SELECTIONOF WATER PURIFIERES BY GRAY RELATIONAL ANALYSIS METHOD (FUZZY NUMBERS)

Jugendra Singh Assistant Professor, Department of Mechanical Engineering IIMT College of Polytechnic, Greater Noida, UP, India jugendrasingh.351@gmail.com

Sanjay mavi Assistant Professor, Department of Mechanical Engineering IIMT College of Polytechnic, Greater Noida, UP, India mavi.sunny@gmail.com

ABSTRACT

Unpurified water plants have been utilized to supply fresh water to people, industries etc. in the world to fulfill their needs of pure water. Unpurified water is a world problem that affects to countries all over the world and it causes a high environmental and economic cost, and poses a high risk to global health. This study is an attempt to identify the most suitable technology for the specific use by soliciting expert opinions desalination technologies. The selection process in this study was limited to seawater feed and five factors and three commercially available desalination technologies, i.e., multi-stage flash, vapor compression and reverse osmosis. In this paper we are using an extended grey relational analysis method of multi criteria decision making to select appropriate optimum purified technologies with interval-valued triangular fuzzy numbers and unknown information on criterion weights to select an optimum purified technology.

Keywords: Desalination, GRA, MCDM, vapor compression

Introduction:

Water is almost as important to life as is the air we breathe. India accounts for 15% of the world's population but has only 3% of the usable fresh water. The total quantity of usable fresh water annually available in India is fixed, but its demand from expanding agriculture and other sectors is increasing at an unprecedented rate.

According to the United Nations, around two to seven billion people will face water shortages by the year 2045 and the amount of water available per person will shrink by a third during the next two decades. The World Water Council estimates that, by 2022, the world will be about 16% short of the fresh water needed to sustain the world population. The World Health Organization (WHO) has estimated that 1000 cubic meters per person per year is the benchmark level below which chronic water scarcity is considered to impede development and harm human health.

1. Purified technologies:

The Purified technologies can be broadly categorized into three general groups of distillation, membrane-based and ion exchange. The basic working principle and working of the processes falling under each of these groups is explained as follows:

1.1 Thermal Distillation processes

In thermal distillation processes the water is transformed into vapor and then is condensed into a liquid state. Commercially available technologies of this type include:

1.1.1.Multi-stage flash distillation

In the Multi-stage flash distillation process, seawater is heated to a high temperature ranging from 90 to 125°C in a vessel called the brine heater. This is generally done by condensing steam on a bank of tubes that carry seawater which passes through the vessel. This heated seawater then flows into another vessel, called a stage, where the ambient pressure is lower, causing the water to immediately boil. The sudden introduction of the heated water into the chamber causes it to boil rapidly, almost exploding or flashing into steam.
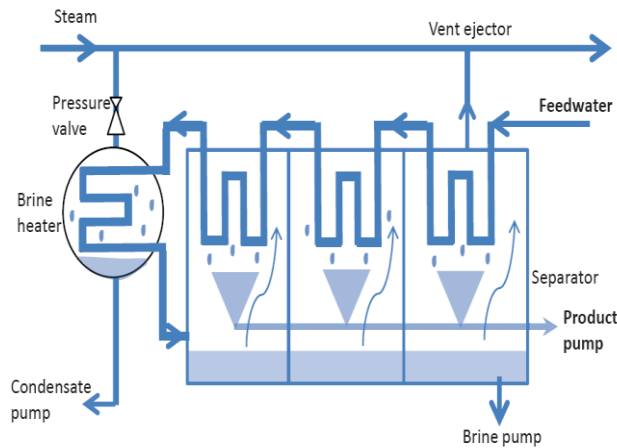
Fig 1- MSF Desalination process (Source: Fichtner,2011)

1.1.2   Vapor compression

The water vapor is collected and compressed. The compression causes the vapor to condense on one side of the tube wall. The heat generated during condensation is transferred back to the feed water in order to continue its evaporation. The heat for evaporating the water comes from the compression of vapor, rather than the direct exchange of heat from steam produced in a boiler (Buros, 2000).Seawater is sprayed on the outside of the heated tube bundle where it boils and partially evaporates, producing more vapor.

With the steam-jet type of VCD unit, called a thermo-compressor, a venturi orifice at the steam jet creates and extracts water vapor from the evaporator, creating a lower ambient pressure.

