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THE STUDY OF NANOFLUID OVER A ROTATING CONE IN THE PRESENCE OF THERMAL RADIATION

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Abstract

In this paper the effects of Prandtl number and radiation parameter on the flow of nanofluid over a rotating cone in the presence of thermal radiation is investigated. Here we are using nanofluid as heat transfer fluid which finds application in industries. The governing nonlinear equations are solved analytically by using homotopy analysis method. Results are plotted to find the effect of radiation parameter and Prandtl number.

Key words: Nanofluid, rotating cone, thermal radiation, homotopy analysis method, Prandtl number

1. Introduction

Using a nano fluid as a heat transfer working fluid has gained much attention in several engineering and industrial applications due to its potential advantages which include higher drop. A nanofluid is a base fluid with suspended metallic nano-size particles specifically nano particles. The nanofluids are extensively used as coolants, lubricants, heat exchangers and micro-channel heat sinks. The theory of convective heat transfer in nanofluids takes an excessive concern for investigators due to higher thermal properties.

The flows over rotating bodies occurs in turbines and turbo machines, in gaseous and nuclear reactors, in approximating the conduit of flight of spinning wheels in the modeling of several geophysical vortices. It is practically seen that whenever some rotating flow collaborates through a surface, a three dimensional complex flow exists which appear in both outward and interior flows. Also the probability of cooling the nose-cone of re-entry vehicles by rotating the nose is an important application (S. Ostrach and W. H Brown [1]).. Dorfman, L.A [3] and Kreith F [4] initially investigated the flow and heat transfer in rotating systems. J.P. Hartnett and E.C. Deland. [5] examined the effects of Prandtl number on the heat transfer by rotating bodies. R G Hering and R J Grosh [6] studied the mixed convection by a rotational non-isothermal cone at low Prandtl number. Vira, N.R. and Fan, D.N [7] have examined the flow and heat transfer on a rotating cone in a rotating fluid. An approximate method of solution for the heat transfer from vertical cones in laminar natural convection was examined by M. Alamgir [8]. M. C Ece [9] studied the time dependent boundary layer flow on an impulsively started translating and spinning rotational symmetric body and obtained analytical and numerical solutions for leading and first order initial value problems. Roy S, Anilkumar D [10] studied the unsteady mixed convection from a rotating cone in a rotating fluid with thermal and mass diffusion. Wael Abbas Emad A., Sayed [11] investigated the effects of Hall current and Joule heating on flow and heat transfer of a nanofluid along a vertical cone in the presence of thermal radiation by subjecting to a uniform strong transverse magnetic field normal to the cone surface. It was found that,

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nanoparticle volume fraction parameter and types of nanofluid play an important role to significantly determine the flow behavior. Vijayalakshmi A R (12) considered unsteady gravity driven convective flow and heat transfer of optically thick nanofluid past an oscillating vertical plate in the presence of magnetic field using laplace transform and have obtained the velocity and temperature profiles for different parameters. Shahzad Ahmad and etal [13] studied the heat and mass transfer in two-dimensional magnetohydrodynamic nanofluid flow over a cone/plate using similarity transformation. They found that compared to the rotating plate, heat and mass transfer properties of the flow over the rotating cone are more evident. K. Padmaja and B. Rushi Kumar [14] investigated numerically the nanofluid flow about a permeable, vertical, rotating cone with Dufour and Soret effects in the presence of thermal radiation, magnetic field, and chemical reaction. Graphically it was found that the Dufour and Soret numbers and the thermal radiation parameter have a significant impact on the rates of heat and mass transfer. Fuzhang Wang and etal [15] studied the impact of nanopaprticles aggregation and thermal radiation by modelling equations for the flow pattern. The modelling equations are converted into ordinary differential equations using similarity transformations. It was found that the dynamics of flow with nanoparticles aggregation case shows better quality heat transport for increased values of radiation parameter.

In this paper we are investigating the effects of Prandtl number and radiation parameter on the flow of nanofluid over a rotating cone using homotopy analysis method.

