Issues of Two Bodies in Magnetic Interactions

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Abstract We offer several two-body potentials with oscillatory forms and magnetic interactions in the current study. Using the Nikiforov-Uvarov approach, the eigenvalues and eigenfunctions are discovered for one of those problems.

Keywords The non-relativistic quantum theory, The relativistic two-body equation, The Nikiforov-Uvarov method

1. Introduction

In the present work, we shall be particularly interested in apotential well as the one shown in figure (1).

characterize the quantum numbers of such possible solutionsusing these potentials.

Let us consider, for example, a relativistic charged spinless particle m in a field of a fixed (quantum) magnetic moment μ [11], or alterentivly, a charged spin 1/2 particle of mass m and ... narge [12,13]. In both cases the radial equation has ...
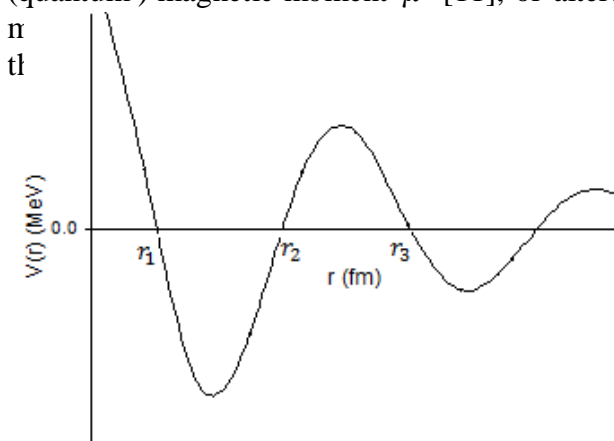
$$\left[-\frac{d^2}{dr^2} + V - \lambda^2\right](r) = 0 \qquad (1)$$

Where the effective potential is given, apart from the Coulomb potential $\frac{\alpha}{r}$, by

$$V(r) = \frac{A}{r^2} + \frac{B}{r^3} + \frac{C}{r^4}$$

Figure (1). The relation between the potential V(r) and the distance r at different distances

Magnetic forces between spin 1/2 particles lead to the effective radial potentials of this type [1-3], with one or more deep narrow wells. Magnetic interactions are studied for various problems ranging from macroscopic to microscopic scales [4-10]. The purpose of our work is to use analytic techniques to study the eigenvalues and eigenfunctions for such cases. Effective potentials between two-fermions taking into account form factors and full relativistic spinor kinematics have similar shapes. The motivation for doing that is as we shall show later on the existence of such potentials in several physical applications. In such cases, one would need to understand better the existence and to

Clearly, if one solves the same problem with a Dirac equation and give also an anomalous magnetic moment a to the particle, then additional terms are added to equation (2).

Further models may also treat the magnetic moment of both particles. The potential in equation (2) is treated in atomic phenomena as well as in the quark models as a perturbation. This is justified only if the energies are of the order of the Coulomb energies. At very short distances, the form of the potentials is quite uncertain, and these potentials must be modified by form factors.

Form factors must then be calculated in a non-perturbative and self-cosistent method from the wave functions which are localized around each well. In the next section, we will sketch some of the physical problems in which case such a form of this potential appears. In section (3) we will give a short description of the relativistic two -body equation which is used to describe two-body systems, and finally, in section (4), we will show a method for solving equation (1) over the whole range of the independent variable.

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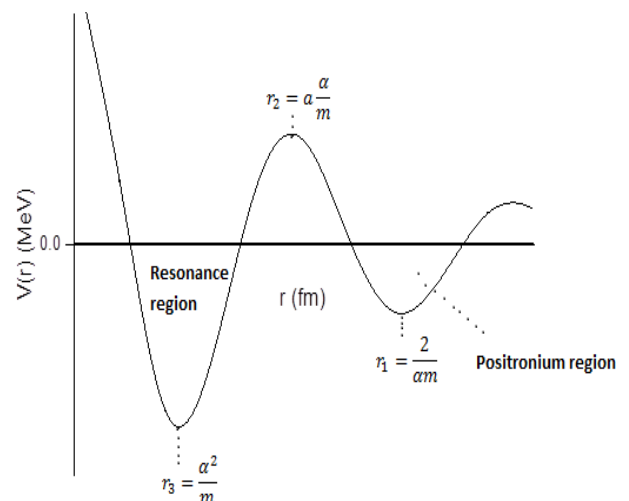
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2. Dynamical Models

