Industrial Engineering Journal
ISSN: 0970-2555
Volume : 54, Issue 1, No.4, January : 2025

LUCAS ANTIMAGIC LABELING OF SOME TREES OF DIAMETER LESS THAN FIVE

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ABSTRACT
A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \to \{1,2, \dots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).
In this paper the Lucas Antimagic Labeling of some trees of diameter less than five are found.
KEYWORDS: Tree,Diameter, Lucas Antimagic graph.

1.INTRODUCTION
In this study, we explore graph G(V,E) characterized as finite, simple, and undirected with p vertices and q edges. Graph labeling stands as a cornerstone concept in graph theory, involving the assignment of integers to vertices or edges. This concept's vast applications across fields such as astronomy, coding theory, and beyond, have brought it to the forefront of research.
The journey into this intriguing research domain was profoundly inspired by the seminal contributions of Gallian, whose comprehensive survey [1] serves as a cornerstone in the field. Gallian's work meticulously catalogues numerous labeling methods and their diverse applications, providing a robust foundation upon which we have embarked on our own research endeavors.
A pivotal and intriguing concept that particularly captivated our attention is Antimagic labeling, introduced by the pioneering efforts of N. Hartsfield and G. Ringel in the year 1990. This innovative labeling method has sparked a surge of interest and subsequent investigations, opening new vistas for exploration within the rich landscape of graph theory. Antimagic labeling, with its unique properties and potential applications, offers a fertile ground for further inquiry and discovery.
Inspired by these groundbreaking contributions, we have introduced the novel concept of Lucas Antimagic labeling. This labeling scheme builds upon the principles of Antimagic labeling, integrating the properties of Lucas numbers to create a distinctive and versatile labeling method. Our research delves into the intricacies of Lucas Antimagic labeling, exploring its applicability on various tree graphs whose diameter is less than five.

2.DEFINITIONS
Definition 2.1: Lucas number is defined by
$L_1 = 2, L_2 = 1, L_n = L_{n-1} + L_{n-2}, if\ n > 2$
The first few Lucas numbers are 2,1,3,4,7,11,18,29,47,…
Definition 2.2:[3] A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \to \{1,2, \dots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).
Definition 2.3:[4] A connected graph without any cycle is called a tree.
LIST OF NOTATIONS
1. T_1^2- the tree of diameter two acquired by attaching n pendant edges to the internal vertex of the path P_3.
2. T_1^3- the tree of diameter three acquired by attaching n pendant edges to the first internal vertex of the path P_4.
3. T_2^3- the tree of diameter three acquired by attaching n pendant edges to the second internal vertex of the path P_4.

4. T_3^3- the tree of diameter three acquired by attaching m,n pendant edges to the first and second internal vertex of the path P_4.
5. T_4^3- the tree of diameter three acquired by attaching n leaves through a bridge to the mid vertex of the path P_3.
6. T_1^4- the tree of diameter four acquired by attaching n pendant edges to the first internal vertex of the path P_5.
7. T_2^4- the tree of diameter four acquired by attaching n pendant edges to the second internal vertex of the path P_5.
8. T_3^4- the tree of diameter four acquired by attaching n pendant edges to the third internal vertex of the path P_5.
9. T_4^4- the tree of diameter four acquired by attaching n,m pendant edges to the first and second internal vertex of the path P_5.
10. T_5^4- the tree of diameter four acquired by attaching n,m pendant edges to the first and third internal vertex of the path P_5.
11. T_6^4- the tree of diameter four acquired by attaching m,n pendant edges to the second and third internal vertex of the path P_5.
12. T_7^4- the tree of diameter four acquired by attaching n,m and l pendant edges to the internal vertices of the path P_5.

Observation:

$T_1^3 \cong T_2^3$, $T_1^4 \cong T_2^4$, $T_4^4 \cong T_6^4$

3.MAIN RESULTS

Tree of Diameter two

Theorem 3.1:

The tree T_1^2 is a Lucas antimagic graph.

Proof:

Let G be T_1^2

Let $V(G) = \{p_i : 1 \leq i \leq 3, v_i : 1 \leq i \leq n\}$

$E(G) = \{p_i p_{i+1} \ : 1 \leq i \leq 2 \ , p_2 v_i : 1 \leq i \leq n\}$

Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(p_1p_2) = L_1$

$f(p_2p_3) = L_2$

$f(p_2v_i) = L_{2+i}, 1 \leq i \leq n$

The induced function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(p_1) = L_1$

$f^*(p_2) = L_1 + L_2 + \sum_{i=1}^{n} L_{2+i}$

$f^*(p_3) = L_2$

$f^*(v_i) = L_{2+i}, 1 \leq i \leq n$

We observe that the vertices are all distinct.

