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STATIC ANALYSES OF CARBON NANOTUBE-REINFORCED COMPOSITE PLATES USING FINITE ELEMENT METHOD WITH FIRST ORDER SHEAR DEFORMATION PLATE THEORY

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Abstract
Static analyses of thin-to-moderately thick composite plates reinforced by single-walled carbon nanotubes using the finite element method based on the first order shear deformation plate theory. Four types of distributions of the uniaxially aligned reinforcement material are considered, that is uniform and three kinds of functionally graded distributions of carbon nanotubes along the thickness direction of plates. The effective material properties of the nanocomposite plates are estimated according to the rule of mixture.
For the static analysis, it is found that both the CNT volume fraction and the width-to-thickness ratio have pronounced effect on the natural static analysis of the CNTRC plate. To increase the strength of plate stiffener are used and for comparison, results are validated with literature. Some new results for stiffened CNTRC plates are presented in terms of parametric studies .A computer program, for the finite element analysis of such carbon nanotube reinforced composites stiffened plates, has been developed in FORTRAN.

Introduction
Conventional metal material stiffened plates are structural components consisting of plates reinforced by a system of ribs or beams to enhance their load-carrying capacity. There are many practical applications of such structures. Carbon nanotube reinforced composite laminates as a primary structural component has increased considerably in recent times for applications in important weight-critical structures. Carbon nanotubes and nanofibers were directly grown on carbon fiber substrate by catalytic decomposition of acetylene precursor using thermal chemical vapor deposition process. These carbon nanotubes and nanofibers coated carbon fibers were used as reinforcement in epoxy matrix for the fabrication of unidirectional composites. The morphology of carbon nonmaterial's grown on carbon fibers was examined by scanning electron microscope (SEM) and high resolution transmission electron microscope (HRTEM). Electron microscopic observations revealed uniform coverage of carbon fibers with carbon nanotubes, nanofibers and filaments.
The composites made of carbon nanotubes coated carbon fibers are showed 69% higher tensile strength as compared to composites made of carbon fiber which had undergone similar heat treatment but without carbon nanotubes growth. The results of tensile test revealed that both the high vacuum condition and choice of an appropriate catalyst precursor strongly influence the fiber properties thereby affect the resultant properties of the composites made of these surface modified carbon fibers To achieve better efficiency in terms of strength and weight-optimization, such structures are frequently appended with beam-like stiffener components. A large amount of literature on analysis of stiffened plates of isotropic materials is available. A reasonable amount of literature on analysis of stiffened plates of Fiber Reinforced Laminated Composites also exists already. Methods of analysis are based mainly on different numerical methods like FEM, FDM, FSM etc. However, the FEM is found to be the most popular one amongst them and perhaps most effective.

2. Carbon Nanotubes
A carbon atom can form various types of allotropes. In 3D structures, diamond and graphite are the allotropes of carbon. Carbon also forms low-dimensional (2D, 1D or 0D) allotropes collectively known as carbon nonmaterial. Examples of such nonmaterials are 1D carbon nanotubes (CNTs) and 0D

fullerenes. In the list of carbon nonmaterial, grapheme is known as 2D single layer of graphite. The sp bonds in grapheme are stronger than sp bonds in diamond that makes grapheme the strongest material. The lattice structure of grapheme in real space consists of hexagonal arrangement of carbon atoms. An isolated carbon atom has four valence electrons in its 2s and 2p atomic orbital. These sp orbital's are in the same plane while the remaining 2 p_z is perpendicular to other orbital .The σ bonds between the adjacent carbon atoms are formed by the sp hybridized orbital, whereas the 2 p_z orbitals form the π bonds that are out of the plane of grapheme .

3.1 Types of Carbon Nanotubes

Depending on the number of concentrically rolled-up grapheme sheets, CNTs are also classified to single-walled (SWNT), double-walled (DWNT), and multiwall CNTs (MWNT) .The structure of SWNT can be conceptualized by wrapping a one-atom-thick layer of grapheme into a seamless cylinder. MWNT consists of two or more numbers of rolled-up concentric layers of graphene. DWNT is considered as a special type of MWNT where in only two concentrically rolled up grapheme sheets are present. There are two models to describe the structures of MWNT. CNT made from grapheme sheet zigzag and armchair.