Magnetic Interactions in Nonrelativistic Quantum Theory

Consider two charged particles e_1 and e_2 with magntic moments $\overline{\mu_1}$ and $\overline{\mu_2}$ respectivly the Hamiltonain in this case is

$$H = \frac{1}{\;\;\;}$$

$[\bar{P}_1 - e_1$

$$\bar{\mu}_1 \times (\bar{r}_1 - \bar{r}_2) \over |\bar{r}_1 - \bar{r}_2|^3 \Big]^2$$

1

1

$$\underline{\quad}$$

$+ \dfrac{1}{2m}$

2

$[\bar{P}_2 - e_2$

$$\mu_1 \times (\bar{r}_2 - \bar{r}_1) \over |\bar{r}_2 - \bar{r}_1|^3 \Big]^2$$

2   2   1

$$+ \frac{e_1 e_2}{|\bar{r}_2 - \bar{r}_1|} + \bar{\mu}_1$$

1

$$\cdot \bar{\mu}_2 \frac{(3\bar{\sigma}_1 \cdot \bar{r}_1)(3\bar{\sigma}_2 \cdot \bar{r}_2) - \bar{\sigma}_1 \cdot \sigma_2}{r^3}$$

2

$$+ \frac{8\pi}{3} (\bar{\mu}_1 \cdot \bar{\mu}_2)(\bar{\sigma}_1 \cdot \bar{\sigma}_2)(\bar{r}_1 - \bar{r}_2) \quad (3)$$

1   2   1   2   1   2

Or In the center of mass frame

$$H = \frac{1}{\quad} P^2 - \bar{P} \cdot \bar{a}^{\times r} + A + e_1 \cdot e_2 - \mu_1 \cdot \mu_2 S_{12}(\bar{r}) \qquad (4)$$

Figure (2). The trend of the potential V(r) with distance r at maxima and minima for different regions

$\dfrac{2\mu}{r^3}$

$r^4 \quad r$

Where $a = \frac{1}{\mu} \bar{\mu}_1 + \frac{e_2}{\quad} \bar{\mu}_2, \ \bar{P} = \frac{m_2 \bar{P}_1 - m_1 \bar{P}_2}{}$,

2.3. The 4-Component Neutrino with a Magnetic

$m_1 \quad 2$

$m_2 \quad 1$

$m_1 + m_2$

Moment

$e_1^2 \quad 2$

$$A = \frac{1}{m_1} \mu_2$$

$$+ \frac{e_2{}^2}{m_2} \mu_1 \,, \, \mu = \frac{1}{m_1} + \frac{1}{m_2}$$

An interesting limit of the equation in the previous section is when

And $_{12}(r)$ is the so-called tensor and dipole-dipole interaction potential

In some special cases, the problem becomes easier to solve e.g.

$$e \to 0, m \to 0 \text{ such that } \frac{e}{m} = \mu = \text{magntic moment}$$

One gets an equation which is exactly solvable [14,15]; viz,

a) $m_2 \gg m_1 . \mu_1 = 0 \, \sigma$

$$d\,2$$

$$2 \quad \underline{(k+1)}$$

$$\underline{2(k+1)\,1}$$

b) $m_2 = m_1 \; (e \cdot g \; pp, p\bar{p}, e$

$e , e e$
, ...)

$$2 + \lambda - \left[\frac{dy}{y\,2} + \epsilon \right.$$
$$\left. y\,3 + \right.$$
$$y\,4]] = 0 \quad (8)$$

