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A NUMERICAL ANALYSIS OF CONVECTIVE HEAT TRANSFER IN A VERTICAL WAVY CHANNEL WITH CHEMICAL REACTION

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Abstract
The effect of chemical reaction on unsteady combined heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel with oscillatory flux. The non-linear governing equations are solved by employing a regular perturbation technique with the slope δ of the wavy wall as a perturbation parameter. The velocity, the temperature and the concentration are analyzed for different variations of the governing parameters. The rate heat and mass transfer are evaluated for different variations.

Keywords: Heat Transfer, Mass Transfer, Chemical reaction, Wavy channel

I. Introduction

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing .Das et al[1] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy[2] has studied the effects of reaction on a long surface with suction. Radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

Kandaswamy et al[3] have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.

The study of heat transfer and mixed convection flow in enclosures of various shapes has received attention [4] due to its practical applications. Interest in these convection flow and heat transfer in porous medium has been motivated by a broad range of applications to geothermal systems, crude oil production, storage of nuclear waste materials, ground water pollution, fiber and granular insulations solidification of castings. In a wide range of such problems, the physical system can be modeled as a two-dimensional rectangular enclosure with vertical walls held at different temperatures and the connecting adiabatic horizontal walls. Convective heat transfer in a rectangular porous duct whose vertical walls are maintained at two different temperatures and horizontal walls insulated received attention by several investigators [5]. Furthermore, in references [6 and 7] numerical results are being presented.

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage, evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed .Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[8],Lai and Kulacki[9].The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al[10]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably , Nelson and Wood[11].

II. Mathematical model

We consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at y =L while the boundary at y = -L is maintained at constant temperature T_1 while both the walls are maintained at uniform concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. We choose a rectangular Cartesian system 0(x ,y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y=±L.

The equations governing the unsteady flow, heat and mass transfer in terms of stream function ψ.

$$[(\nabla^2\psi)_t+\psi_x(\nabla^2\psi)_y-\psi_y(\nabla^2\psi)_x]=\nu\nabla^4\psi-\beta g(T-T_0)_y- -\beta^* g(C-C_0)_y-(\frac{\nu}{k})\nabla^2\psi \tag{2.1}$$

$$\rho_e C_p(\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=\lambda\nabla^2\theta-Q(T-T_o)+Q_1(C-C_o) \tag{2.2}$$

$$(\frac{\partial\phi}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y})=D\nabla^2\phi-k_1(C-C_o) \tag{2.3}$$

The boundary conditions for the velocity and temperature fields are

$$\frac{\partial\psi}{\partial y}=0,\quad \frac{\partial\psi}{\partial x}=0,\quad T=T_1,\quad C=C_1 \qquad \text{on } y=-L$$

$$\frac{\partial\psi}{\partial y}=0,\quad \frac{\partial\psi}{\partial x}=0,\quad T=T_2+\Delta T_e\ Sin(mx+nt),\quad C=C_2 \qquad \text{on } y=L \tag{2.4}$$

Introducing the non-dimensional variables as

$$x'=mx,\ y'=y/L,t'=t\nu m^2,\Psi'=\Psi/\nu,\theta=\frac{T-T_2}{T_1-T_2},\phi=\frac{C-C_2}{C_1-c_2} \tag{2.5}$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$\delta R(\delta(\nabla_1^2\psi)_t+\frac{\partial(\psi,\nabla_1^2\psi)}{\partial(x,y)})=\nabla_1^4\psi+(\frac{G}{R})(\theta_y+N\phi_y)-D^{-1}\nabla_1^2\psi-M^2\frac{\partial^2\psi}{\partial y^2} \tag{2.6}$$

$$\delta P(\delta\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=\nabla_1^2\theta-\alpha\theta+Q_2\phi \tag{2.7}$$

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$$\delta Sc(\delta\frac{\partial\phi}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y})=\nabla_1^2\phi-\gamma\phi \quad (2.8)$$

where

$R=\frac{UL}{\nu}$ (Reynolds number) $G=\frac{\beta g\Delta T_e L^3}{\nu^2}$ (Grashof number)

$P=\frac{\mu c_p}{k_1}$ (Prandtl number), $D^{-1}=\frac{L^2}{k}$ (Darcy parameter),