Hence G is a Lucas antimagic graph.

Trees of Diameter three

Theorem 3.2:

The tree T_1^3 is a Lucas antimagic graph.

Proof:

Let G be T_1^3

Let $V(G) = \{p_i : 1 \leq i \leq 4, v_i : 1 \leq i \leq n\}$

$E(G) = \{p_i p_{i+1} \ : 1 \leq i \leq 3 \ , p_2 v_i : 1 \leq i \leq n\}$

Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(p_1p_2) = L_2$
$f(p_2p_3) = L_1$
$f(p_3p_4) = L_3$
$f(p_2v_i) = L_{3+i}, 1 \leq i \leq n$
The induced function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ is given by
$f^*(p_1) = L_2$
$f^*(p_2) = L_1 + L_2 + \sum_{i=1}^{n} L_{3+i}$
$f^*(p_3) = L_1 + L_3$
$f^*(p_4) = L_3$
$f^*(v_i) = L_{3+i}, 1 \leq i \leq n$
We observe that the vertices are all distinct.
Hence G is a Lucas antimagic graph.
Theorem 3.3:
The tree T_3^3 is a Lucas antimagic graph.
Proof:
Let G be T_3^3
Let $V(G) = \{p_i : 1 \leq i \leq 4, v_i : 1 \leq i \leq n, u_i : 1 \leq i \leq m\}$
$E(G) = \{p_ip_{i+1} : 1 \leq i \leq 3, p_2v_i : 1 \leq i \leq n, p_3u_i : 1 \leq i \leq m\}$
Define a function $f : E(G) \to \{L_1, L_2, \dots L_q\}$ by
$f(p_1p_2) = L_2$
$f(p_2p_3) = L_1$
$f(p_3p_4) = L_3$
$f(p_2v_i) = L_{3+i}, 1 \leq i \leq n$
$f(p_3u_i) = L_{3+n+i}, 1 \leq i \leq m$
The induced function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ is given by
$f^*(p_1) = L_2$
$f^*(p_2) = L_1 + L_2 + \sum_{i=1}^{n} L_{3+i}$
$f^*(p_3) = L_1 + L_3 + \sum_{i=1}^{m} L_{3+n+i}$
$f^*(p_4) = L_3$
$f^*(v_i) = L_{3+i}, 1 \leq i \leq n$
$f^*(u_i) = L_{3+n+i}, 1 \leq i \leq m$
We observe that the vertices are all distinct.
Hence G is a Lucas antimagic graph.
Theorem 3.4:
The tree T_4^3 is a Lucas antimagic graph.
Proof:
Let G be T_4^3
Let $V(G) = \{p_i : 1 \leq i \leq 3, p, v_i : 1 \leq i \leq n\}$
$E(G) = \{p_ip_{i+1} : 1 \leq i \leq 2, p_2p, pv_i : 1 \leq i \leq n\}$
Define a function $f : E(G) \to \{L_1, L_2, \dots L_q\}$ by
$f(p_1p_2) = L_1$
$f(p_2p_3) = L_2$
$f(p_2p) = L_3$
$f(pv_i) = L_{3+i}, 1 \leq i \leq n$
The induced function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ is given by
$f^*(p_1) = L_1$
$f^*(p_2) = L_1 + L_2 + L_3$
$f^*(p_3) = L_2$

Industrial Engineering Journal
ISSN: 0970-2555
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$$f^*(p) = L_3 + \sum_{i=1}^{n} L_{3+i}$$

$f^*(v_i) = L_{3+i}, 1 \leq i \leq n$

We observe that the vertices are all distinct.

Hence G is a Lucas antimagic graph.

Trees of Diameter four

Theorem 3.5:

The tree T_1^4 is a Lucas antimagic graph.

Proof:

Let G be T_1^4

Let $V(G) = \{p_i : 1 \leq i \leq 5, v_i : 1 \leq i \leq n\}$

$E(G) = \{p_i p_{i+1} \ : 1 \leq i \leq 4, p_2 v_i : 1 \leq i \leq n\}$

Define a function $f : E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(p_1 p_2) = L_1$

$f(p_2 p_3) = L_4$

$f(p_3 p_4) = L_2$

$f(p_4 p_5) = L_3$

$f(p_2 v_i) = L_{4+i}, 1 \leq i \leq n$

The induced function $f^* : V(G) \to \{1, 2, \dots \sum L_q\}$ is given by

$f^*(p_1) = L_1$

$f^*(p_2) = L_1 + L_4 + \sum_{i=1}^{n} L_{4+i}$

$f^*(p_3) = L_4 + L_2$

$f^*(p_4) = L_2 + L_3$

$f^*(p_5) = L_3$

$f^*(v_i) = L_{4+i}, 1 \leq i \leq n$

We observe that the vertices are all distinct.

