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A PRECISE STUDY OF STENOSIS IN THE PRESENCE OF RELEVANT FACTORS BY USING MAGNETIC EFFECT

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Abstract
The paper likely introduces a mathematical model to represent the behavior of blood flow through a vessel with a bell-shaped stenosis (narrowing). The blood is assumed to be homogeneous, incompressible, and a non-Newtonian fluid as Homogeneous implies uniform composition, Incompressible implies that the density of the fluid remains constant and Non-Newtonian fluid suggests that the blood does not follow Newton's law of viscosity. The model considers the radial velocity to be very small. Radial velocity refers to the velocity of fluid particles in the radial direction (perpendicular to the central axis). Various parameters are considered in the model, and their variations are likely explored to understand their impact on the results. These parameters could include dimensions of the stenosis, fluid properties, and other relevant factors. The mention of obtaining precise results suggests that the study aims for accuracy and reliability in its findings.
Keywords: Power law fluid model, shear stress, pressure drop, stenosis height.

Introduction
Stenosis refers to the narrowing of the arteries, which can impede the normal flow of blood. This condition is common in humans and is associated with various chronic diseases. The paper recognizes the significance of hemodynamic factors in the formation and progression of arterial stenosis. Hemodynamics refers to the study of blood flow and its related mechanical aspects. Chronic arterial disease involves thickening, hardening, and loss of elasticity in arterial walls, leading to impaired blood circulation. Stenosis is attributed to the accumulation of calcium deposits in the artery lumen. The factors contributing to this buildup include aging, hypertension, diabetes, hyperlipidemia, and other diseased conditions affecting the arteries. The paper is part of a body of research that employs mathematical models to understand blood flow in stenosed arteries. Some of the referenced studies include those by D.S. Sankar and Ahmad Izani Md. Ismail in 2009, Pankaj Mathur and Surekha Jain in 2013, and Sapna Ratan Shah, Rohit Kumar, and Anamika in 2017. D.S. Sankar and Ahmad Izani Md. Ismail (2009) Compared the two-fluid Casson model and the two-fluid Herschel-Bulkley model and Found that in the Casson model, pressure drop, plug core radius, wall shear stress, and resistance to flow were significantly lower compared to the Herschel-Bulkley model. Sahu et al. (2010) Observed that resistive impedance increases with increasing tube radius for a constant stenosis height and Concluded that resistive impedance and wall shear stress increase for a particular stenosis height. Khan et al. (2010) Indicated that wall shear stress increases with the increase of stenosis height and Noted that wall shear stress increases with the increase of length stenosis when other parameters are held constant. Gaurav Varshney, V.K. Katiyar, Sushil Kumar (2010) explored the influence of a magnetic field on various fluid parameters in the presence of multiple stenosis in arteries and Investigated the impact of the magnetic field on blood velocity, flow rate, wall shear stress, flow resistance, and flow acceleration. They also found that all these flow characteristics were affected by both the applied magnetic field and the presence of multiple stenosis. Laskar et al. (2011) concluded that blood flow characteristics are influenced by non-Newtonian rheology and observed changes in flow patterns and an increase in shear stress at the wall when compared with a Newtonian model. They found that non-Newtonian rheology refers to the behavior of fluids that do not follow Newton's law of viscosity, which is a common feature of blood.

Saktipada Nanda and Ratan Kumar Bose (2012) observed, through theoretical investigation, that the flow resistance at the stenosed portion of the artery increases marginally with the increase of the average flow velocity and suggests a relationship between flow resistance and average flow velocity in stenosed arteries. Pankaj Mathur and Surekha Jain, 2013 developed a mathematical model for studying the non-Newtonian flow of blood through a stenosed arterial segment and concluded that pressure drop and shear stress increase as the size of stenosis increases for a given non-Newtonian model of blood and emphasizes the impact of stenosis size on pressure and shear stress in the context of non-Newtonian blood flow. Alimohamadi et al. (2014) demonstrated that changes in temperature and pressure reduction are significant and related to the value of magnetic field intensity and suggest that the magnetic field intensity plays a role in influencing temperature and pressure changes in blood flow.

Literature

Noreen Sher Akbar (2015) observed that resistive impedance in a diverging tapering appears to be smaller than that in converging tapering along explained that this difference is due to the higher flow rate in diverging tapering compared to converging tapering, leading to lower resistive impedance and noted that impedance resistance attains its maximum values in symmetric stenosis. Nehad Ali Shah and A. Al-Zubaidi (2021) focus on the influence of pulsatile pressure gradient and a transverse magnetic field on unsteady blood flow through an inclined tapered cylindrical tube with a porous medium and The fractional calculus technique is employed to develop a mathematical model of blood flow, incorporating fractional derivatives in the governing equations. Mallinath Dhange (2022) observed the primary purpose of the study is to investigate blood flow through an inclined pipe with stenosis and subsequent expansion and the investigation is conducted under the influence of a constant incompressible Casson fluid and a magnetic field. Wafa F. Alfwzan (2023) described it as long, hollow, and thin silicone tubes that are inserted into various parts of the body, such as the bladder, veins, and arteries and the study is to investigate entropy formation in blood flow through a non-concentric catheterized artery with stenosis.