The Dirac Particle with an Anomalous Magnetic

Where $k = \pm(j + {}^1)$

Moment in a Dipole Field

Also for a
2

vv system, we have the Hamiltionian which

Consider now a relativistic Dirac particle with an anomalous magnetic moment a, charge e in the field of a

includes the recoil and spin-spin terms

$$H = {}^{(1)} \cdot \bar{p} + {}^{(2)} \cdot \bar{p}{}^{\to}$$

fixed quantum dipole moment $\bar{\mu}$ and charge e_2 the

$$1 \qquad 2$$

$$r\,3 \qquad \overline{3}$$

equation (in natural units) [1,2] is

$$[\gamma^\mu (p - e A) - m] = -a \, e^1 \, \gamma^\mu \gamma^v \, F$$

(5)

$$+ \frac{3(\overline{\sigma}_1 \cdot \overline{r})(\overline{\sigma}_2 \cdot \overline{r}) - \overline{\sigma}_1 \cdot \overline{\sigma}_2}{} + 8\pi \overline{\sigma}_1 \cdot \overline{\sigma}_2 \, \delta(r) \tag{9}$$

$$\mu \quad 1 \quad \mu$$
$$4\pi$$
$$\mu v$$

This Hamiltonian is interesting to study. To this one may

With $= (e^2, \mu_2 \frac{\overline{\sigma_1 \times r}}{r^3})$

add also the Hamiltonian for a dyonium i.e two objects having each a magnetic and an electric charge

Or in the Hamiltonian form

$$\rightarrow \qquad \alpha$$

$$\frac{\alpha \quad 1}{}$$

$$H = {}^{(1)} \cdot (\overline{p} - e_1 \overline{A_1}) + {}^{(2)} \cdot (\overline{p_2} - e_2 \overline{A_2}) \tag{10}$$

$$[\alpha \cdot (p - e_1 A) - [E -] + \beta m] = -a\epsilon \, \frac{1}{2m} \, r \, 2 \, i\beta\alpha_r \tag{6}$$

$${}^{(2)} \times \overline{r}$$
$${}^{(1)} \times \overline{r}$$

Where $\epsilon = sign \, e$

$\cdot e$) and α

$= \overline{\alpha \cdot r}$

$$\overline{A_1} = \mu_2$$
$$3 \quad \cdot \overline{A_2} = \mu_1$$

$$r^3$$

$$1 \quad 2 \qquad r \qquad r$$

When $\rightarrow = 0$, the above equation can be transformed to two- uncoupled second order Sturm-Liouville eigen value equations [1,15]; viz;

Magnetic monopoles have not yet been seen [17,18]. They have influenced fields like high energy physics e.g. quark confinement, superstring theory and supersymmetric

$$[\, {}^2 + k^2 - V(r)]$$

$= 0 \qquad (7)$

quantum field theories. This has guided condensed matter

$$dr \, 2$$

Where $k^2 = E^2 - m^2$

$i \quad i$

physicists to discover anomalous states and excitations

in systems such as Bose-Einstein condensates [19] and

And the effective energy and angular momentum potential (r) will have the same shape as shown in figure (2)

Next one must include the dipole field A and the spin-spin terms due to the anomalous magnetic moment. In such case one obtains 4 -coupled first order equations [16]. spinices [20].

The above-mentioned models may be treated within the framework of relativistic two-body problem in the so-called one-time formulation [21]. Also one must add the various potentials obtained from the Behe-Salpeter type of

equations which should be treated in a nonperturbative manner. In the next section, we will discuss the two-body equations.

3. The Relativistic Two-Body Equations

In a previous works, we derived a new set of the relativistic radial equations for two spin 1/2 particles [22,23]. The 16 amplitudes are re-expressed in terms of four scalars and four vectors which satisfy coupled differential equations. These equations are reduced further to sets of coupled differential equations according to parity and total angular momentum.

The equations are solved in the positronium case and the results obtained are similar to that of Schrödinger equation for the hydrogen-like atom.

In quantum field theory, the work on the two-body problem is based on two approaches. The first approach is a covariant formulation (the so-called Bethe-Salpeter equation) for describing the relativistic two-body systems [24], but owing to the complex structure of this equation no general solution has been found as yet [25-31]. Other approaches have been used by several authors (the one-time formulation) in order to give a physical interpretation for the two-body amplitudes. The reason for using our equations is that they are exactly solvable. A simplification and reduction of the system of coupled equations for the 16 amplitudes were achieved by the explicit imposition of Lorentz and charge conjugation invariance and parity conservation.

