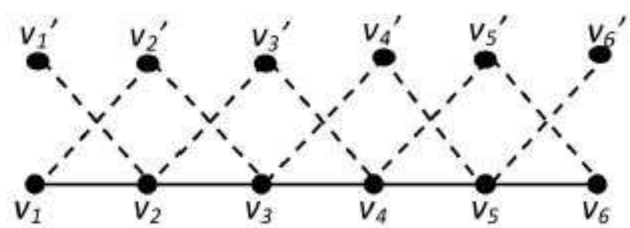

Figure 1: The splitting graph of P_6

3. *Observations on S(G)*:

1. A vertex of G and its duplicate vertices are not adjacent in $S(G)$
2. All duplicate vertices induces a null graph.
3. $d(v_i)\ in\ G = d(v_i')\ in\ S(G)$.

4. $d(v_i)\ in\, S(G) = 2d(v_i')\ in\ S(G)$

5. By definition of splitting graph, $d(v_i, v_j', G) = d(v_i, v_j', S(G))$

Note: As the splitting graph consists of the vertices of given graph and its duplicate vertices, for convenience sake, partition the vertex set of *S*(*G*) into the sets *A*,*B* as below.

$A = \{v_i / v_i \in V(G)\}$

$B = \{v_i^{/} / v_i^{/} \in V(S(G)) - V(G)\}$

In order to demonstrate the validity of the theorem, we adopt the following notation:

$d_A(G,i) =$ *number of pairs of vertices in the set A, at a distance i*

$d_B(G,i) =$ *number of pairs of vertices in the set B, at a distance i*

$d_{AB}(G,i) =$ *number of pairs of vertices of which one is in the set A and the other is in the set B, that are at a distance i*

$d_A(v_i, v_j) =$ *distance between the vertices* v_i, v_j *among the vertices of the set A*

$d_B(v_i, v_j) =$ *distance between the vertices* v_i, v_j *among the vertices of the set B*

$d_{AB}(v_i, v_j) =$ *distance between the vertices* v_i, v_j *among the vertices of the set A, B*

With the above notation, the Wiener polynomial of Splitting graph is given as

$W(S(G,q)) = a_1 q + a_2 q^2 + a_3 q^3 + ...+ a_i q^i$

It can be easily observed that $a_i = d_A(G,i) + d_B(G,i) + d_{AB}(G,i)$

By computing these three terms we found the coefficients of wiener polynomial.

Note: In the following proofs consider j as n whenever $j \equiv 0 (\bmod n)$.

***Theorem*1**:*For a given integer $n \geq 2$ and a path P_n on n vertices, the Wiener polynomial of $S(P_n)$ is given by*

$$W(S(P_n), q) = 3q + 2q^2 + q^3 \text{ for } n = 2$$
$$= 3(n-1)q + 2(n-1)q^2 + (2n-4)q^3 + \sum_{i=4}^{n-1}(n-i)q^i \text{ for } n \geq 3$$

Proof: Since S(P$_2$) is isomorphic to P$_4$.It follows that $W(S(P_2),q) = W(P_4,q) = 3q + 2q^2 + q^3$ We now consider the case when $n \geq 3$

We first observe in this case that the diam(S(P_n)) = n-1.

Let $\{v_1, v_2, v_3, ..., v_n\}$ be the vertices of P_n taken in order and $\{v_1', v_2', ...v_n'\}$ be the corresponding replication of vertices in $S(P_n)$. Then, for each i, $1 \leq i \leq n-1$ and $i \neq j$, we get $1 \leq d_A(v_i, v_j) \leq n-1$ and $d_A(P_n, i) = n - i$ and hence $\sum_{\{v_i, v_j\} \subseteq A} q^{d_A(v_i, v_j)} = \sum_{i=1}^{n-1} d_A(P_n, i) q^i = \sum_{i=1}^{n-1} (n-i) q^i$.