2.1 Development of the Model

The radius of the artery depends upon the geometry of the stenosis and can be written as follows

$$R_o = R\left\{1 - \frac{\sigma}{R} e^{-\alpha z^2}\right\} \quad (1)$$

Where $\alpha = \frac{M^2\beta^2}{{R_o}^2}$

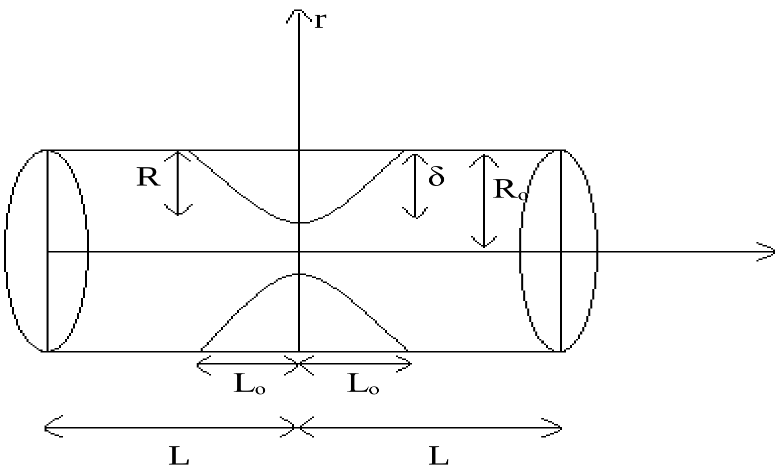

Figure 1: Geometry of a bell-shaped stenosis in the artery

And δ is the height of the stenosis assumed to be much smaller in comparison to the unobstructed radius R of the artery ($\delta \ll R_o$). R_o is the radius of an artery in the stenosed region at the axial distance z. M is a parametric constant, β is the relative length of the constriction, defined as the ratio of the radius to the half length of the stenosis, i.e.,

$$\beta = \frac{R}{L_o}$$

$$H_a{}^2 = \beta_o{}^2 R_o{}^2 \frac{\sigma}{\mu}$$

$$\beta_o{}^2 = \frac{H_a{}^2}{R_o{}^2\left(\frac{\sigma}{\mu}\right)}$$

$$\beta_o = \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}$$

Where β_o is the magnetic effect

Now let us consider the laminar and steady flow of the fluid. When the inertial and entrance effects are neglected, the one-dimensional flow equation is given by

$$0 = \frac{-dp}{dz} + \frac{\mu}{r}\frac{d}{dr}\left\{r\left(\frac{-dw}{dr}\right)^n\right\} + \beta_o \qquad (2)$$

Where w is the axial velocity, p is the fluid pressure. The boundary conditions associated with equation (2) are given as follows:

$$\frac{w}{dr} = 0 \text{ at } r = 0 \qquad 3(a)$$

$$w = 0 \text{ at } r = R_o \qquad 3(b)$$

2.2 Method of Solution

Following Shukla et. al. (1979) and solving equation (2) and using equation (3) we get

$$w = \frac{n}{n+1}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}}\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \qquad (4)$$

Also in case of no stenosis $\delta = 0$

$$w' = \frac{n}{n+1}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}}\left(R_o{}^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \qquad (5)$$

From the above two results we have

$$\bar{w} = \frac{w}{w'} = \frac{R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}}{R_o{}^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}} \qquad (6)$$

The constant flux Q is given by

$$Q = \int_0^R 2\pi r w dr$$

$$Q = \pi\int_0^R r^2\left(\frac{-dw}{dr}\right)dr \qquad (7)$$

From equation (2) $\frac{dp}{dz} = \frac{\mu}{r}\frac{d}{dr}\left\{r\left(\frac{-dw}{dr}\right)^n\right\} + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}$

Integrating both sides w.r.t r

$$\int \frac{r}{\mu}\frac{dp}{dz}dr = \int \frac{d}{dr}\left\{r\left(\frac{-dw}{dr}\right)^n\right\} + \int \frac{H_a\, r}{R_o\sqrt{\mu\sigma}}dr$$

$$\frac{r}{2\mu}\frac{dp}{dz} = \left(\frac{-dw}{dr}\right)^n + \frac{H_a r}{2R_o\sqrt{\mu\sigma}}$$

$$\left(\frac{-dw}{dr}\right) = \left\{\frac{r}{2}\left[\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_o\sqrt{\mu\sigma}}\right]\right\}^{\frac{1}{n}} \qquad (8)$$