$Sc=\frac{\nu}{D_1}$ (Schmidt number) $M^2=\frac{\sigma\mu_e^2 H_o^2 L^2}{\nu^2}$ (Hartmann Number)

$\alpha=\frac{QL^2}{\lambda}$ (Heat source parameter) $Q_2=\frac{Q_1(C_1-C_2)L^2}{(T_1-T_2)}$ (Radiation absorption parameter)

$\gamma_1=\frac{K_1L^2}{D_1}$ (Chemical reaction parameter) $\delta=mL$ (Aspect ratio)

$\gamma=\frac{n}{\nu m^2}$ (non-dimensional thermal wave velocity)

$\nabla_1^2=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ The corresponding boundary conditions are

$$\psi(+1)-\psi(-1)=-1$$

$$\frac{\partial\psi}{\partial x}=0,\ \frac{\partial\psi}{\partial y}=0 \quad at\, y=\pm1 \quad (2.9)$$

$$\theta(x,y)=1 \quad ,C=1 \quad on\ y=-1$$

$$\theta(x,y)=Sin(x+\gamma t) \quad ,C=0 \quad on \quad y=+1 \qquad \theta(x,y)=Sin(x+\gamma t),C=1$$

$$\frac{\partial\theta}{\partial y}=0,\frac{\partial C}{\partial y}=0 \quad at \quad y=0 \quad (2.10)$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis. Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

III. Nusselt number and Sherwood number

Knowing the temperature & concentration the local rate of heat and mass transfer on the walls have been calculated using the formula

$$Nu=\frac{1}{\theta_m-\theta_w}(\frac{\partial\theta}{\partial y})_{y=\pm1}$$

where $\theta_m=0.5\int_{-1}^{1}\theta\,dy$ and $Sh=\frac{1}{C_m-C_w}(\frac{\partial C}{\partial y})_{y=\pm1}$

where $C_m=0.5\int_{-1}^{1}C\,dy$

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where $d_1 . d_2 , \ldots\ldots\ldots , d_{14}$ are constants.

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IV. Discussion of the numerical results

In this analysis we investigate the effect of Chemical reaction on convective Heat and mass transfer flow of a viscous fluid in a vertical wavy channel.

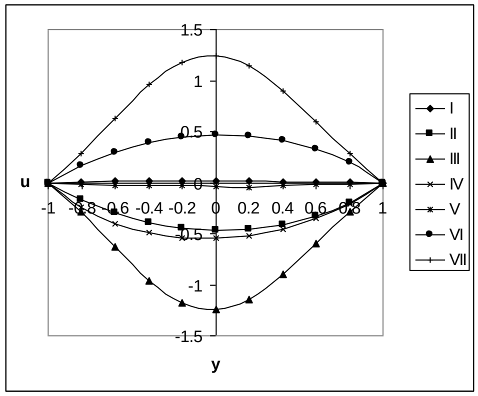

Fig.1 : Variation of u with γ

	I	II	III	IV	V	VI	VII
γ	1	1.5	2.5	3.5	-0.5	-1.5	-2.5

The effect of chemical reaction on u is shown in Fig.1. We find that the velocity exhibits the reversal flow which appears in the entire region at γ=1 disappears everywhere in the region with higher γ>0 (degenerating chemical reaction case) No such phenomena is observed for γ<0. Also |u| enhances with γ≤2.5 and depreciates with higher γ≤3.5, while the variation of u with γ<0 (generating chemical reaction) shows that |u| depreciates with increase in |γ|.

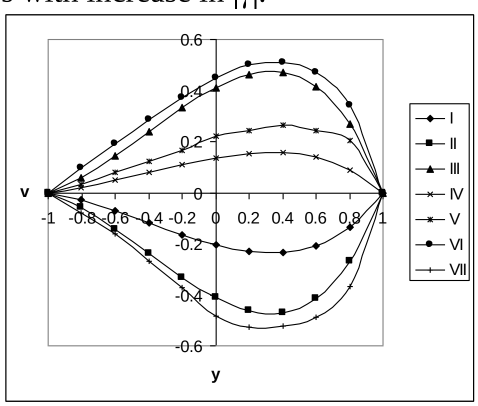

Fig.2 : Variation of v with γ

	I	II	III	IV	V	VI	VII
γ	1	1.5	2.5	3.5	-0.5	-1.5	-2.5

Also enhances with γ≤2.5 and depreciates with higher γ≥3.5, which it depreciates with |γ| everywhere in the region(Fig.2).