The number of the coupled equations is not more than eight and can be fewer depending on the interaction used. For instantaneous potential, the equations reduce to three- sets of tractable coupled differential equations. These equations are easier and analytically solvable [23].

Physically the use of the concept of potential for the relativistic problems at short distances may be questionable, however, one must remember that we are dealing with interactions of highly localized states. In such case for the bound state, the potential concept is proven to be superior to

4. Methods of Solution

In general one may recourse to the numerical solution to any of the above-mentioned problems but, here we will sketch the method for the simple case i.e equations (1) and (2) as shown in figure (3)

For such a potential, we have three zeros at

$$(r) = 0 \qquad (11)$$

and the positions of minima and maxima can be obtained from the equation

$$V^{'}(r) = 0 \qquad (12)$$

Equations (11) and (12) are a cubic equations in the general form, viz;

$$ar^3 + br^2 + cr + d = 0 \qquad (13) \text{ By replacing } r \text{ by } r + y, \text{ this can be}$$

transformed into

$$r^3 + Fr + H \qquad (14)$$

For which the three roots are

$$-a - b \;.\; -aw^2 - \;.\; -aw - bw^2 \qquad (15) \text{Where } w^3 = 1 \;.\; 1 + w + w^2 = 0 \qquad (16)$$

$$2a^3 = H + \sqrt{H^2 + \;^4\; F^3} \qquad (17)$$

$$\frac{}{27}$$

$$2b^3 = H - \sqrt{H^2 + \frac{4}{27} F^3} \qquad (18)$$

And from here one can proceed to find the solutions around the extremum points and the WKB like solutions as in a previous work [33,34]. For the solutions around the zeros one can use a generalized procedure which has been adopted by Shorupski [35].

In the present work, we will use the Nikiforov -Uvarov method as described in the next section.

The Nikiforov -Uvarov Method [36-38]

The Nikiforov-Uvarov (NU) method is based on solving the hypergeometric-type second-order differential equation.

$$T''(s) + \frac{\tilde{r}(s)}{} T'(s) + \frac{\tilde{\sigma}(s)}{} T(s) = 0 . \qquad (19)$$

the use of the perturbation theory [32]. The interset in such

$$\sigma(s)$$

$$\sigma^2(s)$$

class of interactions stems from the existence of high mass resonances e.g. the superpositrium in QED.

The effective static potential between two fermions is known and leads to the Briet- equation which is valid for weak binding. However, because of the strong binding nature of the potentials one may need some extrapolation of such potentials.

To do that one may introduce form factors at the vertices to take into account the self-energy and radiative corrections in an already renormalized form. Such a program has been discussed previously [19], where they incorporated form

Where (s) and $\tilde{\sigma}(s)$ are second-degree polynomials,

$r(s)$ is a first-degree polynomial and $\psi(s)$ is a function of the hypergeometric-type.

By taking $(s) = (s)Y(s) \qquad (20)$ And substituting in equation (19), we get

$$Y''(s) + [2 \frac{\varphi'(s)}{\varphi(s)} + \frac{r(s)}{\sigma(s)}] Y'(s)$$

$$+ [\frac{\varphi''(s)}{\varphi(s)} + \frac{\varphi'(s) \tilde{r}(s)}{\varphi(s)} + \frac{\tilde{\sigma}(s)}{\sigma(s)\sigma(s)}] Y(s) = 0 . \qquad (21)$$

By taking $2 \frac{\varphi'(s)}{} + \frac{\tilde{r}(s)}{} = r(s)$ and $\frac{\varphi'(s)}{} = \frac{\pi(s)}{}$.(22)

factors for both particles using a relativistic two-body

$\varphi(s)$

$\sigma(s)$

$\sigma(s)$

$\varphi(s)$

$\sigma(s)$

one-time formalism. The effective potential is given by equation (2.12) in that reference. The effect of form factors has been derived in previous works [22,23].

We get $\quad (s) = r(s) + 2\pi(s)$. $\qquad$ (23)

Where both, (s) and $\tau(s)$ are polynomials of degree atmost one.