Using equation (8) in (7)

$$Q = \pi\int_0^R r^2\left(\frac{-dw}{dr}\right)dr$$

$$Q = \pi\left[\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_o\sqrt{\mu\sigma}}\right]^{\frac{1}{n}} \int_0^R \frac{1}{2^n} r^{\left(2+\frac{1}{n}\right)} dr$$

$$Q = \pi\left[\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_o\sqrt{\mu\sigma}}\right]^{\frac{1}{n}} \int_0^R \frac{1}{2^n} r^{\left(\frac{2n+1}{n}\right)} dr$$

$$Q = \frac{\pi}{2^n}\left[\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_o\sqrt{\mu\sigma}}\right]^{\frac{1}{n}} \left(\frac{r^{\frac{3n+1}{n}}}{\frac{3n+1}{n}}\right)_0^R$$

$$Q = \frac{\pi n R^{\frac{3n+1}{n}}}{(3n+1)2^n}\left[\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_o\sqrt{\mu\sigma}}\right]^{\frac{1}{n}} \qquad (9)$$

$$\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_o\sqrt{\mu\sigma}} = \left\{\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right\}^n$$

$$\frac{dp}{dz} = \mu\left\{\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right\}^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}} \qquad (10)$$

$$dp = \left\{\mu\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\right\} dz$$

Integrating equation (9) along with the condition $p = p_o$ at $z = -L$ and $p = p_L$ at $z = L$

$$\int_{p_0}^{p_L} dp = \left\{\mu\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\right\}\int_{-L}^{L} dz$$

$$p_L - P_0 = \left\{\mu\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\right\} 2L \qquad (11)$$

The shearing stress at the wall is given by

$$\tau = \mu(r)\left(\frac{-dw}{dr}\right)^n$$

$$\tau = \mu\left[\frac{r}{2}\left(\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_o\sqrt{\mu\sigma}}\right)\right] \qquad \text{[Using equation (8)]}$$

$$\tau = \frac{r}{2}\left\{\mu\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\right\} - \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}$$

$$\tau = \frac{r\mu}{2}\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\left(\frac{r}{2} - 1\right) \qquad (12)$$

2.3 Discussion and Results

The behavior of the flow rate with the help of equation (9) is shown in graph (1) to (3). Graph (1) indicates the flow rate decreases with increasing pressure gradient and Hematocrit factor. Graph (2) and (3) show that the blood flow rate also decreases with increasing value of pressure gradient, fluid index as well as viscosity of blood. The behavior for the pressure gradient given by equation (10) is plotted in graph (4) to (6). Graph (4) shows that the pressure gradient increases with increasing flow rate and fluid index. Similarly, graphs (5) and (6) show that the pressure gradient also increases with increasing flow rate and Hematocrit factor as well as the viscosity of blood. The expression for shearing stress is found by equation (12) which is plotted in graph (7) to (9). Graph (7) indicates that the shearing stress increases with increasing flow rate and fluid index. Similarly graph (8) and (9) shows that the shearing stress also increases with increasing flow rate and Hematocrit as well as the viscosity of blood.