<u>The distance between the vertices of *A* and *B* is</u>

$$d_{AB}(v_i, v_j') = \begin{cases} 2 \ \ for \ \ i = j \\ |i-j|, 1 \leq i, j \leq n \end{cases}$$

And $d_{AB}(P_n, i) = \begin{cases} 2(n-i) \ \ for \ \ i \neq 2 \\ 3n - 2i \ \ for \ i = 2 \end{cases}$

$$\sum_{\{v_i, v_j'\} \in V(AB)} q^{d(v_i, v_j')} = nq^2 + 2\left((n-1)q^1 + (n-2)q^2 + (n-3)q^3 + \ldots \qquad + 1q^{n-1}\right) = \ nq^2 + 2\sum_{i=1}^{n-1} (n-i) q^i$$

The distance between the vertices of *B* is

$$d_B(v'_i, v_j') = \begin{cases} 3 \ \ for \ \ j = i+1, 1 \leq i \leq n-1 \\ j - i, 1 \leq i < j \leq n \end{cases}$$

and $d_B(P_n, i) = \begin{cases} 2n - 4 \ \ for \ \ i = 3 \\ n - i \ \ for \ 2 \leq i \leq n-1 \end{cases}$

$$\sum_{\{v'_i, v_j'\} \in V(B)} q^{(v'_i, v_j')} = (n-1)q^3 + \left((n-2)q^2 + (n-3)q^3 + \ldots \qquad + 1q^{n-1}\right) = \ (n-1)q^3 + \sum_{i=2}^{n-1} (n-i) q^i$$

By adding all the polynomials ,we get

$$W(S(P_n, q)) = 4W(P_n, q) + (1-n)q + nq^2 + (n-1)q^3 \ \ for \ \ n \geq 4$$

Theorem2: The wiener polynomial of splitting graph of cycle $S(C_n)$ is

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$$W(S(C_n,q)) = 4W(C_n,q) + n(q^2+q^3-q), n \geq 4$$
$$= 9q + 3q^2 + 3q^3 \ for\ n = 3$$

Proof: For ***odd n***

Let $\{v_1, v_2, v_3 ..... v_n\}$ be the vertices of C_n and $\{v_1^{/}, v_2^{/}, ... v_n^{/}\}$ be the corresponding duplicate vertices in $S(C_n)$

The distance between the vertices of A is

$$d_A(v_i, v_j) = x, 1 \leq x \leq \frac{n-1}{2}, 1 \leq i, j \leq n, i < j, j = x + i (\bmod n)$$

and $d_A(C_n, i) = n, 1 \leq i \leq \frac{n-1}{2}$

$$\sum_{\{v_i, v_j\} \in V(A)} q^{(v_i, v_j)} = nq^1 + nq^2 + nq^3 + ... \qquad + nq^{\frac{n-1}{2}} \ = \ n\sum_{i=1}^{\frac{n-1}{2}} q^i$$

The distance between the vertices of A,B is

$$d_{AB}(v_i, v_j^{/}) = 2 \ for\ i = j$$
$$= x, 1 \leq x \leq \frac{n-1}{2}, 1 \leq i, j \leq n, \ j = i + x \ (\bmod n)$$
$$= n - x + i (\bmod n)$$

$$d_{AB}(C_n, i) = \begin{cases} 3n \ for\ \ i = 2 \\ 2n \ for\ 1 \leq i \leq \frac{n-1}{2}, \ i \neq 2. \end{cases}$$

$$\sum_{\{v_i, v_j^{/}\} \in V(AB)} q^{(v_i, v_j^{/})} = nq^2 + 2\left(nq + nq^2 + nq^3 + ... \qquad + nq^{\frac{n-1}{2}} \right) = nq^2 + \ 2n\sum_{i=1}^{\frac{n-1}{2}} q^i$$

The distance between the vertices of B is

$$d_B(v_i^{/}, v_j^{/}) = \begin{cases} 3 \ for\ i = j+1, 1 \leq i \leq n-1 \\ x, \ 2 \leq x \leq \frac{n-1}{2}, \ 1 \leq i, j \leq n, \ j = i + x (\bmod n) \end{cases}$$

$$d_B(C_n,i)=\begin{cases}2n \ \ for \ \ i=3\\ n \ \ for \ \ 2\le i\le \dfrac{n-1}{2},\ i\ne 3.\end{cases}$$

The wiener polynomial in this case becomes

$$\sum_{\{v_i^{/},v_j^{/}\}\in V(AB)} q^{(v_i^{/},v_j^{/})}=nq^3+\left(nq^2+nq^3+\ldots \qquad +nq^{\frac{n-1}{2}}\right) = \quad nq^3+n\sum_{i=2}^{\frac{n-1}{2}}q^i$$