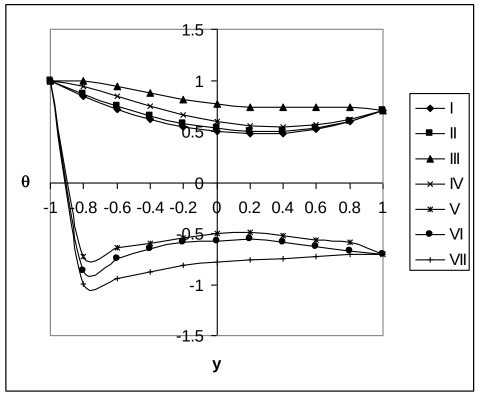

Fig.3 : Variation of θ with γ

	I	II	III	IV	V	VI	VII
γ	1	1.5	2.5	3.5	-0.5	-1.5	-2.5

While an increase in γ<0 (generating chemical reaction case) smaller the actual temperature in the flow region(Fig.3).

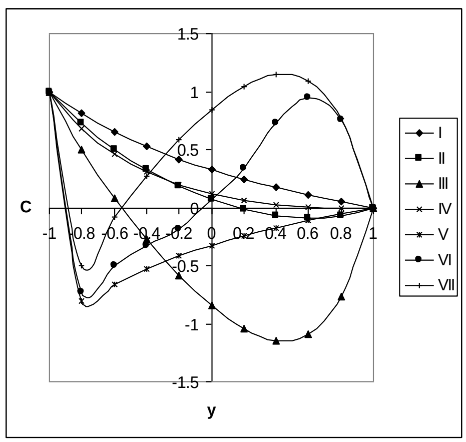

Fig.4 : Variation of C with γ

	I	II	III	IV	V	VI	VII
γ	1	1.5	2.5	3.5	-0.5	-1.5	-2.5

While an increase n γ<0 enhances the actual concentration in the entire flow region (Fig.4).

The average Nusselt number(Nu) which represents the rate of heat transfer at y=±1 is shown in tables.1and 2 for different values of parameters. The Sherwood Number(Sh) which measures the rate of mass transfer at y=±1 is shown in Tables.3 and 4 for different variations.

Table-1 Nusselt Number Nu_1 at y = 1

G	I	II	III	IV	V	VI
10^3	-3.9534	-3.8856	-4.0793	-4.1058	-3.2133	-3.67302
$3x10^3$	-3.9625	-3.8935	-4.0906	-4.1176	-3.3967	-3.84741
10^3	-3.9908	-3.9184	-4.1266	-4.1545	-4.1923	-4.58446
$3x10^3$	-3.9812	-3.91002	-4.114	-4.1419	-3.8738	-4.29321
N	1	2	-0.5	-0.8	1	1
γ1	2	2	2	2	4	6

Table-2 Nusselt Number Nu_2 at y = -1

G	I	II	III	IV	V	VI
10^3	7.8348	7.7106	8.0655	8.1138	6.2046	6.8681
$3x10^3$	7.8519	7.7256	8.0866	8.1359	6.5066	7.1264
10^3	7.9054	7.7724	8.1528	8.205	7.817	8.2177
$3x10^3$	7.887	7.7565	8.1303	8.1815	7.2924	7.7864
N	1	2	-0.5	-0.8	1	1
γ1	2	2	2	2	4	6

Table-3 Sherwood number (Sh) at y = 1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-0.2943	-0.28389	-0.31362	-0.3176	6.74864	10.43639	-0.30233	-0.3343	-0.295
$3x10^3$	-0.29536	-0.28481	-0.31497	-0.3190	-3.147	12.54275	-0.30341	-0.3359	-0.296
10^3	-0.29866	-0.28768	-0.31915	-0.3234	-0.61564	-1.10815	-0.30676	-0.3408	-0.3
$3x10^3$	-0.29754	-0.28671	-0.31773	-3.3219	-0.83107	-1.73405	-0.30563	-0.3393	-0.298
N	1	2	-0.5	-0.8	1	1	1	1	1
γ1	2	2	2	2	4	6	2	2	2
x+γt	π/4	π/4	π/4	π/4	π/4	π/4	π/2	π	2π