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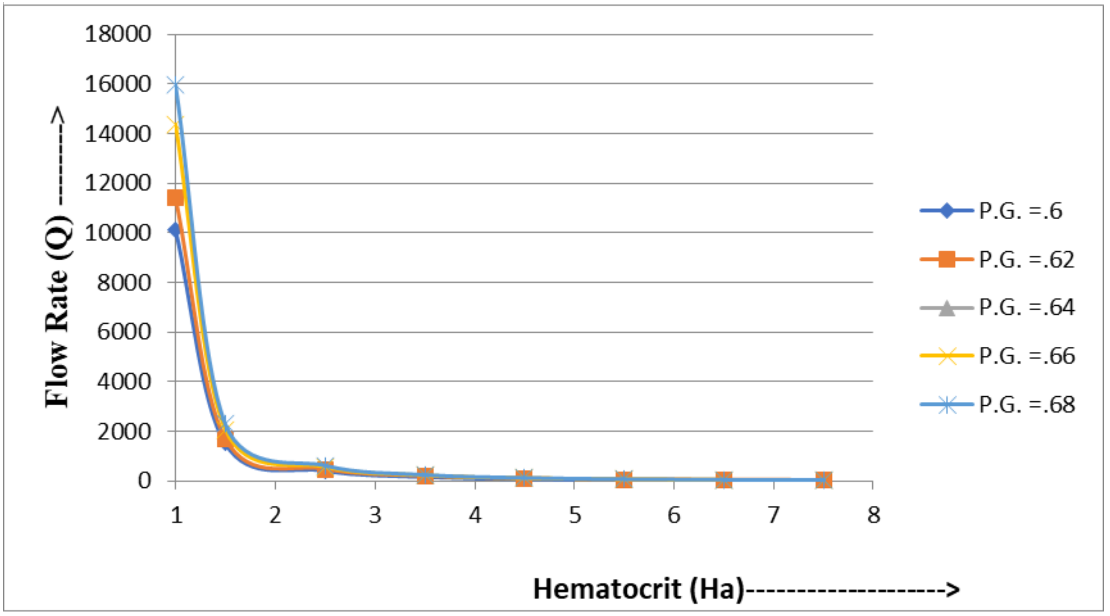
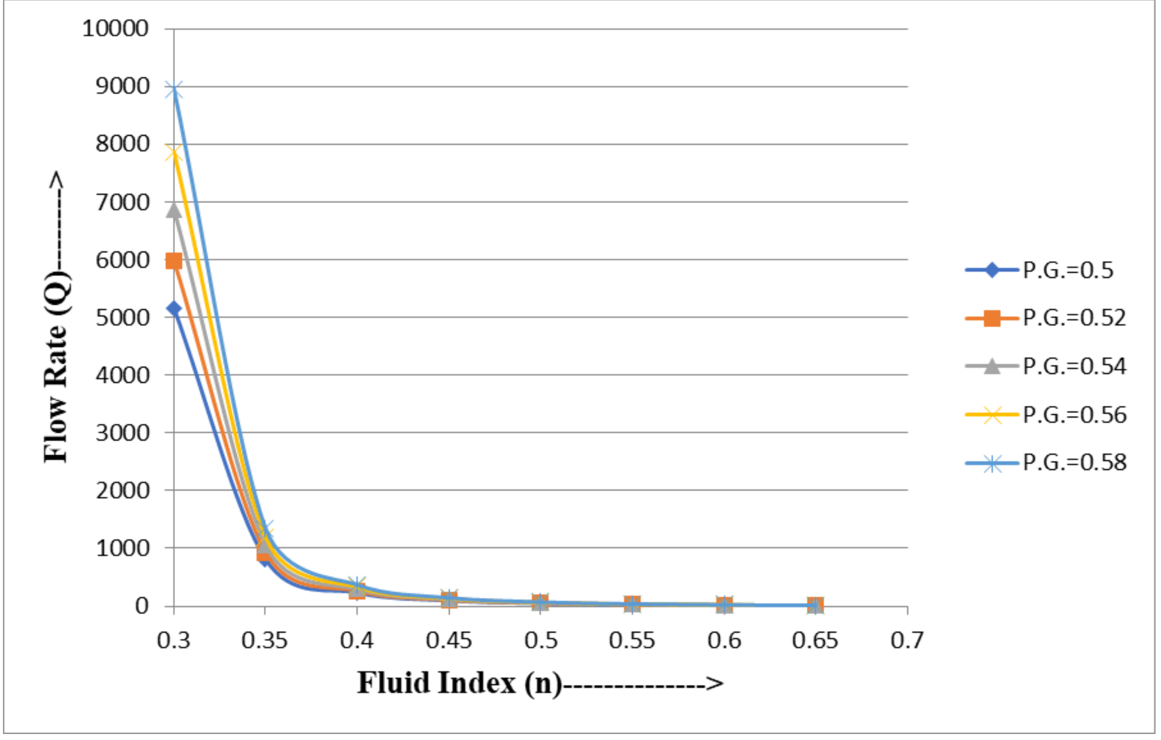
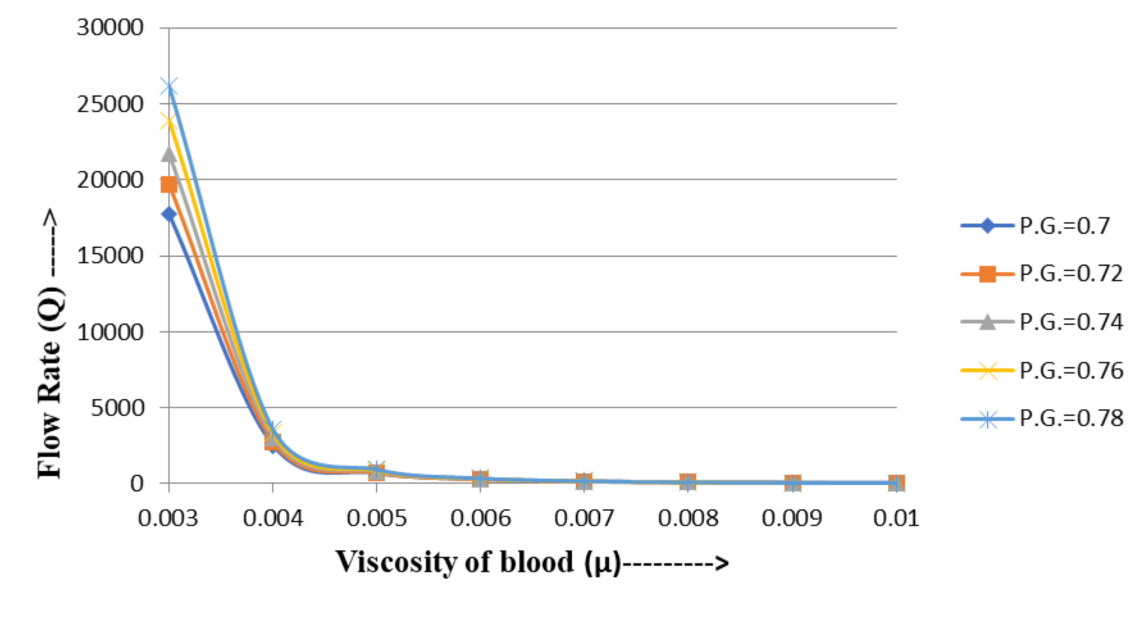