The extracted water vapor is compressed by the steam jet. This mixture is condensed on the tube walls to provide the thermal energy, heat of condensation, to evaporate the seawater being applied on the other side of the tube walls in the evaporator

1.2. Membrane-based desalination

Membrane technologies rely on utilizing the surface properties of membranes. The selective membranes are used to separate out the dissolved salts from water molecules. The most popular membrane-based desalination technologies are:

1.2.1   Reverse osmosis (RO)

RO is a pressure-driven process that separates two solutions with different concentrations across a semi-permeable membrane. When pressure is applied to the solution with the higher salt concentration solution, the water will flow in a reverse direction through the semi-permeable membrane, leaving the salt behind.
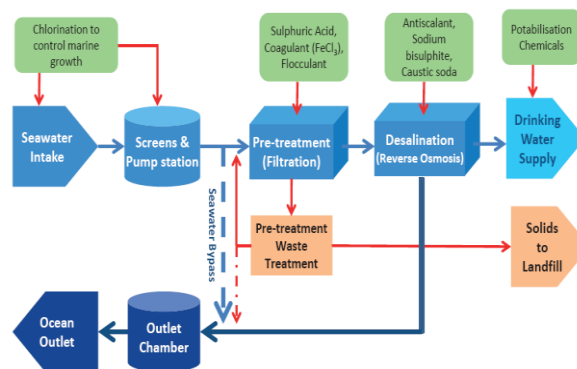

Fig:2- RO Desalination process (Source: Fichtner, 2011)

Methodology:

In fuzzy MCDM problems, performance rating values are usually characterized by fuzzy numbers. In this paper, criteria values are considered as linguistic variables. The concept of a linguistic variable is very useful in dealing with situations that are too complex or too ill-defined to be amenable for description in conventional quantitative expressions. These linguistic variables can be expressed as interval-valued triangular fuzzy numbers given in Table 1.

Definitions of linguistic variables for the ratings

Linguistic variables	Interval-valued triangular fuzzy numbers
Very poor (VP)	[(0, 0);0; (1, 1.5)]
Poor (P)	[(0, 0.5);1; (2.5, 3.5)]
Medium poor (MP)	[(0, 1.5); 3;(4.5, 5.5)]
Medium (M)	[(2.5, 3.5); 5;(6.5, 7.5)]
Medium good (MG)	[(4.5, 5.5); 7;(8, 9.5)]
Good (G)	[(5.5, 7.5); 9;(9.5, 10)]
Very good (VG)	[(8.5, 9.5);10;(10,10)]

Consider a MCDM problem, let A = {A_1,A_2, . . . ,A_m} be a finite set of feasible alternatives, C = {C_1,C_2, . . . ,C_n} be a finite set of criteria. The weight vector of the criteria w = (w_1,w_2, . . . ,w_n) is unknown, but it satisfies $w_j \geq 0$; j = 1, 2, . . . , n, $\sum_{j=1}^{n} w_j = 1$.

Suppose that the performance of alternative A_i with respect to criterion C_j is denoted as $\tilde{x}_{ij}$, then, $\tilde{X} = \left[\tilde{x}_{ij}\right]_{m \times n}$ is a fuzzy decision matrix. As illustrated in Fig. 1, $\tilde{x}_{ij}$ is expressed in interval-valued triangular fuzzy number: $x = \left\{(x_1, x_2, x_3), (x_1', x_2', x_3')\right\}$.

The basic principle of the GRA method (Wei, 2010) is that the chosen alternative should have the ''largest degree of grey relation'' from the reference solution. Obviously, for the weight vector given, the larger the values $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$, the better the alternative A_i is. Butthe information on criterion weights is unknown. So, in order to get the values $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$, we must first calculate the weight information. For this purpose, we can establish the following multiple objective optimization model to obtain the weight information:

Since each alternative is non-inferior, so there exists no preference relation on all the alternatives. Therefore, we can aggregate the above multiple objective optimization model with equal weights into the following single-objective optimization model:

To solve the above model, referring to (Wu & Chen, 2007), we construct the Lagrange function of the constrained optimization problem (M-2):

Where λ is the Lagranges multiplier, and it is a real number.