Also one can take $\dfrac{\ddot{(s)}}{\dot{\varphi}^{()}s} + \dfrac{\dot{\varphi}^{()}(s)}{\dot{\varphi}^{()}s^{()}} \dfrac{\tilde{r}(s)}{\sigma s^{2()}} + \dfrac{\tilde{\sigma}(s)}{\dot{\sigma}^{2()}s} = \dfrac{\bar{\alpha}(s)}{\sigma s}$. (24)

Applying the same systematic way for equation (28), we get

Where $\dfrac{\ddot{\varphi}(s)}{\varphi(s)} = [\dfrac{\dot{\varphi}(s)}{\varphi(s)}]^{\cdot} + [\dfrac{\dot{\varphi}(s)}{\varphi(s)}]^{2} = [\dfrac{\pi(s)}{}]^{\cdot} + [\dfrac{(s)}{}]^{2}$. (25)

$\lambda = -nr^{\cdot}(s) - \dfrac{n(n-1)}{}\ddot{\sigma}(s) \qquad$ (33)

$\varphi(s)$

$\varphi(s)$

$\varphi(s)$

$\sigma(s)$

$\sigma(s)$

$n \qquad\qquad 2$

And $\bar{}(s) = \tilde{\sigma}(s) + \pi^{2}(s) + \pi(s)[r(s) - \sigma^{\cdot}(s)]$

$\qquad\qquad + \pi^{\cdot}(s)\sigma(s)$. $\qquad$ (26)

So equation (21) becomes

$\qquad Y^{\cdot}(s) + \dfrac{(s)}{}(s) + \dfrac{\bar{}(s)}{} Y(s) = 0$. $\qquad$ (27)

Where n is the principle quantum number.

By comparing equations (31) and (33), we get the energy eigenvalues equation.

To get the eigenfunction (s), one must know (s) and $Y(s)$.

$\sigma(s)$

$\sigma^{2}(s)$

We can obtian (s) from equation (22).

An algebraic transformation from equation (19) to $\qquad$ Now, we will obtian (s) by equation (27) is systematic. Hence one can divide $\bar{}(s)$ by

$(s) = \dfrac{Bn}{}\ d^{n}\ [\sigma^{n}(s)\,(s)] \qquad$ (34)

(s) to obtain a constant λ . i.e.

$\qquad\qquad \bar{\alpha}(s) = \lambda\,\sigma(s)$. $\qquad$ (28) Equation (27) can be reduced to a hypergeometric

$$\rho(s)\ ds^n$$

We can use the Rodrigues' formula of the associatedLaguerre polynomials, where equation in the form

$L^z(r) \equiv (s)$ and $B = \dfrac{1}{\underline{\quad}}$

(35)

$n \qquad\qquad n$

$n!$

$(s)Y''(s) + (s)Y(s) + \lambda\, Y(s) = 0 .$ (29)If one substitutes from equation (28) in equation (26) and

B_n is a normalization constant and the weight function

(s) must satisfy the condition below

solve the quadratic equation for $\pi(s)$, we get

$\dfrac{\rho'(s)}{\rho(s)} = \dfrac{r(s) - \sigma'(s)}{\sigma(s)}$

(36)

$\pi^2(s) + \pi(s)[r(s) - \sigma'(s)] + \tilde{\sigma}(s) - k\sigma(s) = 0 .$ (30)

Where $k = \lambda - \pi'(s) .$ (31)

Now, we will apply this method to solve equation (1) inour case.

$(s) = \dfrac{\sigma'(s) - \tilde{r}(s)}{2} \pm \sqrt{\left(\dfrac{\sigma'(s) - \tilde{r}(s)}{2}\right)^2 - \tilde{\sigma}(s)}$

$+ (s)$ (32)

First, we will give the solution for a general inversepolynomial potential form using Nikiforov-Uvarov (NU)

The possible solutions for (s) depend on the parameter k according to the plus and minus signs of $\pi(s)$. Since $\pi(s)$

method.

let $U(r) = r\, R(r)$

and subtituting in equation (1)

is a polynomial of degree at most one, so the expression under the square root must be the square of a polynomial. Inthis case, an equation of the quadratic form is available for [39-47], we get

$\dfrac{d^2 R}{dr^2} + \dfrac{2}{r} * \dfrac{dR}{dr} + [\lambda^2 - (r)] = 0$ (37)

$\overline{\quad}_2 \; - \; -\; \overline{\quad}$

the constant k. To determine the parameter k one must set the discriminant of this quadratic expression to be equal to zero. After determining the values of k one can find the values of (s), $and\ (s)$.