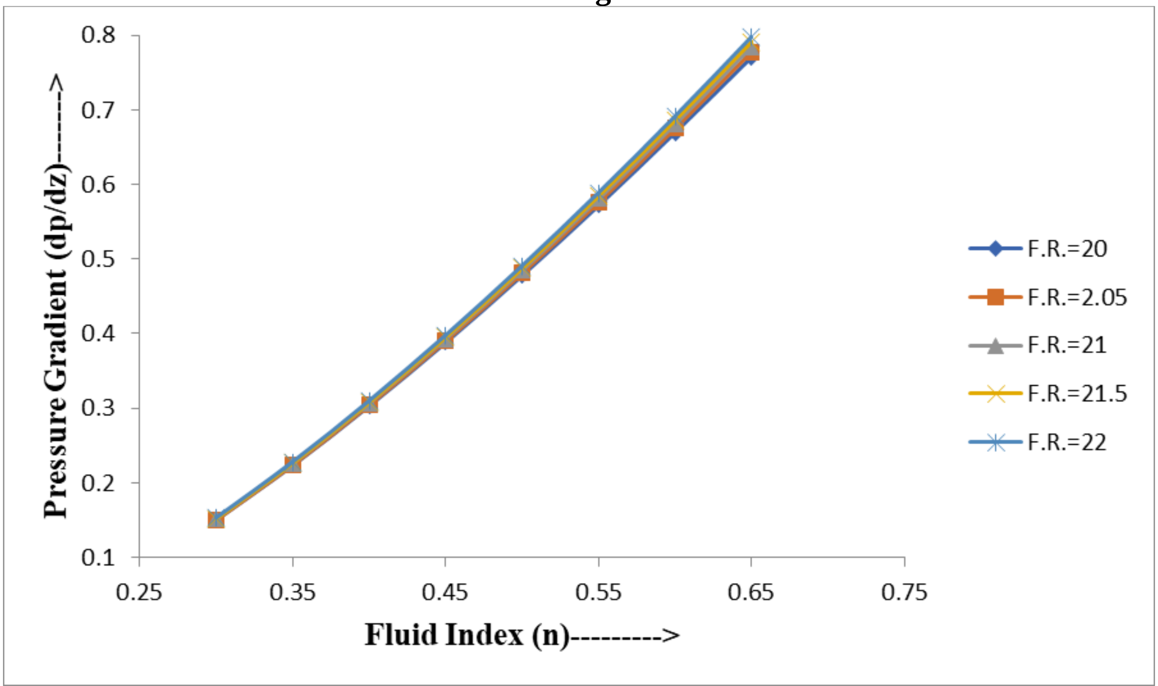
Graph 1: Variation of blood flow rate with distinct Hematocrit (Ha) and pressure gradient



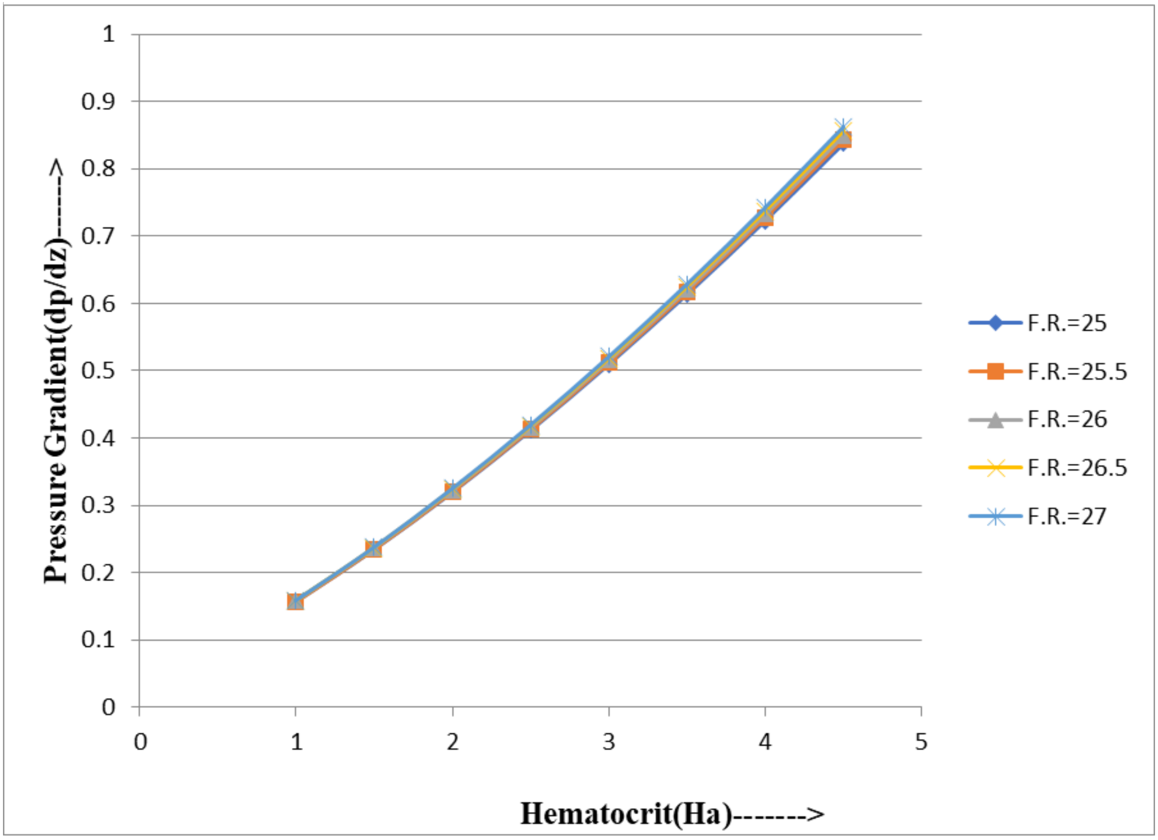
Graph 2: Variations of blood flow rate with distinct fluid index (n) and pressure gradient