Differentiating Eq. (16) with respect to wj (j = 1, 2,. . . ,n) and λ, and setting these partial derivatives equal to zero, we obtain the following set of equations:

$$\begin{cases} \dfrac{\partial L}{\partial w_j} = \sum_{i=1}^{m} (\xi_{ij}^{(1)} + \xi_{ij}^{(2)}) + \lambda w_j = 0 \\ \dfrac{\partial L}{\partial w_j} = \sum_{j=1}^{n} w_j^2 - 1 = 0 \end{cases}$$

By solving Eq. (17), we get a simple and exact formula for determining the criteria weights as follows:

By normalizing $w_j^{(*)} (j = 1; 2; \ldots ; n)$ be a unit, we have

$$w_j = \frac{\displaystyle\sum_{i=1}^{m} (\xi_{ij}^{(1)} + \xi_{ij}^{(2)})}{\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n} (\xi_{ij}^{(1)} + \xi_{ij}^{(2)})}$$

The weight vector of criteria is w = (w₁,w₂, . . . ,wₙ). Then, we can get cð1Þ i and cð2Þ i ði ¼ 1; 2; . . . ;mÞ by Eq. (15). That is to say, the grey relational grade between the reference series and comparison series is an interval value _ ci ¼ cð1Þ i ; cð2Þ I h I ði ¼ 1; 2; . . . ;mÞ.

$$p(A_s \succ A_t) = p(\bar{\gamma}_s \geq \bar{\gamma}_t)$$

$$= \max\left\{1 - \max\left\{\frac{\gamma_t^{(2)} - \gamma_s^{(1)}}{L(\bar{\gamma}_s) + L(\bar{\gamma}_t)}, 0\right\}, 0\right\},$$

$\bar{\gamma}_s = \left[\gamma_s^{(1)}, \gamma_s^{(2)}\right], \bar{\gamma}_t = \left[\gamma_t^{(1)}, \gamma_t^{(2)}\right], L(\bar{\gamma}_s) = \gamma_s^{(2)} - \gamma_s^{(1)}$ and $L(\bar{\gamma}_t) = \gamma_t^{(2)} - \gamma_t^{(1)}$.

Thus, the likelihood matrix can be obtained and expressed as follows:

As the matrix P is a fuzzy complementary judgement matrix, optimal degrees of membership for alternatives $A_i (i = 1,2, ,\ldots,m)$ can be defined as follows (Li et al., 2009):

Thus, a sort vector V = (V₁,V₂, . . . ,Vₘ) of the alternatives can be obtained.

Rank all the alternatives $A_i(i = 1,2,\ldots,m)$ and select the best one(s) in accordance with the value V_i (i = 1,2,\ldots,m). The bigger the value V_i, the better the alternative.

Calculation Analysis:

Consider a MCDM problem, let A = {A₁,A₂, . . . ,Aₘ} be a finite set of feasible alternatives, C = {C₁,C₂, . . . ,Cₙ} be a finite set of criteria. The weight vector of the criteria w = (w₁,w₂, . . . ,wₙ) is unknown, but it satisfies $w_j \geq 0; j = 1, 2, \ldots, n, \sum_{j=1}^{n} w_j = 1$.

Five criteria: C_1, C_2, C_3, C_4, C_5

C_1 = Available technology (AT), only commercially available ones.

C_2 = Plant capacity (PC), the higher the better ($\leq 10\%$ of the total available capacity).

C_3 = Energy consumption rate per unit water product (EC), the lower the rate the better.

C_4 = Equipment efficiency and type of energy utilization (EE),

C_5 = Total cost (TC).

Three alternatives: A_1, A_2, A_3

Where

A_1 = Vapour compression

A_2 = Reverse osmosis

A_3 = MSF

DECISION MAKERS :

Table 1

	A_1	A_2	A_3
C_1	**G[(5.5,7.5);9;(9.5,10)]**	**G[(5.5,7.5);9;(9.5,10)]**	**MG[(4.5,5.5);7;(8,9.5)]**
C_2	**MG[(4.5,5.5);7;(8,9.5)]**	**MG[(4.5,5.5);7;(8,9.5)]**	**M[(2.5,3.5);5;(6.5,7.5)]**
C_3	**G[(5.5,7.5);9;(9.5,10)]**	**G[(5.5,7.5);9;(9.5,10)]**	**MG[(4.5,5.5);7;(8,9.5)]**
C_4	**MG[(4.5,5.5);7;(8,9.5)]**	**G[(5.5,7.5);9;(9.5,10)]**	**VG[(8.5,9.5);10;(10,10)]**
C_5	M[(2.5,3.5);5;(6.5,7.5)]	MG[(4.5,5.5);7;](8,9.5)]	MG[(4.5,5.5);7;(8,9.5)]