Table-4Sherwood number (Sh) at y = -1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	-1.2823	-1.2748	-1.2935	-1.2956	-2.1453	-3.235	-1.5842	-2.0275	-1.1589
$3x10^3$	-1.2827	-1.2753	-1.2939	-1.2959	-2.1015	-3.3947	-1.5822	-2.0218	-1.1604
10^3	-1.2841	-1.2768	-1.2949	-1.2969	-1.9092	-5.2827	-1.5766	-2.0055	-1.1646
$3x10^3$	-1.2836	-1..2763	-1.2946	-1.2966	-1.9866	-4.1981	-1.5784	-2.0109	-1.1632
N	1	2	-0.5	-0.8	1	1	1	1	1
γ1	2	2	2	2	4	6	2	2	2
x+γt	π/4	π/4	π/4	π/4	π/4	π/4	π/2	π	2π

An increase in the chemical reaction parameter $\gamma_1 \leq 4$ reduces $|Nu|$ and enhances with higher $\gamma_1 \geq 6$ in the heating case at both the walls.An increase in x+γt reduces the rate of heat transfer at y=+1 and fluctuates at y=-1.(Tables.1 and 2)

An increase in the chemical reaction parameter γ_1 leads to an enhancement in n$|Sh|$ at both the walls.With reference to the phase x+γt we find that the rate of mass transfer enhances with increase in $x+\gamma t \leq \pi$ and depreciates with $x+\gamma t \geq 2\pi$ at both the walls.(Tables.3 and 4)

References.

[1] U.N. Das, R.K. Deka, and V.M. Soundalgekar, Effects of mass transfer on flow Pst an impulsively started infinte vertical plate with constant heat flux and chemical reaction.,Forschung in Ingenieurwesen, V.60(10), p.284-287, 1994

[2] R. Muthukumaraswamy, Effects of a chemical reaction on a moving isothermal surface with suction., Acta Mechnica,V.155,p.65, 2002

[3] P. Kandaswamy, Wahid Abd, B.Md.Raj, B. Azme Khamis, Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection, Theoret. Appl. Mech., V.33. No.2, pp.123-148, 2006

[4] Hiroxhi Iwai, Kazuyoshi nakabe, Kenjiro Suzuki: Flow and Heat transfer characteristics of backward-facing step laminar flow in a rectangular duct., Int.J.Heat and Mass transfer,V.43, pp.457-471(2000)
[5] Teoman Ayhan, Hayati Olgum : Betul Ayhan : Heat transfer and flow structure in a Rectangualr channel withwing -1, type vortex Generator. Tr. J. of Engineering and Environmental Science, pp, 185-195, 22 (1998).
[6] Cheng K.S. and J.R. Hi.: Steady, Two-dimensional, natural convection in rectangular enclosures with differently heated walls transaction of the ASME, v. 109, p, 400, (1987).
[7] Chan, B.K.C, Ivey, U.M and Barry, J.M: Natural convection in enclosed porous medium with rectangular boundaries ASME journal of heat transfer, v. 92, pp, 21-27 (1970).
[8] Bejan,A and Khair,K.R:Heat and Mass transfer by natural convection in a porous medium, Int. J. Heat Mass transfrt,V.28,pp.908-918(1985).
[9] Lai,F.C and Kulacki,F.A : Coupled heat and mass transfer by natural convection from vertical surfaces in porous medium.,Int.J.Heat Mass Transfer, V.34, pp.1189-1194(1991).
[10] Angirasaa,D,Peterson,G.P and Pop, I :Combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium, Int. J. Heat Mass Transfer,V.40,pp.2755-2773(1997).
[11] Nelson,D.J and Wood,B.D:Combined heat and mass transfer by natural convection between vertical plates ,Int.J.,Heat Mass transfer,V.82,pp.1789-1792(1989).