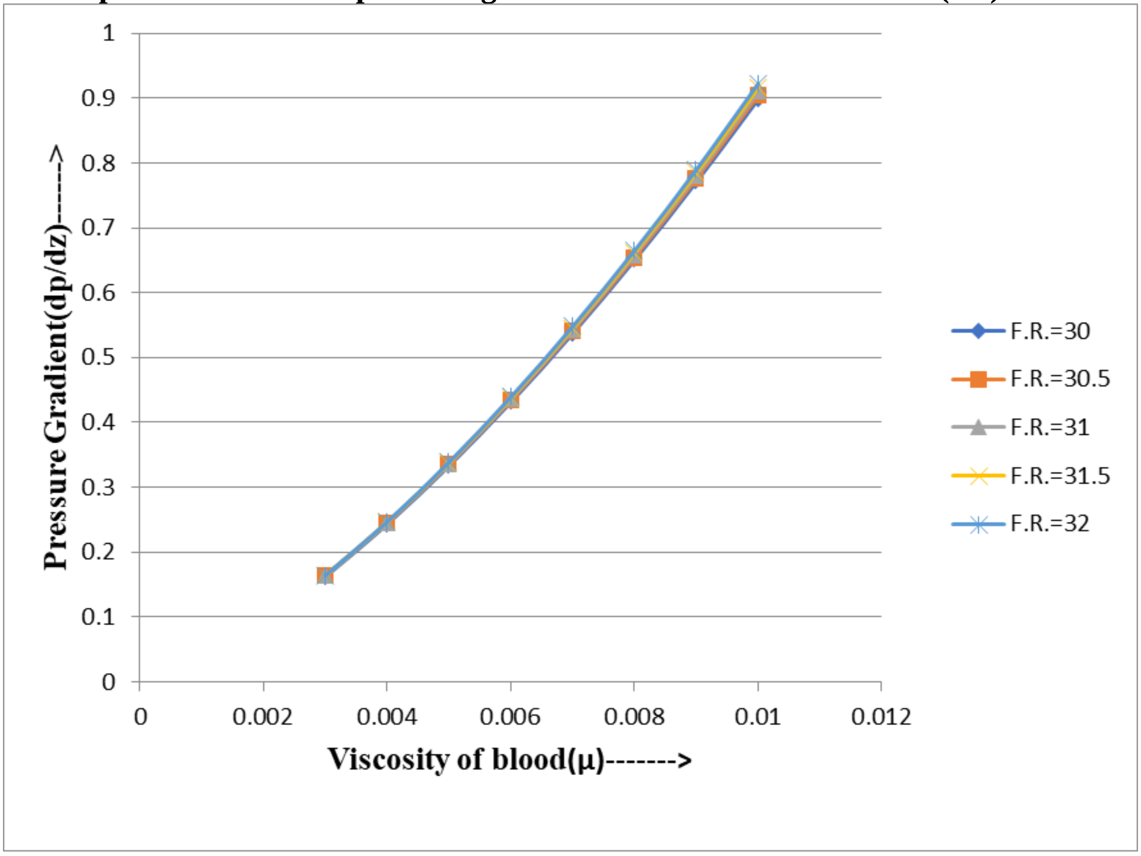
Graph 3: Variations of blood flow rate in terms of distinct viscosity of blood (μ) and pressure gradient



Graph 4: Variations of pressure gradient with distinct fluid index (n) and flow rate