By adding all the polynomials ,we get the wiener polynomial of splitting graph of cycle for odd n is

$$W(S(C_n,q))=4W(C_n,q)+n(q^2+q^3-q)$$

***For even n*:**

The distance function between the vertices of$A$is given as

$$d_A(v_i,v_j)=\begin{cases}x,\ 1\le x\le \dfrac{n-2}{2},\ 1\le i,\ j\le n,\ \ j=i+x(\text{mod}\, n)\\ \dfrac{n}{2}, 1\le i\le \dfrac{n}{2}, \dfrac{n}{2}+1\le j\le n\end{cases}.$$

$$d_A(C_n,i)=\begin{cases}n, 1\le i\le \dfrac{n-2}{2}\\ \dfrac{n}{2}, i=\dfrac{n}{2}\end{cases}$$

$$\sum_{\{v_i,v_j\}\in V(A)} q^{(v_i,v_j)}=nq^1+nq^2+nq^3+\ldots \qquad +nq^{\frac{n-2}{2}} \quad = \quad n\sum_{i=1}^{\frac{n-2}{2}}q^i+\frac{n}{2}q^{\frac{n}{2}}$$

Distance between the vertices of the set A,B of the cycle is given as

$$d_{AB}(v_i,v_j^{/})=\begin{cases}2 \ \ for \ \ i=j\\ x,\ 1\le x\le \dfrac{n}{2},\ 1\le i,\ j\le n,\ \ j=i+x \ (\text{mod}\, n)\\ \qquad\qquad\qquad\qquad\qquad =n-x+i(\text{mod}\, n)\end{cases}$$

$$d_{AB}(C_n,i)=\begin{cases}3n \;\; for \;\; i=2\\ 2n \;\; for \; 1\le i\le \dfrac{n-2}{2},\, i\ne 2.\\ n \;\; for\, i=\dfrac{n}{2}\end{cases}$$

The corresponding wiener polynomial is given as

$$\sum_{\{v_i,v_j^{/}\}\in V(AB)} q^{(v_i,v_j^{/})}=nq^2+2\left(nq+nq^2+nq^3+\ldots \qquad +nq^{\frac{n-2}{2}}\right)+nq^{\frac{n}{2}}=nq^2+\;\; 2n\sum_{i=1}^{\frac{n-2}{2}}q^i+nq^{\frac{n}{2}}$$

The distance between the vertices of B is defined as

$$d_B(v_i^{/},v_j^{/})=\begin{cases}3 \;\; for \;\; i=j+1\\ x,\; 2\le x\le \dfrac{n}{2},\; 1\le i,\, j\le n,\;\; j=i+x(\mathrm{mod}\, n)\\ \dfrac{n}{2}, 1\le i\le \dfrac{n}{2},\;\; \dfrac{n}{2}+1\le j\le n\end{cases}$$

$$d_B(C_n,i)=\begin{cases}2n \;\; for \;\; i=3\\ n \;\; for \;\; 2\le i\le \dfrac{n-2}{2},\, i\ne 3.\\ \dfrac{n}{2} \;\; for\, i=\dfrac{n}{2}\end{cases}$$

Thus the wiener polynomial in this case becomes

$$\sum_{\{v_i^{/},v_j^{/}\}\in V(B)} q^{(v_i^{/},v_j^{/})}=nq^3+\left(nq^2+nq^3+\ldots \qquad +nq^{\frac{n-2}{2}}\right)+\frac{n}{2}q^{\frac{n}{2}} = \;\; nq^3+n\sum_{i=2}^{\frac{n-2}{2}}q^i+\frac{n}{2}q^{\frac{n}{2}}$$

Thus the corresponding Wiener polynomial is given as

$$W(S(C_n,q))=4W(C_n,q)+n(q^2+q^3-q)$$

***Theorem*3**: *The wiener polynomial of* $S(W_{1,n})$ *is*

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$W(S(W_{1,n},q)) = 6nq + (n+1)^2 q^2,\ n \geq 4$

Proof: Let v_0 be the centralvertex and $v_1, v_2, \ldots v_n$ be the rim vertices of the wheel $W_{1,n}$ .And $v_0{}^{/}, v_1{}^{/}, v_2{}^{/}, \ldots, v_n{}^{/}$ be the corresponding duplicate vertices in $S(W_{1,n})$. To find the wiener index we find the distance between the vertices as in the following .