.**3.2 First-Order Shear Deformation Theory** The present first- order shear deformation theory, proposed herein and used in present thesis. In this theory yields a constant value of transverse shear strain through the thickness of the smart plate, and thus requires shear correction factors. The shear correction factors are dimensionless quantities introduced to account for the discrepancy between the constant states of shear strains in FSDT. For composite laminates, the shear correction factors, in general, depend on the constituent ply properties, lamination scheme, and type of structure (i.e., geometry and boundary conditions). As already mentioned, the objective has been to find an optimal choice between accuracy and complexity, since it is further required to be extended to stiffened configurations. Hence, first- order shear deformation theory of continuity requirement would make the next set of formulation.

Various Assumptions are considered for the plate analysis:

a) The material of the plate is elastic, homogenous and Isotropic in nature.

b) The plate is initially flat.

c) The deflection of the mid plane of the plate is small as compared to its thickness.

d) The straight line initially perpendicular to the mid plane of the plate will remain perpendicular even after bending of the plate.

e) Normal component of the stress will be assumed small as compared to other component.

f) With reference to the assumption b, c, &d it will be assumed that middle plane of the plate will remain unstiffened.

g) The welding effect of the stiffeners to the plate shall be ignored.

3.3 Plate Geometric and Constitutive Relations

3.3.1 Displacement Relation

Considering moderately thick CNTRC plates, the first order shear deformation plate theory (FSDT) was employed to account for the displacement field {u, v, w} within a plate domain, according to displacements and rotations of the mid-plane of the plate.

$$\left.\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned}\right\} \qquad (3.1)$$

From this, the spatial displacement field in terms of the reference plane variables, may be written in a compact form as

$$\{\Delta\} = [G]\{d\} \qquad \ldots(3.2)$$

$$\{d\} = \lfloor u_0 v_0 w_0 \theta_x \theta_y \rfloor$$

$$[G]=\begin{bmatrix}1&0&0&Z&0\\0&1&0&0&Z\\0&0&1&0&0\end{bmatrix}$$

3.3.2 Stress-Strain Relation

The linear in-plane and transverse shear strains are given by

$\varepsilon_{xx}= u_{,x}=u_{o,x}+z\theta_{x,x}= \bar{\varepsilon}_{xx} + Z\bar{\kappa}_{xx}$

$\varepsilon_{yy}= v_{,y}=v_{o,y}+z\theta_{y,y}= \bar{\varepsilon}_{yy} + Z\bar{\kappa}_{yy}$

$\varepsilon_{zz}= w_{,z}= 0$

$$\gamma_{xy}= u_{,y}+v_{,x}=(u_{o,y}+v_{o,x})+z(\theta_{x,y}+\theta_{y,x}) \qquad ...(3.\ 3)$$

$\gamma_{xz}= u_{,z}+w_{,x}= \theta_x+w_{,x}= \Phi_x$

$$\gamma_{yz}= v_{,z}+w_{,y}= \theta_y+w_{,y}= \Phi_y$$

Where, the conventions,(...),x=(…)/∂x…etc is used above and hereafter. The over-barred quantities represent relevant generalized/laminate strain components defined at the reference plane. Hence, the nonzero strain components derived from the displacement assumptions.

$$\{\varepsilon\}_k^T= \{\varepsilon_{xx}\varepsilon_{yy}\gamma_{xy} : \gamma_{xz}\gamma_{yz}\}= \{\{\varepsilon_p\}\ \{\varepsilon_t\}\} \qquad \dots (3.4)$$

Where the subscripts p and t collectively represent the in-plane and transverse strain components, respectively; superscript T represents transposition.

3.4 Carbon Nano-Tube Reinforced Composite Plates:

The effective material properties of the two-phase nanocomposites, mixture of CNTs and an isotropic polymer, can be estimated according to the rule of mixture. Due to the simplicity and convenience, study the rule of mixture was employed by introducing the CNT efficiency parameters and the effective material properties of CNTRC plates can thus be written as

$$E_{11} = \eta_1 V_{cnt} E_{11} + V_m E^m$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_m}{E_m}$$

$$E_{22}\left(E_m V_{cnt} + E_{22}^{cnt} V_m\right) = \eta_2 E_m E_{22}^{cnt}$$

$$E_{22} = \frac{\eta_2 E_m E_{22}^{cnt}}{\left(E_m V_{cnt} + E_{22}^{cnt} V_m\right)}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_m}{G^m}$$

$$G_{12}\left(V_{cnt} G^m + G_{12}^{cnt} V_m\right) = \eta_3 G^m G_{12}^{cnt}$$

$$G_{12} = \frac{\eta_3 G^m G_{12}^{cnt}}{\left(V_{cnt} G^m + G_{12}^{cnt} V_m\right)}$$

Where E_{11}^{cn}, E_{22}^{cnt} and G_{12}^{cnt} indicates the young's moduli and shear modulus, respectively. E_m and G^m represent the corresponding properties of the isotropic matrix.