The generalized potential

$$(r) = \sum_{h=0}^{h} A_{-h} \, r^{-h} = A_0 + A_{-1} \, r^{-1} + A_{-2} \, r^{-2} + A_{-3} \, r^{-3} + A_{-4} \, r^{-4} + \cdots \quad (38)$$

Substituting into equation (37), one gets

$$\frac{d^2 R}{dr^2} + 2 * \frac{dR}{r \, dr} + [\lambda^2 - \sum_{h=0}^{h} A_{-h} \, r^{-h}] = 0 \qquad (39)$$

$$\frac{d^2 R}{dr^2} + 2 * \frac{dR}{r \, dr} + \frac{V_1(r)}{r^2} R = 0 \qquad (40)$$

Where

$$V_1(r) = -[B_0 r^2 + A_{-1} r + A_{-2} + A_{-3} r^{-1} + A_{-4} r^{-2} + A_{-5} r^{-3} + \cdots] \qquad (41)$$

And $B_0 = -\lambda^2 + A_0$

By substituting equation (41) in equation (40), we get

$$\frac{d^2 R}{dr^2} + \frac{2}{r} * \frac{dR}{dr} + \frac{1}{r^2} [-B_0 r^2 - A_{-1} r - A_{-2} - \sum_{h=1}^{h} A_{-h-2} \, r^{-h}] R = 0 \qquad (42)$$

Let $r = y + r_0$ (where r_0 is the smallest distance from the origin to the particle) and take Maclurin expansion for the summation terms to the second order, one gets

$$h \qquad 1 \; h \qquad h \quad A$$

$$h$$

$$A \qquad y \; ^{-h}$$

$$\sum A_{-h-2} \; (\;)$$

$$= \sum \frac{-h-2}{\underline{\quad}} = \sum \frac{-h-2}{\underline{\quad}} [1 \overline{+}]$$

$$(43)$$

$$h=1$$

$$r$$

$$h=1$$

$$(y + r_0)^h$$

$$h=1$$

$$r_0^h$$

$$r_0$$

$$h \qquad\qquad h$$

$$A \qquad y \; ^{-h} \qquad A$$

$$h \qquad h(h + 1)$$

$$\sum \frac{-h-2}{0} [\overline{1} +$$

$$\qquad\qquad\qquad\qquad\qquad \frac{\quad\quad}{}$$

$$h=1 \qquad\qquad\qquad\qquad\qquad\qquad r \; h$$

$$r_0]$$

$$\approx \sum \frac{-h-2}{^0 r \; h} [1 -$$

$$h=1$$

$$\overline{r_0}$$

$$y +$$

$$2r_0^2$$

$$y^2]$$

$$(44)$$

$$h \qquad 1 \; h \qquad h \quad A$$

$$(h + 2)(h + 1)$$

$$h(h + 2)$$

$$h(h + 1)$$

$$\sum A_{-h-2} \; (\frac{\;}{r})$$

$$h=1$$

$$\approx \sum_{h=1} \frac{-h-2}{r^h} \, [[$$

$$]-[$$

$$r_0$$

$$] \, r +$$

$$2 r_0^2$$

$$r^2 \,]$$

(45)

By substituting equation (45) into equation (42), we get

$$\frac{d^2 R}{dr^2}$$

$$2 \quad \frac{dR}{} - 1 \quad A \quad \frac{(h+2)(h+1)}{h(h+2)}$$

$$h(h+1)$$

$$+ r * \frac{}{dr}$$

$$+ r^2$$

$$[-B_0 r^2 - A_{-1} r - A_{-2} - \sum_{h=1} \frac{-h-2}{r_0^h} \, [[$$

$$-[$$

$$r_0$$

$$] \, r +$$

$$2 r_0^2$$

$$r^2 \,]] \, R = 0$$ (46)