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2. Analysis of the Flow

We have considered the unsteady incompressible fluid flow over a rotating cone in a viscous fluid with nanoparticles. The cone is rotating with time-dependent angular velocity about the axis of symmetry. The non-rotating curvilinear coordinate system is given in fig 1. It is assumed that u, v and w are the velocity components in tangential, azimuthal and normal directions, respectively. The wall temperature T_W and wall nano particle volume fraction C_W are linear functions of distance x and the free stream temperature T_∞ and nano particle volume fraction C_∞ are constant. The time dependent angular velocity Ω_0 of the cone causes the unsteadiness in the flow field. The boundary layer equations for flow, heat and mass transfer over a rotating cone in a nanofluid are

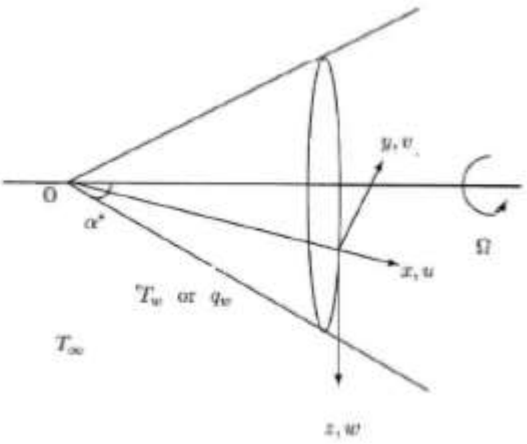

Fig1. Schematic diagram of the physical model and coordinate system

$$\frac{1}{x}\frac{\partial(xu)}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-\frac{v^2}{x}=\nu\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}+\frac{uv}{x}=\nu\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\frac{k_0}{\rho c_f}\frac{\partial^2 T}{\partial z^2}-\frac{1}{\rho c_f}\frac{\partial q_r}{\partial t}+\tau\left[D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right] \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+w\frac{\partial C}{\partial z}=D_B\frac{\partial^2 C}{\partial z^2}+\frac{D_T}{T_\infty}\frac{\partial^2 C}{\partial z^2} \tag{5}$$

The boundary conditions applicable to the problem are

$$u(x,0,t)=w(x,0,t)=0,\ v(x,0,t)=\Omega_0 x\sin\alpha^* R(t^*)$$

$$T(x,0,t)=T_W, C(x,0,t)=C_W, u(x,\infty,t)=0, v(x,\infty,t)=0$$

$$T(x,\infty,t)=T_\infty, C(x,\infty,t)=C_\infty \tag{6}$$

Here ρ is the density, t and $t^*(=\Omega_0\sin\alpha^* t)$ are the dimensional and dimensionless times, respectively, α^* is the semi-vertical angle of the cone, υ is the kinematic viscosity, T is the temperature, C is the species concentration, the subscripts w and ∞ denote the conditions at the wall and the ambient conditions, respectively. κ_0 is the thermal conductivity of the fluid. q_r is the radiative heat flux, τ is the ratio of nano particle heat capacity and the base fluid heat capacity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient, By using the Rosseland approximation $q_r=-\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial z}$ in which σ^* denotes the Stefan–Boltzman constant and κ^* the Rosseland mean absorption coefficient, the fluid phase temperature differences within the flow are sufficiently small so that T^4 may be described as a linear function of temperature.Expanding T^4 in Taylors series about the free-stream temperature T_∞ and neglecting higher order terms, we obtain $T^4\cong 4T_\infty^3 T-3T_\infty^4$. Using these in equation (4) we get

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\left(\frac{k_0}{\rho c_f}+\frac{16\sigma^* T_\infty^3}{3\rho c_f\kappa^*}\right)\frac{\partial^2 T}{\partial z^2}+\tau\left[D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right] \tag{7}$$