The distancebetween the vertices of A is given as

$$d_A(W_{1,n}, i) = \begin{cases} 2n \ \ for \ \ i = 1 \\ \dfrac{n(n-3)}{2} \ \ for \ \ i = 2 \end{cases}$$

$$\sum_{\{v_i, v_j\} \in V(A)} q^{(v_i, v_j)} = 2nq + \left(\binom{n-1}{2} - 1 \right) q^2$$

The central vertex v_0 is at a distance 2 $v_0{}^{/}, v_i{}^{/}$ respectively. v_0 is at a distance one to $v_i{}^{/}$ and $v_0{}^{/}$ is at a distance one to v_i. Further each v_i is at a distance one to $v_{i-1}{}^{/}, v_{i+1}{}^{/}, (2 \leq i \leq n-1)$ and v_1 to $v_n{}^{/}$, and v_n to $v_1{}^{/}$. Thus there are 4n pairs of vertices with distance one and all other pairs of vertices $\left((n-1)^2 - 1\right)$are at a distance 2 .Thus

$$d_{AB}(v_i, v_j{}^{/}) = \begin{cases} 2 \ \ for \ \ i = j \\ 1 \ for \ |i - j| = 1, (1,n) \,\&\, (n,1) \\ 1 \ \ for\, i = 0, 1 \leq j \leq n \ \&\ \ j = 0, 1 \leq i \leq n \end{cases}$$

$$d_{AB}(W_{1,n}, i) = \begin{cases} 4n \ \ for \ \ i = 1 \\ (n-1)^2 \ \ for \ \ i = 2 \end{cases}$$

$$\sum_{\{v_i, v_j{}^{/}\} \in V(AB)} q^{(v_i, v_j{}^{/})} = 4nq + (n-1)^2 q^2$$

Each $v_i{}^{/}, (0 \leq i \leq n)$ is at a distance 2 to $v_j{}^{/}$ and there are $\binom{n+1}{2}$ such $v_i{}^{/}s$ .Thus the distance function and the polynomial for the vertices belongs to B is given as

$$d_B(v_i{}^{/}, v_j{}^{/}) = 2 \ \ for \ \ i < j, 1 \leq i \leq n, i < j$$

$$d_B(W_{1,n}, i) = \binom{i+1}{2}$$

$$\sum_{(v_i,v_j)} q^{(v_i,v_j)} = \binom{n+1}{2} q^2$$

By adding all above $W(S(W_{1,n}, q)) = 3W((W_{1,n}, q)) - \binom{n+2}{2} q^2$.

***Theorem4* :** The wiener polynomial of $S(K_n)$ is

$$W(S(K_n, q)) = 3W(K_n, q) + \binom{n+1}{2} q^2$$

Proof: Let $v_1, v_2, ..., v_n \in V(K_n)$.Since all v_i s are at a distance one and there are $\binom{n}{2}$ such pairs, the corresponding wiener polynomial is $\binom{n}{2} q$.

Further $d(v_i, v_j^{/}) = \begin{cases} 1, i \neq j \\ 2, other\ wise \end{cases}$

Thus the corresponding wiener polynomial is $2\binom{n}{2} q + nq^2$.All duplicate vertices are at a distance 2 between them and there are $\binom{n}{2}$ such pairs. Thus the wiener polynomial in this case is $\binom{n}{2} q^2$.

Thus by adding all above $W(S(K_n, q)) = 3\binom{n}{2} q + \left(\binom{n}{2} + n\right) q^2 = W(K_n, q) + \binom{n+1}{2} q^2$

***Theorem* 5**: The wiener polynomial of splitting graph of sunlet graph $S(C_n \otimes K_2)$ is $\sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$

$$W(S(C_n \otimes K_2)) = 4W(C_n \otimes K_2) + 2n\left(-q + q^2 + q^3\right), n \geq 5$$

Proof: Let $v_1, v_2, ... v_n \in V(C_n), a_1, a_2, ..., a_n$ be the pendent vertices that are adjacent to v_i in $(C_n \otimes K_2)$. $v_1^{/}, v_2^{/}, ... v_n^{/} \in V(C_n), a_1^{/}, a_2^{/}, ... a_n^{/}$ be the corresponding duplicate vertices. For convenience sake we partition the vertex set into the following subsets andfind the distances between them.

$A = Set\, of\ vertices\ of\ sunlet\, graph$

$B = Set\, of\ duplicate\, vertices\ of\ sunlet\, graph$

Further the sets A,B are partitioned as

$A1 = Set\, of\ vertices\ of\ cycle\ of\ sunlet\, graph \quad B1 = Set\, of\ duplicate vertices\, of\ cycle.$

$A2 = Set\, of\ pendent\, vertices\ of\ sunlet\, graph\ B2 = Set\, of\ duplicate\, vertices\ of\ \deg ree\ one$

The distances between the vertices from the above sets are given in ten combitions as follows.