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_4\\\sigma_5\\\sigma_6\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\Q_{12}&Q_{22}&0&0&0\\0&0&Q_{44}&0&0\\0&0&0&Q_{55}&0\\0&0&0&0&Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{Bmatrix}$$

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}} \qquad Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}}$$

$$Q_{12}=\frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}}, \quad Q_{44}=G_{23}, \quad Q_{55}=G_{13}, \quad Q_{66}=G_{12}$$

3.5 BASIC ASSUMPTIONS OF STIFFENER

Some of the important assumptions made in the formulations are as follows.

1. The middle plane of the plate, namely the X-Y plane, is considered as the reference plane of the whole stiffened plate system.
2. The stiffeners attached with the plates are of rectangular cross section.
3. The stiffeners / beams are perfectly attached with the plate.
4. The lamination in the stiffeners may be either in vertical direction perpendicular to the plate or in horizontal direction parallel to the plate.
5. The stiffeners may have any curved shape and orientation on the plan view of the system.

3.6 Stiffener Geometric and Constitutive Relation:

3.6.1 Displacement Relation:

The spatial displacement field for the stiffeners is adopted as follows as first order shear deformation theory.

$$\begin{Bmatrix} U'(x',y',z',t) \\ V'(x',y',z',t) \\ W'(x',y',z',t) \end{Bmatrix} = \begin{Bmatrix} u'(x',t)+z'\theta_{x'}(x',t) \\ v'(x',t)+z'\theta_{y'}(x',t) \\ w'(x',t) \end{Bmatrix} \quad \text{... (3.12)}$$

This may also be written in a compact form as

$$\{\Delta'_s\}=[G_s]\{d'_s\} \quad (3.13)$$

Where, the spatial and reference plane displacement vectors for the stiffener, in its local axis$X$, system, are given by

$$\{\Delta'_s\}=[U'V'W']^T, \quad \{d'_s\}=[u'v'w'\theta_{x'}\theta_{y'}]^T$$

And the $[Gs]$ matrix may easily be obtained using Eq. (3.12)

3.6.2 Stress-Strain Relation:

Now, using the well known linear strain-displacement relations, the nonzero spatial strain components for the stiffeners are obtained as

$$\{\varepsilon'_s\}=\begin{Bmatrix} \varepsilon_{x'x's} \\ \gamma_{x'y's} \\ \gamma_{x'z's} \\ \gamma_{y'z's} \end{Bmatrix} = \begin{Bmatrix} U'_{,x'} \\ U'_{,y'}+V'_{,x'} \\ U'_{,x'}+W'_{,x'} \\ V'_{,z'}+W'_{,y'} \end{Bmatrix} = \begin{Bmatrix} u'_{,x'}+z'\theta_{x',x'} \\ v'_{,x'}+z'\theta_{y',x'} \\ w'_{,x'}+\theta_{x'} \\ \theta_{y'} \end{Bmatrix}$$

$$= \begin{Bmatrix} \bar{\varepsilon}_{x'x's}+z'\bar{k}_{x'x's} \\ \bar{\gamma}_{x'y's}+z'\bar{k}_{Tx's} \\ \bar{\varphi}_{x's}+y'\bar{k}_{Tx's} \\ \bar{\varphi}_{y's} \end{Bmatrix} = [H_s]\{\bar{\varepsilon}_s\} \quad (3.14)$$

$$[H_S] = \begin{bmatrix} 1 & 0 & z & 0 & 0 & 0 \\ 0 & 1 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where

$$\{\bar{\varepsilon}_s\} = \{u'_{,x'} \quad v'_{,x'} \quad \theta_{x',x'} \quad \theta_{y',x'} \quad (w'_{,x'} + \theta_{x'}) \quad \theta_{y'}\}^T \tag{3.15}$$

$$= \{\bar{\varepsilon}_{x'x's} \quad \bar{\gamma}_{x'y's} \quad \bar{k}_{x'x's} \quad \bar{k}_{Tx's} \quad \bar{\varphi}_{x's} \quad \bar{\varphi}_{y's}\}^T \tag{3.16}$$