Hence G is a Lucas antimagic graph.

Theorem 3.6:

The tree T_2^4 is a Lucas antimagic graph.

Proof:

Let G be T_2^4

Let $V(G) = \{p_i : 1 \leq i \leq 5, v_i : 1 \leq i \leq n\}$

$E(G) = \{p_i p_{i+1} \ : 1 \leq i \leq 4, p_3 v_i : 1 \leq i \leq n\}$

Define a function $f : E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(p_1 p_2) = L_1$

$f(p_2 p_3) = L_4$

$f(p_3 p_4) = L_2$

$f(p_4 p_5) = L_3$

$f(p_3 v_i) = L_{4+i}, 1 \leq i \leq n$

The induced function $f^* : V(G) \to \{1, 2, \dots \sum L_q\}$ is given by

$f^*(p_1) = L_1$

$f^*(p_2) = L_1 + L_4$

$$f^*(p_3) = L_4 + L_2 + \sum_{i=1}^{n} L_{4+i}$$

$f^*(p_4) = L_2 + L_3$

$f^*(p_5) = L_3$

$f^*(v_i) = L_{4+i}, 1 \leq i \leq n$

We observe that the vertices are all distinct.
Hence G is a Lucas antimagic graph.

Theorem 3.7:

The tree T_4^4 is a Lucas antimagic graph.

Proof:

Let G be T_4^4

Let $V(G) = \{p_i : 1 \le i \le 5, v_i : 1 \le i \le n, u_i : 1 \le i \le m\}$

$E(G) = \{p_i p_{i+1} : 1 \le i \le 4, p_2 v_i : 1 \le i \le n, p_3 u_i : 1 \le i \le m\}$

Define a function $f : E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(p_1p_2) = L_1$
$f(p_2p_3) = L_4$
$f(p_3p_4) = L_2$
$f(p_4p_5) = L_3$
$f(p_2v_i) = L_{4+i}, 1 \le i \le n$
$f(p_3u_i) = L_{4+n+i}, 1 \le i \le m$

The induced function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(p_1) = L_1$
$f^*(p_2) = L_1 + L_4 + \sum_{i=1}^{n} L_{4+i}$
$f^*(p_3) = L_4 + L_2 + \sum_{i=1}^{m} L_{4+n+i}$
$f^*(p_4) = L_2 + L_3$
$f^*(p_5) = L_3$
$f^*(v_i) = L_{4+i}, 1 \le i \le n$
$f^*(u_i) = L_{4+n+i}, 1 \le i \le m$

We observe that the vertices are all distinct.
Hence G is a Lucas antimagic graph.

Theorem 3.8:

The tree T_7^4 is a Lucas antimagic graph.

Proof:

Let G be T_7^4

Let $V(G) = \{p_i : 1 \le i \le 5, v_i : 1 \le i \le n, u_i : 1 \le i \le m, w_i : 1 \le i \le l\}$

$E(G) = \{p_i p_{i+1} : 1 \le i \le 4, p_2 v_i : 1 \le i \le n, p_3 u_i : 1 \le i \le m, p_4 w_i : 1 \le i \le l\}$

Define a function $f : E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(p_1p_2) = L_1$
$f(p_2p_3) = L_4$
$f(p_3p_4) = L_2$
$f(p_4p_5) = L_3$
$f(p_2v_i) = L_{4+i}, 1 \le i \le n$
$f(p_3u_i) = L_{4+n+i}, 1 \le i \le m$
$f(p_4w_i) = L_{4+n+m+i}, 1 \le i \le l$

The induced function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(p_1) = L_1$
$f^*(p_2) = L_1 + L_4 + \sum_{i=1}^{n} L_{4+i}$
$f^*(p_3) = L_4 + L_2 + \sum_{i=1}^{m} L_{4+n+i}$

$$f^*(p_4) = L_2 + L_3 + \sum_{i=1}^{l} L_{4+n+m+i}$$

$f^*(p_5) = L_3$
$f^*(v_i) = L_{4+i}, 1 \le i \le n$

$f^*(u_i) = L_{4+n+i}, 1 \leq i \leq m$
$f^*(w_i) = L_{4+n+m+i}, 1 \leq i \leq l$
We observe that the vertices are all distinct.
Hence G is a Lucas antimagic graph.

4.CONCLUSION:

In this article, we establish that several tree graphs exhibit Lucas Antimagic properties. Additionally, parallel investigations into various other graph structures are currently in progress, expanding the breadth of our research. These findings highlight the potential and applicability of Lucas Antimagic labeling in diverse graph theoretic contexts.

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