Using the following transformations,

$$\eta=\left(\frac{\Omega_0\sin\alpha^*}{\nu}\right)^{1/2} z, t^*=\Omega_0 \sin\alpha^* t,\ u(x,z,t)=-2^{-1}\left(\Omega_0 x\sin\alpha^*\right)R\left(t^*\right)h'\left(\eta,t^*\right),$$

$$v(x,z,t)=\left(\Omega_0 x\sin\alpha^*\right)R\left(t^*\right)g\left(\eta,t^*\right),\ w(x,z,t)=\left(\upsilon\Omega_0 x\sin\alpha^*\right)^{1/2}R\left(t^*\right)h\left(\eta,t^*\right),$$

$$C(x,z,t)-C_\infty=(C_W-C_\infty)\phi\left(\eta,t^*\right), C_W-C_\infty=\left(C_0-C_\infty\right)\left(\tfrac{x}{L}\right), R\left(t^*\right)=1+\varepsilon t^*$$

$$T(x,z,t)-T_\infty=(T_W-T_\infty)\theta\left(\eta,t^*\right), T_W-T_\infty=\left(T_0-T_\infty\right)\left(\tfrac{x}{L}\right) \tag{8}$$

Equations (2)-(5) becomes

$$h'''-R\left(t^*\right)hh''+\frac{1}{2}R\left(t^*\right)(h')^2-2R\left(t^*\right)g^2-\frac{1}{R\left(t^*\right)}\frac{dg}{dt^*}h'-\frac{dh'}{dt^*}=0 \tag{9}$$

$$g''+R\left(t^*\right)(gh'-hg')-\frac{1}{R\left(t^*\right)}\frac{dR}{dt^*}g-\frac{\partial g}{\partial t^*}=0 \tag{10}$$

$$\left(1+\tfrac{4}{3}\kappa\right)\theta''-\Pr\left\{R\left(t^*\right)\left(h\theta'-h'\frac{\theta}{2}\right)-\frac{\partial\theta}{\partial t^*}+Nb\phi'\theta'+Nt\theta'^2\right\}=0 \tag{11}$$

$$\phi''-Le\left\{R\left(t^*\right)+\left(h\phi'-h'\frac{\phi}{2}\right)+\frac{\partial\phi}{\partial t^*}\right\}+\frac{Nt}{Nb}\theta''=0 \tag{12}$$

where

$Nb=\dfrac{(\rho c)_P D_B(C_W-C_\infty)}{\upsilon(\rho c)_f}$ is the Brownian motion parameter, $Le=\dfrac{\upsilon}{D_B}$ is the Lewis number,

$Nt=\dfrac{(\rho c)_P D_T(T_W-T_\infty)}{\upsilon(\rho c)_f T_\infty}$ is the thermophoresis parameter, $\Pr=\dfrac{\upsilon}{\alpha}$ is the Prandtl number.

$\kappa=\dfrac{4\sigma^* T_\infty^3}{\kappa^*\kappa_0}$ is the radiation number,

The boundary conditions take the form

$$\eta\to 0: h\left(\eta,t^*\right)=0=h'\left(\eta,t^*\right),\ g\left(\eta,t^*\right)=1,\ \theta\left(\eta,t^*\right)=\phi\left(\eta,t^*\right)=1 \text{ and}$$

$$\eta\to\infty: h'\left(\eta,t^*\right)=0,\ g\left(\eta,t^*\right)=0, \tag{13}$$

The dimensionless local surface skin friction coefficients in tangential and azimuthal directions, the local Nusselt number and local Sherwood number are given as

$$C_{fx}\operatorname{Re}_x^{\frac{1}{2}}=\frac{2\mu\left(\frac{\partial u}{\partial z}\right)\Big|_{z=0}}{\rho\left(\Omega_0 x\sin\alpha^*\right)^2}=-R\left(t^*\right)h''\left(0,t^*\right) \tag{14}$$

$$2^{-1}C_{fy}\,\mathrm{Re}_x^{\frac{1}{2}} = -\frac{2\mu\left(\frac{\partial v}{\partial z}\right)\Big|_{z=0}}{\rho\left(\Omega_0 x\sin\alpha^*\right)^2} = -R\left(t^*\right)g'\left(0,t^*\right), \tag{15}$$