The spatial stress vector $\{\sigma'_s\}$, energy conjugate to the spatial strain vector $\{\varepsilon'_s\}$, is obtained from the constitutive relation

$$\{\sigma'_s\} = [D'_s]\{\varepsilon'_s\} \tag{3.17}$$

Where the components of the spatial stress vector, are given by

$$\{\sigma'_s\} = \lfloor \sigma_{x'x's} \quad \tau_{x'y's} \quad \tau_{x'z's} \quad \tau_{y'z's} \rfloor^T$$

The explicit form of the constitutive matrix $[D'_s]$, in Eq. (3.17), will depend upon the orientation of the individual plies in the stiffener, for which it is written

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \\ \tau_{xy} \\ \tau_{yz} \end{Bmatrix}_k = \begin{bmatrix} Q_{xx} & 0 & Q_{xs} & 0 \\ 0 & Q^z_{xx} & 0 & Q^z_{xy} \\ Q_{sx} & 0 & Q_{ss} & 0 \\ 0 & Q^z_{yx} & 0 & Q^z_{yy} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \\ \gamma_{xy} \\ \gamma_{yz} \end{Bmatrix}_k$$

3.6.3 Stress-Strain Resultant

$$\begin{Bmatrix} N_{x's} \\ N_{x'y's} \\ Q_{x's} \\ Q_{y's} \end{Bmatrix} = \int_{A_s} \begin{bmatrix} Q_{xx} & 0 & Q_{xs} & 0 \\ 0 & Q^z_{xx} & 0 & Q^z_{xy} \\ Q_{sx} & 0 & Q_{xx} & 0 \\ 0 & Q^z_{yx} & 0 & Q^z_{yy} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{x'x's} \\ \gamma_{x'y's} \\ \gamma_{x'z's} \\ \gamma_{y'z's} \end{Bmatrix}_k dz$$

$$\begin{bmatrix} M_{x's} \\ T_{x's} \end{bmatrix} = \int_{A_s} \begin{bmatrix} Q_{xx} & 0 & Q_{xs} & 0 \\ 0 & Q^z_{xx} & 0 & Q^z_{xy} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x'x's} \\ \gamma_{x'y's} \\ \gamma_{x'z's} \\ \gamma_{y'z's} \end{Bmatrix}_k zdz$$

Now after combined both equations, we get

$$\{\bar{\sigma}'_s\} = \begin{bmatrix} N_{x's} \\ N_{x'y's} \\ M_{x's} \\ T_{x's} \\ Q_{x's} \\ Q_{y's} \end{bmatrix} = \int_{A_s} \begin{bmatrix} Q_{xx} & 0 & Q_{xs} & 0 \\ 0 & Q^z_{xx} & 0 & Q^z_{xy} \\ zQ_{xx} & 0 & zQ_{xs} & 0 \\ 0 & zQ^z_{xx} & 0 & zQ^z_{xy} \\ Q_{sx} & 0 & Q_{ss} & 0 \\ 0 & Q^z_{yx} & 0 & Q^z_{yy} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x'x's} \\ \gamma_{x'y's} \\ \gamma_{x'z's} \\ \gamma_{y'z's} \end{Bmatrix}_k dz$$

After arranging the equation and putting the value of $\{\varepsilon_s'\} = [H_s]\{\bar{\varepsilon}_s\}$, we get

$$\{\bar{\sigma}_s'\} = \left[\int_{A_s}^{A_s} [H_s]^T [D_s'][H_s] dA_s\right]\{\bar{\varepsilon}_s'\} = [\bar{D}_s']\{\bar{\varepsilon}_s'\}$$

Thus the mechanical strain vector at any point, is expressed by

GOVERNING EQUATIONS OF MOTION

Now put the value of $\delta U_P and \delta U_S$, in equation of Hamilton principle, we find

$$\int_{t_1}^{t_2} \left[\{\delta d\}^T \left\{ ([M_P] + [M_S])\{\ddot{d}\} + ([K_P] + [K_S])\{d\} + \{F\} \right\} \right] dt = 0$$

$$\{[M]\{\ddot{d}\} - [K]\{d\} + \{F\}\} = \{0\}$$

In above governing equation to be solved for a general dynamic problem.
For the special cases of static and free vibration this takes the familiar form

$[K]\{d\} = \{F\}$ For static problem.

$\{[M]\{\ddot{d}\} - [K]\{d\}\} = \{0\}$ For free vibration problem.

