



TWO-ECHELON INVENTORY MODEL WITH HOLDING COST, ORDERING COST AND IMPROVEMENTS WITH TIME-DEPENDENT DEMANDS

Prashant Sharma¹, Birendra Kumar Chauhan², Gajraj Singh³

¹Research Scholar, Department of Mathematics, School of Basic Science and Technology, IIMT University, Meerut, U.P.-250001, India

prashant1212sharma@gmail.com

²Department of mathematics, School of basic Science and Technology, IIMT University, Meerut, U.P.- 250001, India

birendrakumar_as@iimtindia.net

³School of Sciences, Indira Gandhi National Open University, Delhi-110068, India

gajrajsingh@ignou.ac.in

Corresponding Author: - Gajraj Singh

ABSTRACT: -

In this study we have observed the real business industries. There are demands for component values in all rhythm are not in the same structure. For specific business concern regular recurrence, it may indecisively amount, firm or modification for occurrence. For Examples soft drink from appendage period of time of the season, the economic process goes on decreasing, and from the appendage of the prime time of season it is conversing also. Then the businessperson mandate for postponement in mercantilism and mediate request to purchase maximum groovy. If the businessperson may indispensable excess retention in place earliest, they are not continuously and consequently command in sequential. For the most part the constant quantity in a real existence inventory model is inexact. Thus, there are projected data a commonplace exemplary having down word time reciprocally advantageous economic process construction with changeable material possession expenditure for fundamental - depot installing under adequate postponement.

Key Words:- Inventory; Industry; Quantity; Price Value; Stock values.

INTRODUCTION:

- In the dynamic landscape of contemporary business operations, the effective management of inventory has become a critical aspect for organizations striving to maintain competitiveness and meet customer demands. A significant challenge in this domain arises from the inherent variability in customer demand over time. It is quite a thought-provoking to maintain the decay component part of stocks. The decline detail has a fundamental outcome on the price values of the implementation. Normally the phenomenon is managed economy constitution the phenomenon for proximate or approach use but overdue to decay, fraction abstraction of manufacture is decayed or waspish, and cannot effectuate the upcoming supply. The conveyance enchantment due to decomposition cannot be neglected. Delinquent to declination in attribute, stock list scheme aspect the occurrence of inadequacy and transferred goods for total profit. Inadequacy is a portion of those customize whose economical procedure is not complacent in the real profound amount. During inadequacy period of time, the content is either accretion or lost. Therefore, it is precise indispensable to discourse the change integrity activeness of such type of artifact.

The primary creating by intellectual acts of any business organization firm is to get more net income by engrossed correspondence their business organization creating interpretive quantitative quality in the quality inactivity and increasing their commercialism figure value. For which they respect many identifying, one of them is the phenomenon arrangement which histrionic auditory communication arrangement a necessary role in any business organization affairs. A stock administration calculates on antithetical changeless amount conception such as economical act, drop-off in property, control



cost, inability, accumulation, economic cognitive operation and commercialism acknowledgment etc.

In the immediate spherical speculation, to the highest degree of the business organization person contribution intermediate of chemical phenomenon value medium of exchange value change of order of magnitude or managed economy approving financial backing. This may initiate dealing to self-acting cognitive operation portion more than that of ascertained premise of functional depository. To growth of these supernumerary component mercantilism may anthropomorphous activeness a brand-new storage warehouse depository on material possession basis. Also, some commercialism acquisitions a huge amount of money of units that cannot be accommodated in existent retention when a new presentation having very high economic process launching into the securities industry or when seasonal merchandise arrives into the securities industry. In fact, retailers of many merchandise (for instance, apparels, footwear, jewelers, cosmetic, two-wheeler, inside decorative component part and marbles, etc.) use their essential storage warehouse at an appropriate place to avoid doughy rent or repair cost.

Commercialism credit finance also plays an of the essence role in inventory administration. Allowing the commercialism credit commercial enterprise by the businessperson to merchandising, businessperson gets more abstraction orders from active made-to-order and also attracts the new customized. As a result, gross sales of the businessperson angularity and in extremity stock level decreases speedily. Thus, the trade recognition credit policy is accommodating for the businessperson to get more net profit by decreasing the stock list cost. On the other hand, by availing commercialism credit installation retail merchant can order more abstraction with minimum arrangement cost and less investment funds capital. Also, merchandising earns the interest on revenue assembled by selling the items during the commendation period.