$\mathrm{Re}_x = \frac{\Omega_0 x^2 \sin\alpha^*}{\upsilon}$ is the local Reynolds number.

$$Nu = \frac{\left[x\left(\frac{\partial T}{\partial z}\right)\right]_{z=0}}{T_w - T_\infty} = \mathrm{Re}_x^{1/2}\,\theta'\left(0,t^*\right), \tag{16}$$

$$Sh = \frac{\left[x\left(\frac{\partial C}{\partial z}\right)\right]_{z=0}}{C_w - C_\infty} = \mathrm{Re}_x^{1/2}\,\phi'\left(0,t^*\right), \tag{17}$$

3. Homotopy analysis

The initial approximations h_0, g_0, θ_0 and ϕ_0 with the respective auxiliary linear operators for the homotopy analysis method solutions are

$$h_0\left(\eta,t^*\right) = 0 \tag{18}$$

$$g_0\left(\eta,t^*\right) = \exp(-\eta) \tag{19}$$

$$\theta_0\left(\eta,t^*\right) = \exp(-\eta) \tag{20}$$

$$\phi_0\left(\eta,t^*\right) = \exp(-\eta) \tag{21}$$

$$l_h = \frac{\partial^3 h\left(\eta,t^*\right)}{\partial\eta^3} - \frac{\partial h\left(\eta,t^*\right)}{\partial\eta} \tag{22}$$

$$l_g = \frac{\partial^2 g\left(\eta,t^*\right)}{\partial\eta^2} + \frac{\partial g\left(\eta,t^*\right)}{\partial\eta} \tag{23}$$

$$l_\theta = \frac{\partial^2 \theta\left(\eta,t^*\right)}{\partial\eta^2} + \frac{\partial \theta\left(\eta,t^*\right)}{\partial\eta} \tag{24}$$

$$l_\phi = \frac{\partial^2 \phi\left(\eta,t^*\right)}{\partial\eta^2} + \frac{\partial \phi\left(\eta,t^*\right)}{\partial\eta} \tag{25}$$

4. Results and Discussions

The governing nonlinear boundary layer partial differential equations (9) − (12) along with the boundary conditions (13) are solved by homotopy analysis method. The graphical results of non-dimensional temperature are computed for different values of Radiation parameter and Prandtl number. Fig. 2 shows the effect of radiation parameter κ on temperature field. It is found that $\theta\left(\eta, t^*\right)$ increases for κ. From fig 3 it is found that the temperature and thermal boundary layer thickness decreases with an increasing values of Prandtl number. Greater Prandtl number fluid (oils), has a lower thermal conductivity which results in thinner thermal boundary layer and therefore the rate of heat transfer rate increases. The analytical results are in acceptable form when compared with the numerical calculation of E.M. Sparrow and R.D.Cess [2]

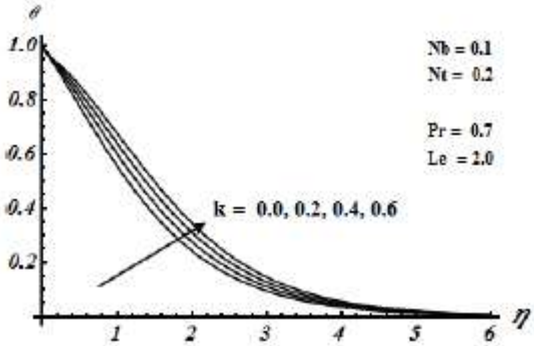

Fig 2. Effects of κ on temperature field $\theta\left(\eta, t^*\right)$ when $R\left(t^*\right) = 1 + \varepsilon t^*, \varepsilon = 0.2$ at t^*=1.