RESULTS AND DISCUSSIONS

STATIC ANALYSIS: BARE PLATES

In this section a CNTRC square plate subjected to a uniformly distributed load has been used. There are four types of distribution in the plate first is UD and other three functionally graded but the stiffener have only UD distribution which is attached with these four types of distribution plate. The results are obtained in terms of non-dimensional central deflection $w^* = -w_o / h$.
Where w_o = central deflection of plate, h= thickness of plate.
We find out the non-dimensional central deflection by FORTRAN

Table 4.1: Non-dimensional central deflection ($x10^{-3}$) of CNTRC square plates (CCCC) under uniformly distributed load. When Volume fraction of CNT=0.11 & b/h=10, 20

V*cnt	b/h		SSSS		Percentage
			Present	P. Zhu	Difference (%)
0.11	10	UD	2.125	2.228	4.66
		FG-V	2.245	2.351	4.50
		FG-O	2.365	2.512	4.85
		FG-X	1.995	2.109	4.65
	20	UD	1.115	1.339	3.93
		FG-V	1.456	1.593	4.73
		FG-O	1.754	1.860	5.69
		FG-X	1.129	1.150	1.82

Table 4.2: Non-dimensional central deflection ($x10^{-3}$) of CNTRC square plates (SSSS) under uniformly distributed load. When Volume of CNT=0.11 & b/h=10, 20

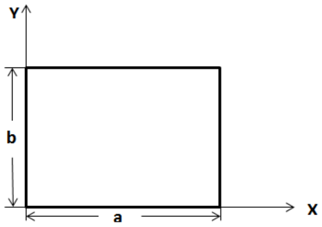
V*cnt	b/h		SSSS		Percentage
			Present	P. Zhu	Difference (%)
0.11	10	UD	3.635	3.739	2.78
		FG-V	4.256	4.466	4.70

	FG-O	5.123	5.230	2.04
	FG-X	3.089	3.177	2.76
20	UD	3.458	3.628	2.65
	FG-V	4.756	4.879	2.49
	FG-O	6.012	6.155	2.32
	FG-X	2.669	2.701	1.18

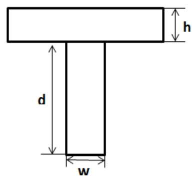
4.3 STATIC ANALYSIS: STIFFENED PLATE

In this section a CNTRC stiffened square plate, attached with stiffener subjected to a uniformly distributed load has been used. There are four types of distribution in the plate first is UD and other three functionally graded but the stiffener have only UD distribution which is attached with these four types of distribution plate. The stiffener distribution is assumed to be vertically oriented in all the cases.

Fig 4.3: Geometry of CNTRC Plate with Stiffener.The mechanical material properties of the plate and the stiffener are considered same, as shown below



(a) Plate



(b) Plate with stiffener

ν^m =0.34 and E^m =2.1GPa.

$$E_{11}^{CNT} = 5.6466 \times 10^3 GPa, E_{22}^{CNT} = 7.08 \times 10^3 GPa, G_{12}^{CNT} = 1.9445 \times 10^3 GPa, \nu_{12}^{CNT} = 0.175$$

In all the cases stiffener have only uniform distribution with all types of distribution plate.

Table 4.3: Non-dimensional central deflection (x10^{-3}) of CNTRC stiffened square plate having one stiffener in y direction under uniformly distributed load.

V*cnt		SSSF	CCCF	CFCF	CFFC
0.11	UD	8.532	8.662	8.652	19.075
	FG-V	8.463	8.562	8.469	19.836
	FG-O	8.536	8.425	8.596	20.143
	FG-X	8.685	8.375	8.635	17.862
0.14	UD	8.658	5.012	8.665	18.423
	FG-V	8.745	5.589	8.514	18.569
	FG-O	8.685	5.895	8.554	19.536
	FG-X	8.569	5.256	8.632	17.125

4.4 STATIC ANALYSIS: ISOTROPIC PLATE

A symmetric square plate simply supported along all its edges, and attached with stiffener placed along the centreline parallel to the X-axis, subjected to a uniformly distributed load. The stiffeners and plate have the same material properties with passion ratio 0.3 in all the case .The elastic modulus for both beam and plate is 17E6 psi and plate is subject to a uniformly distributed load of 1.0 psi.

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Table 4.4: Central deflection (in inches $x10^3$) of stiffened square plate having one Stiffener in y direction under uniformly distributed load.