Rearranging equation (46), we get

$$\frac{d^2 R}{dr^2}$$

$$2 \quad \frac{dR}{} \quad 1$$

$$1 \quad^h \quad (h+2)(h+1) \, A$$

$$^h \quad h(h+2) \, A$$

$$+ \quad * \quad +$$

$$[-(A_{-2} +$$

$$\sum \frac{}{-h-2}) + (\sum \frac{}{-h-2} - A_{-1}) \, r$$

$$r \quad dr$$

1

r^2
h

$h(h + 1)A$
2
$h=1$

$r_0{}^h$

$h=1$

$r_0{}^{h+1}$

$- (B_0 +$

$$\sum_{h=1}^{2} [\frac{-h-2}{r^{h+2}}]) r^2] R = 0 \qquad (47)$$

0

$1 \quad {}^h \quad (h + 2)(h + 1) A$

${}^h \quad h(h + 2) A$

$1 \quad {}^h \quad h(h + 1)A$

let $A_{-2} +$

$$\sum \frac{-h-2}{} = q, \sum \frac{-h-2}{} - A_{-1} = w, B_0 +$$

$$\sum [\frac{-h-2}{}] = z$$

we obtain

2

$h=1$
$r_0{}^h$

$h=1$
$r_0{}^{h+1}$

2

$h=1$
$r_0{}^{h+2}$

$$\frac{d^2R}{dr^2} + \frac{2}{r} * \frac{dR}{dr} + \frac{1}{r^2} [-q + wr - zr^2] = 0$$

$$(48)$$

Now, we follow Nikiforov -Uvarov method

$$r = 2, \sigma = r, \tilde{\sigma} = -q + rw - r^2z$$

Equation (32) is used to obtain (r)

$$- \qquad (r) = -^1 \pm \overline{\sqrt{r^2z} + (k - w) + q} + {}^1$$

(49)

2

Now, we calculate the value of the parameter k

2

4

$(\quad)^2 \qquad 1$

$$\overline{}$$

$$k - w - 4z \left(q + {}_4\right) = 0 \qquad \Delta = b - 4ac = 0 \rightarrow$$

$$(k - w)^2 = 4z \left(q + {}^1\right) \rightarrow k = \overline{\pm\sqrt{4z \left(q + {}^1\right)}} + w$$

(50)

4 4

In equation (50), we will deal with two cases of k.

By substituting the values of k in equation (49) and take the negative value of (r), one gets

$$2$$

$$(r) = -^1 - \sqrt{(\sqrt{z}r \pm \sqrt{q} + ^1)}^{\,2} = -^1 - \sqrt{z}r \mp \sqrt{q} + {}^1$$

(51)

2 4 2 4

Using equation (23), one obtains the value of (r)

$$(r) = 2 - 1 - 2\sqrt{z}r \mp 2\sqrt{q} + {}^1 = 1 - 2\sqrt{z}r \mp 2\sqrt{q} + {}^1$$

(52)

4 4

Also using equation (31) to get λ

$$\lambda = k + \pi \cdot (r) = \pm\sqrt{4z \left(q + {}^1\right)} + w - \sqrt{z} = w \pm \sqrt{4z \left(q + {}^1\right)} - \sqrt{z}$$

(53)

$\overline{}$ 4 $\qquad$ $\overline{}$ 4 $\overline{}$

λ_n can be obtained from equation (33)

$$\lambda_n = 2\pi\sqrt{z}$$

(54)

Comparing equations (53) and (54), one gets the energy eigenvalue equation, hence

$$z = \frac{w^2}{[1+2n\pm2\sqrt{(q+1)}]^2}$$

(55)

By substituting the values of z, w and q, we get the energy eigenvalue equation for a general inverse polynomial potential

$$1^h \quad h(h+1)A$$

$$\begin{array}{c} h \\ h=1 \end{array} \qquad (\Sigma$$

$$\frac{h(h+2)\,A_{-h-2}}{r^{h+1}}$$

$$-A_{-1})^2$$

$$\lambda^2 = A_0 + $$

$$\sum\left[\frac{-h-2}{2}\right] - \frac{0}{\phantom{r_0^{h+2}}}$$

$$2 \qquad r_0^{h+2} \qquad \qquad \qquad 2$$

$$h=1$$

$$[(2n+1)\pm2\sqrt{A}$$

$$+\frac{1}{4}\sum^h$$

$$\frac{(h+2)(h+1)\,A_{-h-2}}{1}$$

$$h=1 \qquad \qquad \qquad \qquad \frac{-2}{r_0^h}$$

$$+_4)]$$

One can also calculate the eigenfunctions for the general inverse polynomial potential from equation (20) using the negative value of k. So first, we calculate $\varphi(s)$ from equation (22)