Case1: For odd n

The distance between the vertices of A1is given as in theorem2

Wiener polynomial in this case is given by

$$\sum_{\{v_i,v_j\}\in V(A1)} q^{(v_i,v_j)} = nq^1 + nq^2 + nq^3 + \ldots \qquad + nq^{\frac{n-1}{2}} \; = \; n\sum_{i=1}^{\frac{n-1}{2}} q^i$$

***case* 2**: The distance function between the vertices of A2 is given as

$$d_{A2}(a_i,a_j) = x,\ 3 \le x \le \frac{n+3}{2},\ 1 \le i, j \le n,\ j = i + x - 2(\bmod n).$$

$$d_{A2}(G,i) = n,\ 3 \le i \le \frac{n+3}{2}$$

$$\sum_{\{a_i,a_j\}\in V(A2)} q^{(a_i,a_j)} = nq^3 + nq^4 + \ldots \qquad + nq^{\frac{n+3}{2}} \; = \; n\sum_{i=3}^{\frac{n+3}{2}} q^i$$

***case*3**:

The distance function ,between the vertices of A1, A2 is given as

$$d_{A1,A2}(v_i,a_j) = x, 1 \le x \le \frac{n+1}{2},\ 1 \le i, j \le n,\ j = i + x - 1\,(\bmod n)$$

$$j = n - x + i + 1(\bmod n), 2 \le x \le \frac{n+1}{2}.$$

$$d_{A1A2}(G,i) = \begin{cases} n\ for\ i = 1 \\ 2n, 2 \le i \le \dfrac{n+1}{2} \end{cases}$$

$$\sum_{\{v_i,a_j\}\in V(A1A2)} q^{(v_i,a_j)} = nq^1 + 2\left(nq^2 + nq^3 + \ldots \qquad + nq^{\frac{n+1}{2}}\right) = nq^1 + \ 2n\sum_{i=2}^{\frac{n+1}{2}} q^i$$

***Case* 4**: Distance function between the vertices of the set B1is given as

$$d_{B1}(v_i^{/}, v_j^{/}) = 3 \ \ for \ \ i = j+1$$
$$= x,\ 2 \le x \le \frac{n-1}{2},\ 1 \le i, j \le n,\ \ j = i + x \ (\mathrm{mod}\, n)$$

$$d_{B1}(G,i) = \begin{cases} 2n \ for\, i = 3 \\ n, 2 \le i \le \dfrac{n-1}{2}, n \ne 3 \end{cases}$$

Thus the wiener polynomial in this case becomes

$$\sum_{\{v_i^{/},v_j^{/}\}\in V(B1)} q^{(v_i^{/},v_j^{/})} = nq^3 + \left(nq^2 + nq^3 + \ldots \qquad + nq^{\frac{n-1}{2}}\right) = \quad nq^3 + n\sum_{i=2}^{\frac{n-1}{2}} q^i$$

***case*5**: Distance function between the vertices B2is given as

$$d_{B2}(a_i^{/}, a_j^{/}) = x,\ 3 \le x \le \frac{n+3}{2},\ 1 \le i, j \le n,\ \ j = i + x - 2(\mathrm{mod}\, n)$$

$$d_{B2}(G,i) = n, 3 \le i \le \frac{n+3}{2}$$

Thus the wiener polynomial in this case becomes

$$\sum_{\{a_i^{/},a_j^{/}\}\in V(B2)} q^{(a_i^{/},a_j^{/})} = nq^3 + nq^4 + \ldots \qquad + nq^{\frac{n+3}{2}} \ = \ n\sum_{i=3}^{\frac{n+3}{2}} q^i$$

***Case* 6**: Distance between the duplicate vertices of the sets B1,B2 is given as distance function

$$d_{B1B2}(a_i^{/}, v_j^{/}) = x,\ 1 \le x \le \frac{n+1}{2},\ 1 \le i, j \le n,\ \ j = i + x - 1(\mathrm{mod}\, n)$$
$$j = n - x + i + 1(\mathrm{mod}\, n), 2 \le x \le \frac{n+1}{2}$$

$$d_{B1B2}(G,i)=\begin{cases} n \; for\, i=1 \\ 2n, 2\le i\le \frac{n+1}{2},, i\ne 3 \end{cases}$$