Plate Thickness (inch)	Stiffener cross-section	SSSS	
		Eccentric	Concentric
.01	.1x.01	.145	.459
.02	.1x.01	.088	.201
.03	.1x.01	.049	.079
.04	.1x.01	.027	.036
.05	.1x.01	.016	.025
.06	.1x.01	.011	.016
.07	.1x.01	.006	.008
.08	.1x.01	.003	.004

Table 4.5: Variation of central deflection (in inches $x10^3$) with varying geometric configuration, keeping the total mass of stiffened square plate unchanged.

Plate Thickness (inch)	Stiffener cross-section	SSSS	
		Eccentric	Concentric
.0923	.090x.006	1.040	1.204
.0876	.110x.012	4.470	4.951
.0753	.214x.024	3.562	4.820
.0624	.316x.044	2 .153	2.856
.0556	.423x.076	2.073	2.256
.0434	.578x.086	1.663	1.985
.0389	.625x.093	1.112	1.356
.0236	.825x.098	0.223	0.246

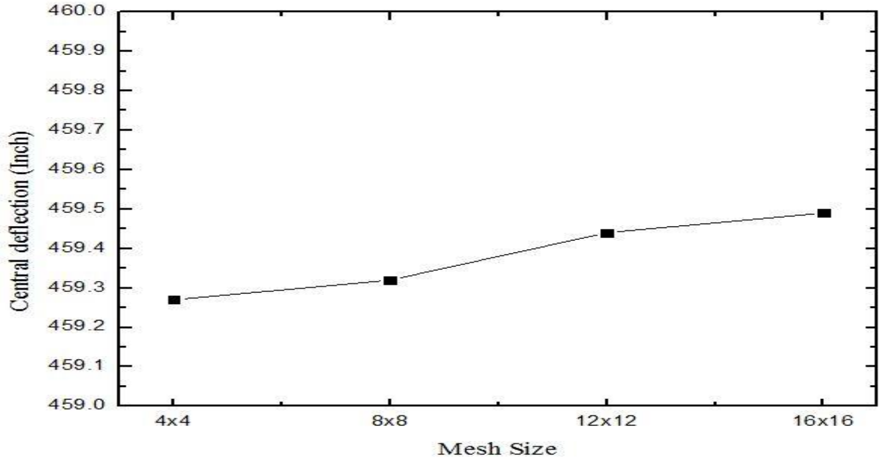

Fig.4.4. Convergence of central deflection of simply supported square plate with one stiffener.

Table 4.6: Central deflection (in inches $x10^3$) of stiffened square plate having one stiffener in y direction under uniformly distributed load.

Plate Thickness (inch)	Stiffener	CCCC	
		Eccentric	Concentric
.01	.1x.01	.032	.086

.02	.1x.01	.021	.048
.03	.1x.01	.018	.022
.04	.1x.01	.012	.018
.05	.1x.01	.008	.012
.06	.1x.01	.005	.010
.07	.1x.01	.002	.005
.08	.1x.01	.001	.002

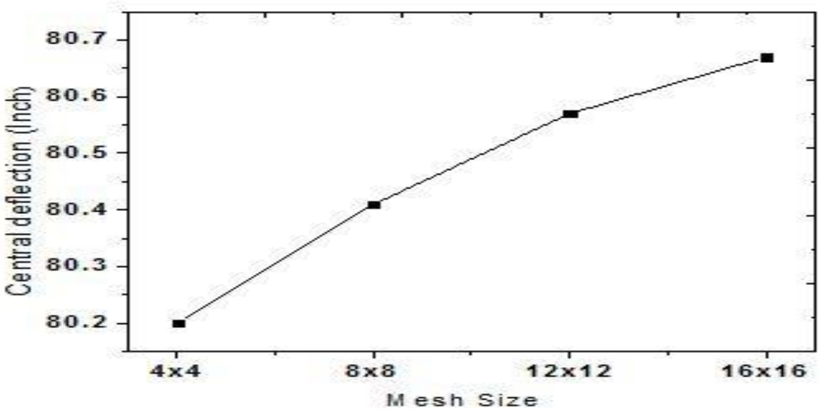

Fig.4.5. **Convergence of central deflection of clamped square plate with one stiffener**.

Table 4.7: Comparison of central deflection (in inches x10^3) for concentric stiffened plate with single stiffener.

Source	**SSSS**	
	Eccentric	**Error (%)**
NASTRAN	.149	2.68
Constraint Method	.136	6.61
Present	.145	

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