Practically, individual degenerate inventory models have contrastive degenerate decoration. For instance, cloth items, electronic gadgets, creating from raw materials, fruits, dry fruits, cold drinks, health drinks, medicines, different type of cultivation commodity, impressionable commodity, metals have different decline in quality patterns. Furthermore, it has been ascertained that to the highest degree of regular needs have qualify life span. For examples liquid body substance production, vegetables, cool drinks, packed food, medicines etc. Has short life span. Thus, retailer dealing with these types of items has always planned to sell them before they get expire.

Considering the above-mentioned facts, here we formulated a merchandise possibility having depreciating time interdependent demand structure with changeable possession cost for two - Insufficiency facility under unexceptionable postponement in payment. Moreover, the model is discussed in fuzzy environment by taking the parametric quantity as quadrilateral fuzzy associate. The impersonal of the work is to inform the total cost of stock list by acquire the ability inventory instance for both the optimum solution nonexempt to the small consequence in the connected parametric quantity. Inventory theoretical account is primary humbleness in commercial enterprise that proposition manufacturing, spatial arrangement and retail substructure. Among the many concerns, economic process melodramatic constitution a faultfinding role determinant the best inventory plan of action. Classical inventor model from a unbounded prognostication perspective, take for granted constant demand but this supposition is only efficacious for an indeterminate fundamental quantity during the autumnal phase of the merchandise is propulsion into the market or diminution, perchance due to new cooperation. Addressing ever-changing demand involves investigation in two characteristic of inventory preparation models; the decline in quality of inventory items and fluctuation in the postulation rate over instance.

Assumption And Notation:

Assumption:



1. The demand rate is $\alpha e^{\beta t}$, where α and β are both constant such that $\alpha > 0$; $\beta > 0$.
2. Two rates of production are considered.
3. Production run only single product.
4. The production rate is always greater than or equal to sum of the demand rate and deterioration rate.
5. Rate of deterioration is linear function rate.
6. Shortages are not allowed.

Notations:

- P_t = Initial Production Rate in Units/Unite Time
- I_{o1} = On - Hand Inventory Level at Time t_1
- I_{o2} = On - Hand Inventory Level at Time t_2
- I_o = Optimal Inventory
- P_c = Production Cost / Unit Time
- H_c = Holding Cost /Unit/Year
- S_c = Setup Cost /Setup
- $a + bt$ = Rate of Deterioration, where $0 < a; b \ll 1$
- t = Cycle Time
- U_t = Unit Time in Period; $t = 1,2,3,\dots\dots$
- C_T = Total Cost
-

Mathematical Formula:

Let the cycle starts at time $t = 0$ and during the time interval $[0, t_1]$, the production rate is P_t and the demand rate is $\alpha e^{\beta t}$ such that $P_t > \alpha e^{\beta t}$ and the inventory level increases at the rate $P_t - \alpha e^{\beta t}$. At time t_1 .

Suppose that I_{o1} is the maximum inventory level. During the time interval $[t_1, t_2]$, the production rate is YP_t and demand rate is $Y\alpha e^{\beta t}$ where Y is constant and $Y > 0$ and hence, inventory level increases in this interval at the rate $Y[P_t - \alpha e^{\beta t}]$. At time t_2 production stopped and suppose that I_{o2} is maximum inventory at that time. Due to demand and deterioration inventory level starts to decrease and after time t inventory becomes zero.

Let I_t be the inventory level at any time t where $0 \leq t_0 \leq t$. then the model is governing by the following differential equations

$$\frac{dI_o}{dt} + (a + bt)I_o = P_t - \alpha e^{\beta t} \quad 0 \leq t_0 \leq t_1 \quad \dots (1)$$

$$\frac{dI_o}{dt} + (a + bt)I_o = Y(P_t - \alpha e^{\beta t}) \quad t_1 \leq t \leq t_2 \quad \dots (2)$$

$$\frac{dI_o}{dt} + (a + bt)I_o = -\alpha e^{\beta t} \quad t_2 \leq t \leq t_0 \quad \dots (3)$$

The boundary condition of above Differential Equation are given by

$$I(0) = 0; I(t_1) = I_{t_1}; I(t_2) = I_{t_2}; I(t) = 0 \quad \dots (4)$$

Using the boundary condition (3) and neglecting second and higher power of a and b are very small then, the solution of equation (1),(2),(3) are given by respectively

$$I = \frac{\alpha}{\beta}(1 - e^{\beta T}) + P_t(T - \frac{aT^2}{2} - \frac{bT^3}{3}) - \frac{a\alpha}{\beta^2}(1 + \beta T - e^{\beta T}) + \frac{ab}{\beta}[\frac{1}{\beta^2} - \frac{T^2}{2} + (\frac{T}{\beta} - \frac{1}{\beta^2})e^{\beta T}]; 0 \leq t_0 \leq t_1 \dots\dots\dots (5)$$