Calculate the normalized decision matrix, $\widetilde{R}$. **Given,** $\widetilde{x}_{ij} = \left[(a_{ij}, a_{ij}^{'}); b_{ij}; (c_{ij}^{'}, c_{ij})\right]$:

NORMALIZED DECISION MATRIX:-

Table 2

	A_1	A_2	A_3
C_1	**[(0.55,0.75);0.9;(0.95,1.00)]**	**[(0.55,0.75);0.9;(0.95,1.00)]**	**[(0.45,0.55);0.70;(0.80,0.95)]**
C_2	**[(0.45,0.55);0.7;(0.8,0.95)]**	**[(0.45,0.55);0.70;(0.80.0.95)]**	**[(0.25,0.35);0.50;(0.65,0.75)]**
C_3	**[(0.55,0.75);0.9;(0.95,1.00)]**	**[(0.55,0.75);0.90;(0.95,1.00)]**	**[(0.45,0.55);0.70;(0.80,0.95)]**
C_4	**[(0.45,0.55);0.7;(0.8,0.95)]**	**[(0.55,0.75);0.90;(0.95,1.00)]**	**[(0.85,0.95);1.00;(1.00,1.00)]**
C_5	[(0.25,0.35);0.5;(0.65,0.75)]	[(0.45,0.55);0.7;(0.8,0.95)]	[(0.45,0.55);0.70;(0.8,0.95)]

$c_i = [(1,1);1;(1,1)]$

Where $i = 1,2,3,\ldots\ldots\ldots m$

Calculate the distance between the reference value and each comparison value:

Because $\widetilde{\delta}_{ij} = \sqrt{1/3[(g-1)^2 + (h-1)^2 + (l-1)^2}$

$\widetilde{\delta}_{ij} = [\delta_{ij}^{(1)}, \delta_{ij}^{(2)}]$

$\delta_{\max}^{(1)} = \max_{ij} \delta_{ij}^{(1)}$, $\delta_{\min}^{1} = \min_{ij} \delta_{ij}^{(1)}$

$\delta_{\max}^{(2)} = \max_{ij} \delta_{ij}^{(2)}$, $\delta_{\min}^{(2)} = \min_{ij} \delta_{ij}^{(2)}$,

Where $i = 1,2,3,\ldots\ldots m$,

$J = 1,2,3,\ldots\ldots n$

Distance from the reference series:-

$\widetilde{\delta}_{ij} = \sqrt{1/3[(g-1)\wedge2 + (h-1)\wedge2 + (l-1)\wedge2}$

so **for** C_1 **to** A_1 **(max)**

$\widetilde{\delta}_{ij} = \sqrt{1/3[(0.75-1)\wedge2 + (0.90-1)\wedge2 + (1-1)\wedge2}$

$\widetilde{\delta}_{ij} = \sqrt{0.02416}$

$\widetilde{\delta}_{ij} = 0.155$

Result Analysis:

$V_1 = \dfrac{1}{m(m-1)}\left\{ \sum_{r=1}^{m} P_{ir} + \dfrac{m}{2} - 1 \right\}$,