Graph 5:Variations of pressure gradient with distinct Hematocrit(Ha) and flow rate



Graph 6:Variations of pressure gradient in terms of viscosity of blood (μ) and flow rate

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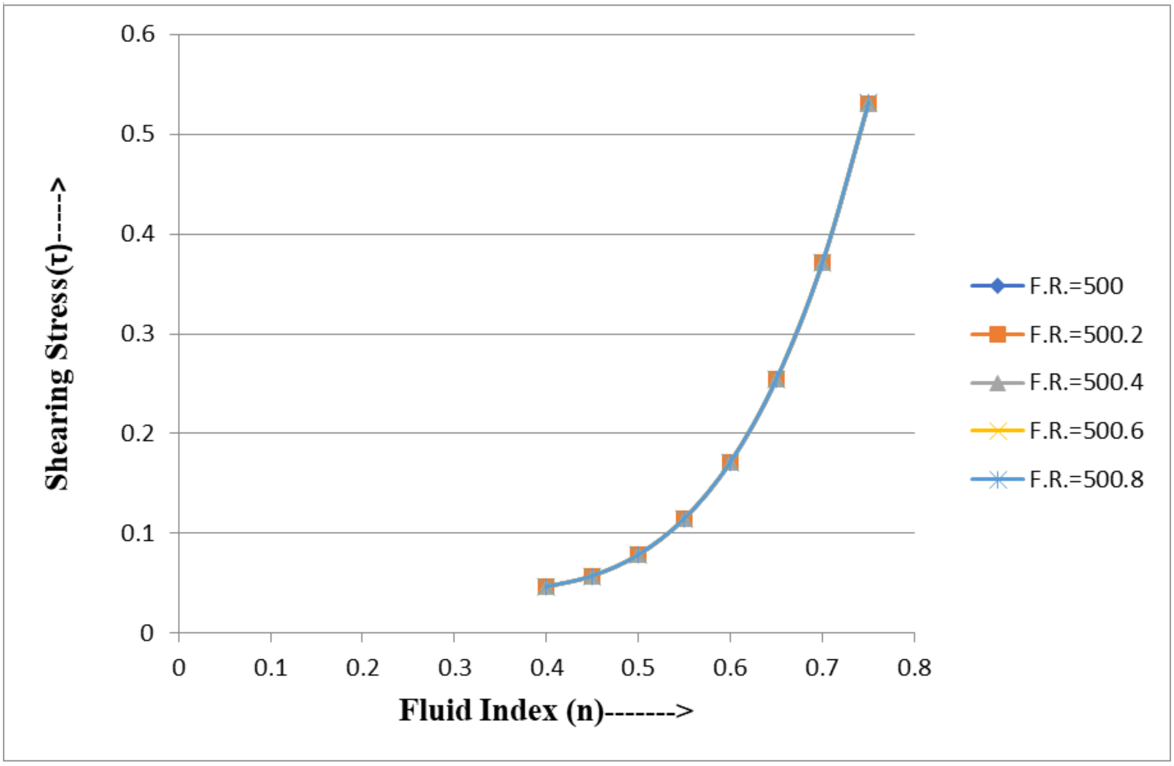
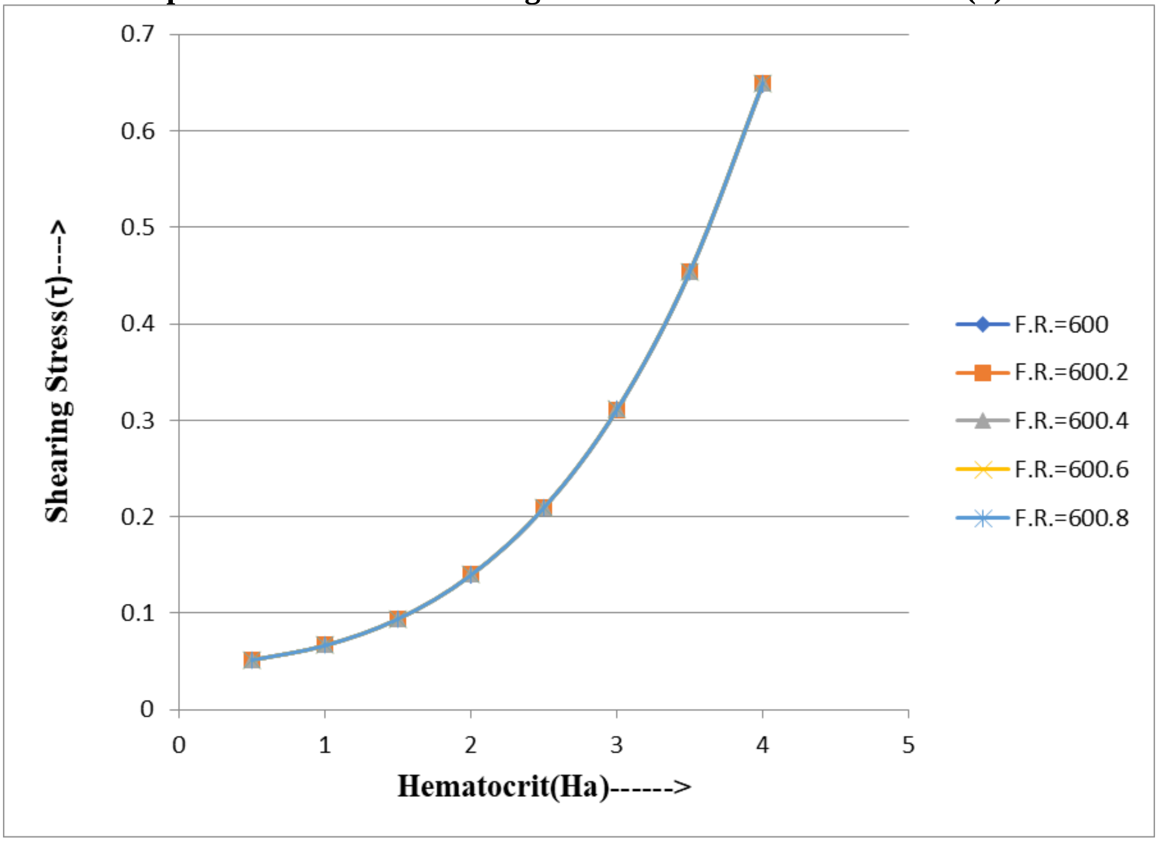
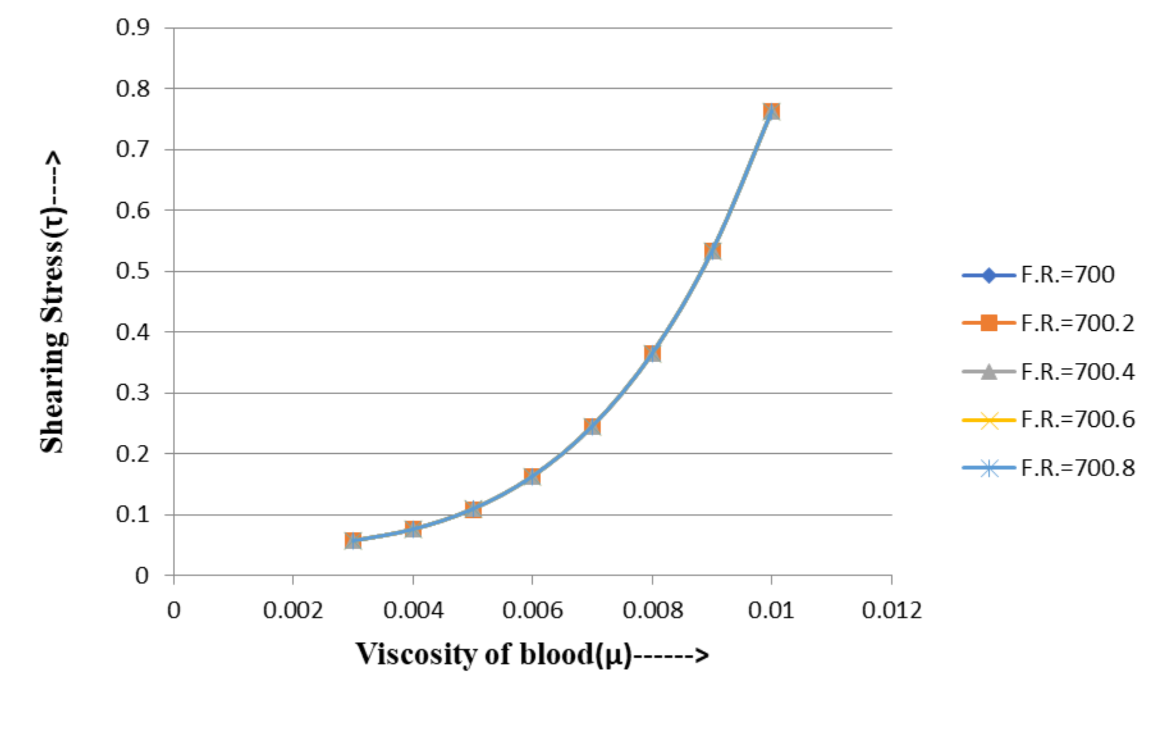


Graph 7:Variations of shearing stress with distinct fluid index (n) and flow rate



Graph 8:Variations of shearing stress with distinct Hematocrit(Ha) and flow rate



Graph 9:Variations of shearing stress in terms of viscosity of blood(μ) and flow rate

Conclusion

The paper explores how stenosis affects blood flow in an artery, specifically considering blood as a power law non-Newtonian fluid. The findings suggest that increasing the fluid index (n) leads to a decrease in flow rate while increasing pressure gradient and shearing stress. These results could have implications for understanding the biomechanics of blood flow in arteries with stenosis.

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