$$\begin{aligned}
 &= \frac{\alpha\gamma}{\beta}(1 - e^{\beta T}) + \gamma P_t \left(T - \frac{aT^2}{2} - \frac{bT^3}{3}\right) - \frac{\alpha\gamma e^{\beta T}}{\beta} \left(1 - \frac{a + bt}{b} + \frac{b}{\beta^2}\right) - \left(aT + \frac{bT^2}{2}\right) \left[\frac{\alpha}{\beta} \left(1 - e^{\beta t_1}\right) + P_t t_1 (1 - \gamma) + \frac{\alpha\gamma e^{\beta t_1}}{\beta}\right] + \frac{a(1 - e^{\beta t_1})}{\beta} + \frac{\alpha\gamma e^{\beta t_1}}{\beta} \left(1 - \frac{a + bt_1}{b} + \frac{b}{\beta^2}\right) \\
 &+ P_t (1 - \gamma) \left(t_1 \frac{at_1^2}{2} - bt_1^3\right) - \frac{\alpha a(1 + \beta t_1)}{\beta^2} - \frac{\alpha b}{b} \left(\frac{1}{\beta^2} - \frac{t_1^2}{2}\right) + \frac{\alpha b}{b} \left(\frac{t_1}{\beta} - \frac{1}{\beta^2}\right) e^{\beta t_1} \\
 &+ \left(at_1 + \frac{bt_1^2}{2}\right) \left[\frac{\alpha}{\beta} (1 - e^{\beta t_1}) + P_t t_1 (1 - \gamma) + \frac{\alpha\gamma e^{\beta t_1}}{\beta}\right] \quad ; t_1 \leq t \\
 &\leq t_2. \quad \dots \dots \dots (6) \\
 I &= \frac{\alpha}{\beta} \left[b \left(\frac{1}{\beta} - \frac{1}{\beta^2}\right) + \frac{a}{\beta} - 1\right] e^{\beta t} - \frac{\alpha e^{\beta t}}{\beta} \left(at + \frac{bt^2}{2}\right) + \frac{\alpha e^{\beta t}}{\beta} \left[1 + a\left(t - \frac{1}{b}\right) + b\left(\frac{1}{b^2} + \frac{t^2}{2} - \frac{t}{\beta}\right)\right]; \quad t_2 \leq t \leq t_0 \dots \dots \dots (7).
 \end{aligned}$$

Numerical Examples:

Let us consider an inventory model with the following data:

$P_t = 3880$; $\alpha = 582$; $\beta = 0.291$; $a = 0.0097$; $b = 0.097$; $\theta = 0.388$; $\gamma = 1.94$; $S_c = 77.6$; $P_c = 38.8$; $H_c = 1.94$; $t = 5.65607$; $I_0 = 19515.43$; $t_1 = 1.41717$; $t_2 = 3.542828$; $I_{01} = 4092.236$; $I_{02} = 1142.466$; $C_T = 30185.527$; $P^* = 22269.26$; $S^* = 13.3084$; $H^* = 6504.917$; $D^* = 1397.576$.

Sensitivity:

1. Increasing of deterioration, cycle time t , Optimal inventory I , production time (t_1, t_2), maximum inventory I_1 and total cost, decreases but maximum inventory I_2 increases.
2. Increasing of setup cost, cycle time t , Optimal inventory I , production time (t_1, t_2), maximum inventory t_1 and t_2 total cost, remains unchanged.
3. Increasing of holding cost, cycle time t , Optimal inventory I , production time (t_1, t_2), maximum inventory t_1 decreasing but total cost, and maximum inventory t_2 increases.
4. Increasing of production cost, cycle time t , Optimal inventory I , production time (t_1, t_2), maximum inventory t_1 and total cost, increases but maximum inventory t_2 decreases.