$$= \frac{1}{3(3-1)} \{ (0.5+0+0) + \frac{3}{2} - 1 \},$$

$$V_1 = 0.1666,$$

similarly

$$V_2 = \frac{1}{m(m-1)} \{ \sum_{r=1}^{m} P_{ir} + \frac{m}{2} - 1 \},$$

$$= \frac{1}{3(3-1)} \{ (0.5+0.5807+0) + \frac{3}{2} - 1 \},$$

$$V_2 = 0.4300,$$

similarly

$$V_3 = \frac{1}{m(m-1)} \{ \sum_{r=1}^{m} P_{ir} + \frac{m}{2} - 1 \},$$

$$= \frac{1}{3(3-1)} \{ (0.4193+0.5+0) + \frac{3}{2} - 1 \},$$

$$V_3 = 0.4031.$$

Rank all the alternatives $A_i(i = 1,2,\ldots,m)$ and select the best one(s) in accordance with the value V_i $(i = 1,2,\ldots,m)$. **The bigger value V_i, the better alternative.**
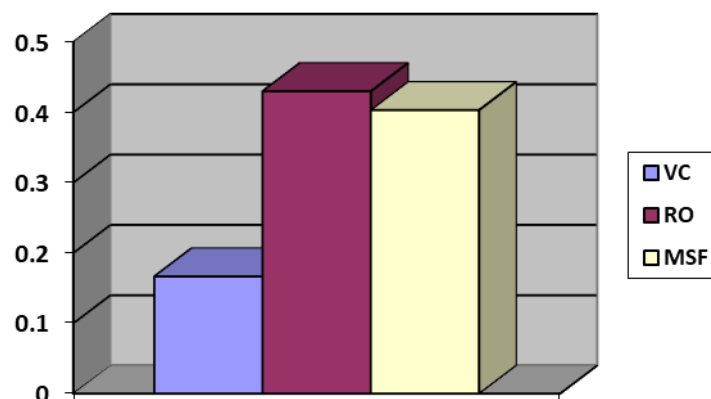
$$A_2 \succ A_3 \succ A_1$$

A_1 =Vapour compression

A_2 =Reverse osmosis

A_3 =MSF

Table 3

Comparison of different technologies	Value
A_1	0.1666
A_2	0.4300
A_3	0.4031

Comarison of DesalinationTtechnologies

Conclusions:

In the world there are acute shortages of fresh water, projects to build desalination plants that are very energy demanding and extremely costly should only be embarked upon after conducting elaborate

studies to find out the most suitable technology to adopt. Our study showed that cost is the most important criteria so we have to select most Optimum technology for better desalination water; therefore, This study investigated single desalination technology processes. We suggest similar studies to be conducted on hybrid desalination plants which utilize more than one technology. In addition, we recommend that other decision-making techniques are used in future studies.

References

1. Lior, N. Advances in Water Desalination; Lior, N., Ed.; John Wiley & Sons, Inc.: Hoboken, NJ, USA, 2012; ISBN 9781118347737.

2. Wetterau, G.; Moch, I.; Frenkel, V.; Huehmer, R.; Hunt, H.; Kiefer, K.; Pankratz, T.; Sethi, S.; Silverman, G.; Trussel, S.; et al. Desalination of Seawater—M61, 1st ed.; American Water Works Association: Denver, CO, USA, 2011; ISBN 978-1-58321-833-4.

3. Al-Othman, A.; Darwish, N.N.; Qasim, M.; Tawalbeh, M.; Darwish, N.A.; Hilal, N. Nuclear desalination: A state-of-the-art review. Desalination 2019, 457, 39–61.

4. Alnajdi, O.; Wu, Y.; Calautit, J.K. Toward a sustainable decentralizedwater supply: Review of adsorption desorption desalination (ADD) and current technologies: Saudi Arabia (SA) as a case study. Water 2020, 12, 1111.

5. Cipollina, A.; Micale, G.; Rizzuti, L. A critical assessment of desalination operations in Sicily. Desalination 2005, 182, 1–12.

6. El-Ghonemy, A.M.K. Performance test of a sea water multi-stage flash distillation plant: Case study. Alexandria Eng. J. 2018, 57, 2401–2413.

7. Lee, K.P.; Arnot, T.C.; Mattia, D. A review of reverse osmosis membrane materials for desalination-Development to date and future potential. J. Membr. Sci. 2011, 370, 1–22.

8. Rognoni, M. La Dissalazione Dell'acqua di Mare. Descrizione, Analisi e Valutazione Delle Principali Tecnologie, 1st ed.; Dario Flaccovio Editore s.r.l.: Palermo, Italy, 2010; ISBN 9788857900308.

9. Weir, W.V. The Weir Group: The History of a Scottish Engineering Legend; Profile Books: London, UK, 2008; ISBN 978-1861978868.

10. Al-Mutawa, A.M.; Al Murbati, W.M.; Al Ruwaili, N.A.; Al Oraif, A.S.; Al Oraif, A.; Al Arafati, A.; Nasrullah, A.; Al Bahow,