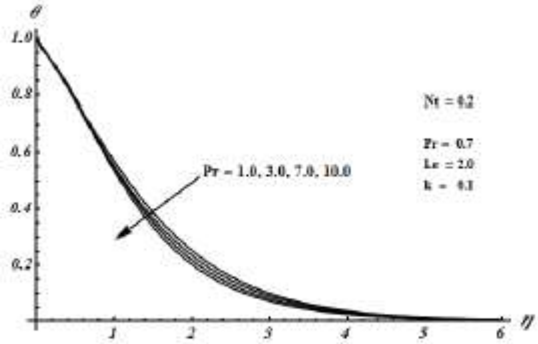

Fig 3. Effects o $R\left(t^*\right) = 1 + \varepsilon t^*, \varepsilon = 0.2$ **at t^*=1.**

5. Conclusions

Graphically it is found that the temperature and thermal boundary layer thickness decreases with an increasing values of Prandtl number. For engineering applications the heat transfer rate should be low which can be obtained by keeping the low temperature difference between the surface and the free stream fluid, using a low Prandtl number fluid (liquid metals), maintaining the surface at a constant temperature instead of at a constant heat flux i,e., room temperature liquid metals (Gallium based liquid metals) can be used as lubricant instead of engine oils which could work only in a narrow temperature range due to their poor thermal stabilities.

6. References

1. S. Ostrach and W. H Brown, Natural Convection Inside a Flat Rotating Container, NACA Technical Note, 4323, (1958).
2. E.M. Sparrow and R.D.Cess , Magnetohydrodynamic flow and heat transfer about a rotating disk, Journal of applied mechanics, 29(1),181-187(1962)
3. L. A. Dorfman, Hydrodynamic resistance and heat loss of rotating solids. Oliver and Boyd. Edinburgh, (1963).
4. Kreith F Convection heat transfer in rotating systems. Adv Heat Tran 5:129–251 (1968).
5. J.P. Hartnett and E.C. Deland, The influence of Prandtl number on the heat transfer from rotating non-isothermal disks and cones, Journal of Heat Transfer, Transactions of the ASME, C83, 95-96 (1961).
6. R. G. Hering and R. J. Grosh, Laminar combined convection from a rotating cone, ASME J. Heat Transfer, 85, 29-34 (1963).
7. N.R. Vira and D.N.Fan, Heat transfer from a cone spinning in a co-rotating fluid, Journal of Heat Transfer,103(4), 815-817(1981).
8. M. Alamgir, Over-All heat transfer from vertical cones in laminar free convection: An approximate method, Transactions of ASME Journal of Heat Transfer,101(1),174-176 (1979).
9. M.C. Ece, The initial boundary-layer flow past a translating and spinning rotational symmetric body, Journal of Engineering Mathematics, 26, 415-428 (1992).
10. Roy S and Anilkumar D, Unsteady mixed convection from a rotating cone in a rotating fluid due to the combined effects of thermal and mass diffusion. Int J Heat Mass Transfer, 47: 1673–1684 (2004).
11. Wael Abbas Emad A., Sayed Hall current and joule heating effects on free convection flow of a nanofluid over a vertical cone in presence of thermal radiation, Thermal Science 2 Science 21(00):83-83(2017).
12. Vijayalakshmi A.R., Study of unsteady gravity-driven convective flow and heat transfer of optically thick nanofluid past an oscillating vertical plate in presence of magnetic field, Elixir Computational physics,128,52927-52931 (2019)
13. Shahzad Ahmad, Kashif Ali, Rabia Saleem, and Hina Bashir, Thermal analysis of nanofluid flow due to rotating cone/plate—A numerical study, AIP Advances 10, 7 (2020).
14. K. Padmaja & B. Rushi Kumar , Dufour and soret effects on MHD flow of Cu-water and Al2O3-water nanofluid flow over a permeable rotating cone , Fuzzy Mathematical Analysis and Advances in Computational Mathematics, 221–236. (2022).
15. Fuzhang Wang, S. Prasanna Rani, Konduru Sarada, R.J. Punith Gowda, Umair khan f,g, Heba Y. Zahran ,Emad E. Mahmoud, The effects of nanoparticle aggregation and radiation on the flow of nanofluid between the gap of a disk and cone, Case studies in thermal engineering,33,101930 (2022).