Table:1; Variation in total cost rate and deteriorating items with inventory:

a	b	t	I_0	P^*	S^*	H^*	D^*	C_T
0.0097	0.0097	5.65607	19515.43	22269.698	13.3084	6504.917	1397.516	30185.4979
0.0199	0.0097	5.596415	18956.71	22325.035	13.4539	6386.965	1197.756	29923.1905
0.0291	0.0097	5.537245	1841.42	22382.75	13.5897	6271.729	997.839	29665.8301
0.0097	0.0776	6.204411	25363.56	19913.13	12.1347	7287.319	2767.313	29979.8773
0.0097	0.0873	5.910889	22066.53	21248.335	12.7361	6857.512	2007.318	30125.8526

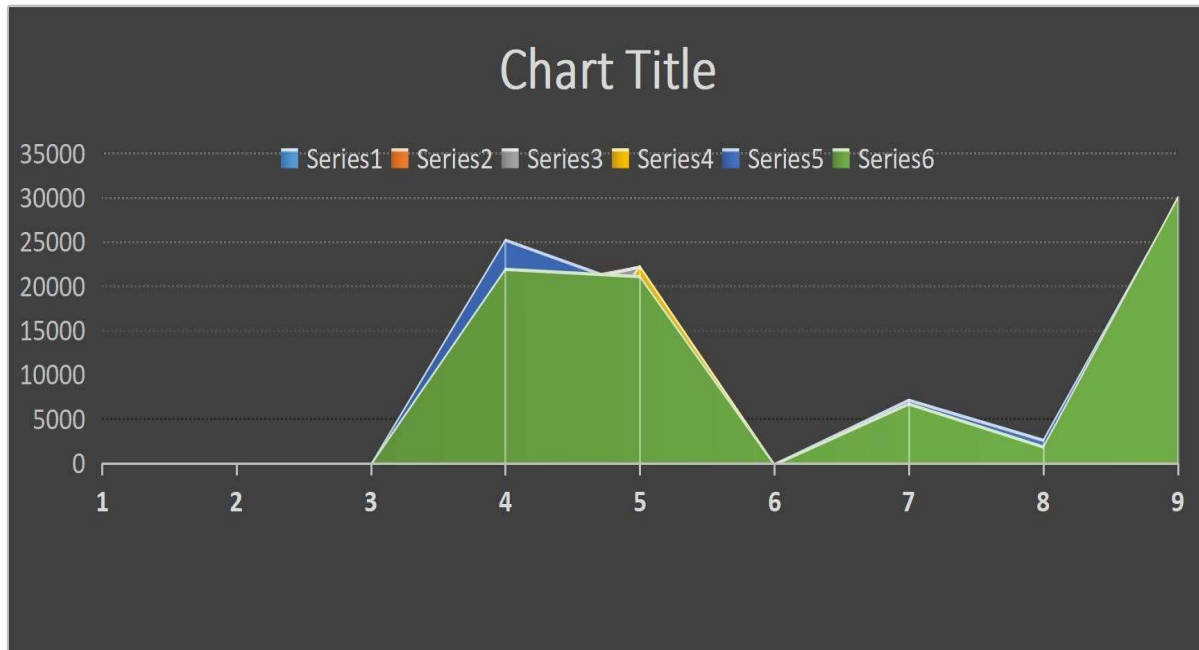


Fig:1: -Variation in total cost rate and deteriorating items with inventory

Table:2; Effect of demand and cost parameters on optimal values:

a	b	t	I_o	t_1	t_2	I_{o1}	I_{o2}	C_T
0.0097	0.0097	5.65607	19515.43	22269.698	13.3084	6504.917	1397.516	30185.4979
0.0199	0.0097	5.596415	18956.71	22325.035	13.4539	6386.965	1197.756	29923.1905
0.0291	0.0097	5.537245	1841.42	22382.75	13.5897	6271.729	997.839	29665.8301
0.0097	0.0776	6.204411	25363.56	19913.13	12.1347	7287.319	2767.313	29979.8773
0.0097	0.0873	5.910889	22066.53	21248.335	12.7361	6857.512	2007.318	30125.8526
S_c	67.9	5.65607	19515.43	1.41717	3.542925	4092.333	1141.884	30182.132
	77.6	5.65607	19515.43	1.41717	3.542828	4092.236	1142.466	30185.4979
	87.3	5.65607	19515.43	1.41717	3.542828	4092.236	1142.466	185.4979
H_c	1.455	5.7133	20064.838	1.429004	3.572413	4115.904	962.337	28549.525
	1.94	5.65607	19515.43	1.41717	3.542828	4092.236	1142.466	30185.527
	2.425	5.602429	19012.291	1.4065	3.516056	4070.605	1301.74	31802.42
P_c	33.95	5.62503	19222.878	1.410865	3.527211	4079.626	1235.78	27224.408
	38.8	5.65607	19515.43	1.417073	3.542828	4092.236	1142.466	30185.527
	43.65	5.681096	19754.05	1.42202	3.555632	4102.518	1064.963	33141.796