Thus the wiener polynomial in this case becomes

$$\sum_{\{a_i^{/},v_j^{/}\}\in V(B1B2)} q^{(a_i^{/},v_j^{/})} = nq^1+2\left(nq^2+nq^3+\ldots \quad +nq^{\frac{n+1}{2}}\right) = nq^1+2n\sum_{i=2}^{\frac{n+1}{2}} q^i$$

***Case*7**: Distance between the vertices of A1,B2 is given as

$$d_{A1B2}(v_i,a_j^{/}) = x, 1\le x\le \frac{n+1}{2},\; 1\le i,j\le n,\; j=i+x-1\,(\mathrm{mod}\, n)$$

$$j=n-x+i+1(\mathrm{mod}\, n), 2\le x\le \frac{n+1}{2}$$

$$d_{A1B2}(G,i)=\begin{cases} n \; for\, i=1 \\ 2n, 2\le i\le \frac{n+1}{2} \end{cases}$$

$$\sum_{\{v_i,a_j\}\in V(A1B2)} q^{(v_i,a_j)} = nq^1+2\left(nq^2+nq^3+\ldots \quad +nq^{\frac{n+1}{2}}\right) = nq^1+\; 2n\sum_{i=2}^{\frac{n+1}{2}} q^i$$

***Case* 8**: Distance between the vertices of A1,B1 is given as

$$d_{A1B1}(v_i,v_j^{/}) = 2 \;\; for \;\; i=j$$

$$= x,\; 1\le x\le \frac{n-1}{2},\; 1\le i,j\le n,\; j=i+x\; (\mathrm{mod}\, n)$$

$$j=n-x+i(\mathrm{mod}\, n),\; ,2\le x\le \frac{n-1}{2}$$

$$d_{A1B1}(G,i)=\begin{cases} 3n \; for\, i=2 \\ 2n, 1\le i\le \frac{n-1}{2}, i\ne 2 \end{cases}$$

$$\sum_{\{v_i,v_j^{/}\}\in V(A1B1)} q^{(v_i,v_j^{/})} = nq^2+2\left(nq+nq^2+nq^3+\ldots \quad +nq^{\frac{n-1}{2}}\right) = nq^2+\; 2n\sum_{i=1}^{\frac{n-1}{2}} q^i$$

***Case* 9**: Distance between thesets A2,B1 is

$$d_{A2B1}(a_i,v_j^{/}) = \begin{cases} 3 \ for \ i=j \\ x, \ 2 \le x \le \frac{n+1}{2}, \ 1 \le i, j \le n, \ j = i+x-1 \ (\mathrm{mod}\, n) \\ \qquad = n-x+i+1 (\mathrm{mod}\, n) \end{cases}$$

$$d_{A2B1}(G,i) = \begin{cases} n \ for \ i=3 \\ 2n, 2 \le i \le \frac{n+1}{2} \end{cases}$$

$$\sum_{\{v_i,a_j\}\in V(A2B1)} q^{(v_i,a_j)} = nq^3 + 2\left(nq^2 + nq^3 + \ldots \qquad + nq^{\frac{n+1}{2}} \right) = nq3 + \ 2n\sum_{i=2}^{\frac{n+1}{2}} q^i$$

***case*10**: Distance between the sets A2,B2 is

$$d_{A2B2}(a_i,a_j^{/}) = \begin{cases} x, 2 \le x \le \frac{n+3}{2}, \ 1 \le i, j \le n, \ j = i+x-2 \ (\mathrm{mod}\, n) \\ x, \ j = n-x+i+2 (\mathrm{mod}\, n) \end{cases}$$

$$d_{A2B2}(G,i) = \begin{cases} n \ for \ i=2 \\ 2n, 3 \le i \le \frac{n+3}{2} \end{cases}$$

$$\sum_{\{a_i,a_j^{/}\}\in V(A2B2)} q^{(a_i,a_j^{/})} = nq^2 + 2\left(nq^3 + nq^4 + \ldots \qquad + nq^{\frac{n+3}{2}} \right) = nq^2 + \ 2n\sum_{i=3}^{\frac{n+3}{2}} q^i$$

By adding all the polynomials ,we get the wiener polynomial of splitting graph of sun let graph for odd n is

$$W(S(C_n \otimes K_2)) = 4W(C_n \otimes K_2) + 2n\left(-q + q^2 + q^3\right), \ n \ge 5$$

For even n:

***case*1**: The distance between the vertices of A1 is given as in theorem 2

$$\sum_{\{v_i,v_j\}\in V(A1)} q^{(v_i,v_j)} = nq^1 + nq^2 + nq^3 + \ldots \qquad + nq^{\frac{n}{2}-1} + \frac{n}{2}q^{\frac{n}{2}} = \ n\sum_{i=1}^{\frac{n}{2}} q^i + \frac{n}{2}q^{\frac{n}{2}}$$

***case* 2**: Distance between the vertices of the set A2

$$d_{A2}(a_i,a_j)=x,\ 3\le x\le \frac{n}{2}+2,\ 1\le i,j\le n,\ j=i+x-2(\bmod n).$$

$$d_{A2}(G,i)=n\ for\ 3\le i\le \frac{n+4}{2}$$

$$\sum_{\{a_i,a_j\}\in V(A2)} q^{(a_i,a_j)}=nq^3+nq^4+\ldots\qquad +nq^{\frac{n+4}{2}}\ =\ n\sum_{i=3}^{\frac{n+2}{2}}q^i+\frac{n}{2}q^{\frac{n+4}{2}}$$

***case*3**: Distance between the vertices of A1,A2 is given as

$$d_{A1A2}(v_i,a_j)=x, 1\le x\le \frac{n+2}{2},\ 1\le i,j\le n,\ j=i+x-1\,(\bmod n)$$
$$=n-x+i+1(\bmod n), 2\le x\le \frac{n+2}{2}.$$

$$d_{A1A2}(G,i)=\begin{cases} n\ for\ i=1 \\ 2n, 2\le i\le \frac{n+2}{2}\end{cases}$$

$$\sum_{\{v_i,a_j\}\in V(A1A2)} q^{(v_i,a_j)}=nq^1+2\left(nq^2+nq^3+\ldots\qquad +nq^{\frac{n+2}{2}}\right)=nq^1+\ 2n\sum_{i=2}^{\frac{n+2}{2}}q^i$$

***Case* 4**: Distance between the vertices of B1 is given as

$$d_{B1}(v_i^{/},v_j^{/})=\begin{cases} 3\ for\ i=j \\ x,\ 2\le x\le \frac{n}{2},\ 1\le i,j\le n,\ j=i+x(\bmod n) \\ \frac{n}{2}, 1\le i\le \frac{n}{2}, \frac{n}{2}+1\le j\le n,\end{cases}$$

$$d_{B1}(G,i)=\begin{cases} 2n\ for\ i=3 \\ n, 2\le i\le \frac{n-2}{2}, i\ne 3 \\ \frac{n}{2}\ for\ i=\frac{n}{2}\end{cases}$$

Thus the wiener polynomial in this case becomes

$$\sum_{\{v_i^/, v_j^/\} \in V(B1)} q^{(v_i^/, v_j^/)} = nq^3 + \left(nq^2 + nq^3 + \ldots \quad + nq^{\frac{n-2}{2}} \right) + \frac{n}{2} q^{\frac{n}{2}} = nq^3 + n \sum_{i=2}^{\frac{n-2}{2}} q^i + \frac{n}{2} q^{\frac{n}{2}}$$

***case*5**: Distance function between the vertices of B2 is given as

$$d_{B2}(a_i^/, a_j^/) = x,\ 3 \le x \le \frac{n+4}{2},\ 1 \le i, j \le n,\ j = i + x - 2(\mathrm{mod}\, n).$$

$$d_{B2}(G, i) = n\ for\ 3 \le i \le \frac{n+4}{2}$$

$$\sum_{\{a_i^/, a_j^/\} \in V(B2)} q^{(a_i^/, a_j^/)} = nq^3 + nq^4 + \ldots \quad + nq^{\frac{n+4}{2}} = n \sum_{i=3}^{\frac{n+2}{2}} q^i + \frac{n}{2} q^{\frac{n+4}{2}}.$$

***case* 6**: Distance function between the vertices of B1,B2 is given as

$$d_{B1B2}(a_i^/, v_j^/) = \begin{cases} 3\ for\ i = j \\ x,\ 2 \le x \le \frac{n+2}{2},\ 1 \le i, j \le n,\ j = i + x - 1(\mathrm{mod}\, n) \end{cases}$$

$$d_{B1B2}(G, i) = \begin{cases} 3n\ for\ i = 3 \\ 2n, 2 \le i \le \frac{n+2}{2} \end{cases}$$

Thus the wiener polynomial in this case becomes

$$\sum_{\{a_i^/, v_j^/\} \in V(B1B2)} q^{(a_i^/, v_j^/)} = nq^3 + 2\left(nq^2 + nq^3 + \ldots \quad + nq^{\frac{n+1}{2}} \right) = nq^3 + 2n \sum_{i=2}^{\frac{n+2}{2}} q^i$$

***Case*7**: Distance between the vertices of A1,B2 is given as

$$d_{A1B2}(v_i, a_j^/) = x, 1 \le x \le \frac{n+2}{2},\ 1 \le i, j \le n,\ j = i + x - 1\ (\mathrm{mod}\, n)$$
$$= n - x + i + 1(\mathrm{mod}\, n).$$

$$d_{A1B2}(G, i) = \begin{cases} n\ for\ i = 1 \\ 2n, 2 \le i \le \frac{n+2}{2} \end{cases}$$

$$\sum_{\{v_i,a_j'\}\in V(A1B2)} q^{(v_i,a_j')} = nq^1 + 2\left(nq^2 + nq^3 + \ldots \quad + nq^{\frac{n+2}{2}} \right) = nq^1 + \ 2n\sum_{i=2}^{\frac{n+2}{2}} q^i$$

***Case*8**: Distancebetween the vertices of A1,B1

$$d(v_i, v_j^{/}) = \begin{cases} 2 \ \ for \ \ i = j \\ x, 1 \le x \le \frac{n}{2}, \ 1 \le i \le n, \ \ j = i + x \ (\bmod n) \\ \qquad\qquad\qquad\qquad = n - x + i(\bmod n) \end{cases}$$

$$d_{A1B1}(G,i) = \begin{cases} 3n \ \ for \, i = 2 \\ 2n, 1 \le i \le \frac{n}{2}, i \ne 2 \\ n, i = \frac{n}{2} \end{cases}$$

The corresponding wiener polynomial is given as

$$\sum_{\{v_i,v_j^{/}\}\in V(A1B1)} q^{(v_i,v_j^{/})} = nq^2 + 2\left(nq + nq^2 + nq^3 + \ldots \quad + nq^{\frac{n}{2}} \right) = nq^2 + \ 2n\sum_{i=1}^{\frac{n-2}{2}} q^i + nq^{\frac{n}{2}}$$

***Case* 9**: Distance function between thevertices of A2,B1 is same as that of A1,B2. Thus the polynomial is

$$\sum_{\{v_i,a_j'\}\in V(A2B1)} q^{(v_i,a_j')} = nq^1 + 2\left(nq^2 + nq^3 + \ldots \quad + nq^{\frac{n+2}{2}} \right) = nq^1 + \ 2n\sum_{i=2}^{\frac{n+2}{2}} q^i$$

Case 10: Distance between the vertices of A2,B2 is given as

$$d(a_i, a_j^{/}) = \begin{cases} x, \ 2 \le x \le \frac{n+4}{2}, \ 1 \le i, j \le n, \ \ j = i + x - 2 \ (\bmod n) \\ x, 3 \le x \le \frac{n+1}{2} \ , \ 1 \le i, j \le n, j = n - x + i + 2(\bmod n) \end{cases}$$

$$d_{A2B2}(G,i)=\begin{cases} n\ for\ i=2 \\ 2n, 3\le i\le \frac{n+2}{2} \\ \frac{n}{2}, i=\frac{n}{2} \end{cases}$$

$$\sum_{\{a_i,a_j^{/}\}\in V(A2B2)} q^{(a_i,a_j^{/})}=nq^2+2\left(nq^3+nq4\ldots \qquad +nq^{\frac{n+4}{2}}\right)=nq^2+\ 2n\sum_{i=3}^{\frac{n+2}{2}}q^i+nq^{\frac{n+4}{2}}$$

By adding all the polynomials ,we get the wiener polynomial of splitting graph of sun let graph for odd n is

$$W(S(C_n\otimes K_2))=4W(C_n\otimes K_2)+2n\left(-q+q^2+q^3\right), n\ge 5$$

4. Conclusion:

In this paper we have obtained the relation between the distance between two vertices and the diameter of $S(G)$. Further Wiener polynomial of the splitting graph of a graph *G* where *G* is isomorphic to P_n, C_n ,K_n, $W_{1,n}$, $C_n\otimes K_2$ are obtained. Further we will establish the relation between the wiener index of splitting graph of an arbitrary graph *G* and the graph *G* in our future work.

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