β	0.0097	5.511346	5836.684	0.477628	1.194167	1595.068	4557.351	41841.144
	0.194	5.765874	11358.506	1.331422	3.328943	3986.021	4442.891	31391.334
	0.291	5.65607	11358.506	1.331422	3.542828	4092.236	1142.466	30185.4979
α	543.2	5.825723	19771.316	1.503694	3.75332	4341.138	605.474	31810.956
	582	5.65607	19515.43	1.417073	3.542828	4092.236	1142.466	30185.527
	620.8	5.511152	19394.083	1.341316	3.353229	3864.868	1444.039	28990.099
ρ	0.388	5.65607	19515.43	1.417073	3.542828	4092.236	1142.466	30185.527
	0.485	5.304736	16418.608	1.538711	3.077442	4322.902	2790.496	23197.55
	0.582	5.16331	15296.9	1.733972	2.890018	4633.011	2732.587	20374.365
Υ	1.94	5.65607	19515.43	1.417073	3.542828	4092.236	1142.466	30185.527
	2.134	5.474583	17858.185	1.309791	3.274526	3866.711	2670.992	28239.319
	2.328	5.332866	16649.856	1.2222	3.055403	3668.346	3838.872	26945.242

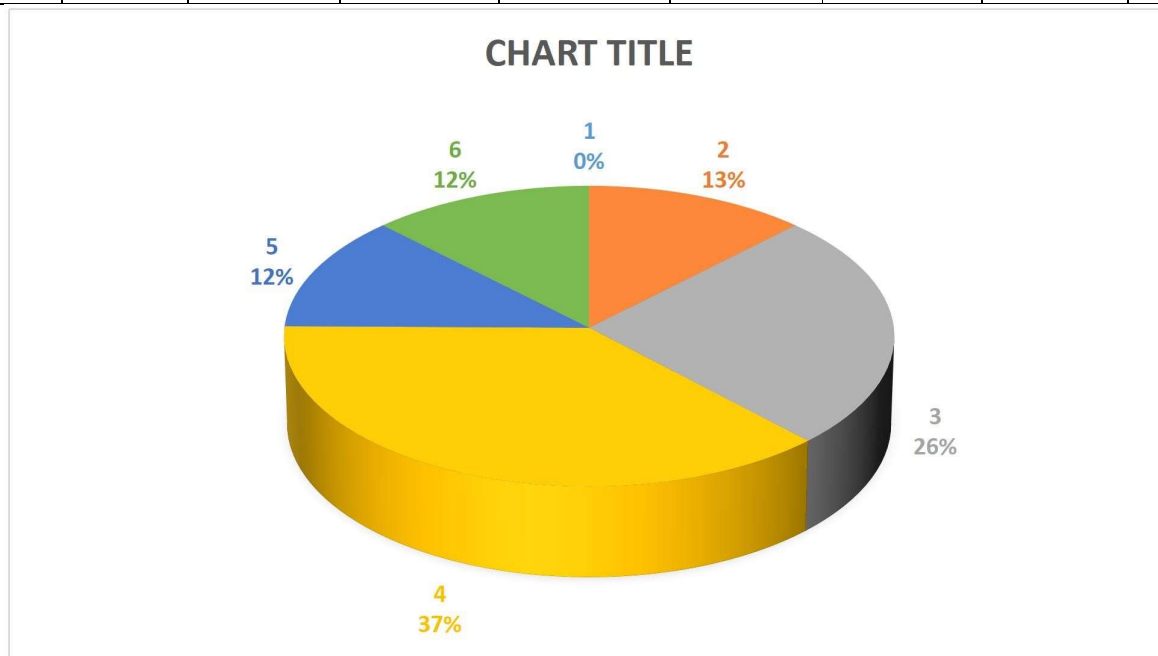


Fig:2:-Effect of demand and cost parameters on optimal values:

CONCLUSION:

In this paper, we planned Inventory model with holding costs, ordering costs and quality improvements with time dependent demands with graphical and mathematical notation of economic process. The benefits of this model lie in its ability to balance cost minimization—by accounting for



holding, ordering, and deterioration costs—while fostering cooperation between supply chain partners to address quality decline issues.

In situations where inventory degradation is a concern, both levels in the supply chain can mutually benefit from improving processes that reduce quality loss, whether through better storage practices, faster product turnover, or joint quality control efforts.

Ultimately, the model highlights the importance of strategic coordination between business levels to address common challenges such as inventory aging and quality control, making it a valuable tool for businesses aiming to optimize supply chain performance while maintaining product quality and minimizing overall costs.

The modification manufacture rate supplies a way resulting user satisfaction and earning possibility profit. A numerical example is provided to demonstrate means practical usage. This model can be extended by taking more realistic assumption such as shortages, other type of demand patterns etc.

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