$$d\varphi(r)$$

$$-1+\sqrt{q+1}$$

$$\int \quad \sqrt{}$$

$$\int \varphi(r)$$
$$r$$

$$= [- \quad z + \quad ^2$$

$$^4] \, dr \qquad (56)$$

$$\varphi(r)_{\overline{z}}= \quad _{\overline{4}} \quad ^- \qquad (57)$$
$$r^{-1+\sqrt{q+1}}$$
$$e^{-\sqrt{z}r}$$

Second, we calculate Y (r) from equation (36) and equation (34)

$$d\rho(r)$$

$$2\sqrt{q+1}$$

$$\int \rho(r)$$

$$= \int [-2\sqrt{z} +$$
$$^4] \, dr \qquad (58)$$
$$r$$

$$Y (r) = Y$$

$$\rho(r) = r^{2\sqrt{q+1}} \, e^{-2\sqrt{z}r}$$

$$_{\overline{4}} \qquad ^-$$

$$(r) = j \; r^{-2\sqrt{q+1}} \, e^{2\sqrt{z}r} \; d^{\,\overline{n}} \; [r^{n+2\sqrt{q+1}} \, e^{-2\sqrt{}} \,]$$

(59)

$$n \qquad n$$

4 \qquad 4

$$ds^n$$

(60)

where j_n is a normalization constant of $Y(r)$ which can be expressed as an associated Laguerre polynomial, where

$$2\sqrt{q+1}$$

$$1 \qquad -2\sqrt{q+1}$$

$$d^n$$

$$n+2\sqrt{q+1}$$

And $_n j = {}^1$

$$n!$$

$$L_n$$
$${}^4(r) =$$
$$r$$
$$n!$$
$$4 \, e^{2\sqrt{z}r}{}_{-[r}$$
$$ds$$
$$4 \, e^{-2\sqrt{}}]$$

$$n$$

(61)

From equation (20), the eigenfunction becomes

$$-1+\sqrt{q+1}$$

$2\sqrt{q}+1$

‒

‒

$$\Psi(\text{r}) = \varphi(\text{r})Y(\text{r}) = N_n\, r^{\,2}$$

$4 \quad e^{-\sqrt{z}r}\, L_n$

$^4(r)$ (62)

Where N_n is a normalization constant of the eigenfunction $\Psi(\text{r})$

By substituting the values of $z, w\ and\ q$, we get the eigenfunction for a general inverse polynomial potential

$\Psi(\text{r}) = (r)$

————————— ———————— ——————————

$-1 \qquad 1 \quad h$

$(h+2)(h+1)\ A-h-2 \quad 1$

$1 \quad h(h+1)A$

$2 \quad A$

$\frac{1}{}\sum h$

$(h+2)(h+1)\ A-h-2 \quad 1$

$+\sqrt{}\ A-2+$

$\sum h=1 \qquad\qquad +$

$-\sqrt{A}0-\lambda 2+ \ \sum h \quad$ [

$-h-2]\ r$

$\sqrt{}\ -2+2$

$h=1$

$r\ h \qquad +4$

$2 \qquad 2$

$= N_n\, r$

$r0h \quad 4 \quad e$

$2 \quad h=1$

$r0h+2$

L_n

$_0 \qquad (r)$

Hence,

1

$$\frac{1}{2} \quad h \quad \frac{(h+2)(h+1) \ A_{-h-2}}{1} \quad 1$$

$$\frac{1}{2} \quad \frac{1}{h}$$

$$h(h+1)A_{-h-2}$$

$$2\sqrt{A_{-2}+\frac{1}{4}} \sum h$$

$$\frac{(h+2)(h+1) \ A_{-h-2}}{2+\sqrt{A_{-2}+\frac{1}{4}}}+1$$

$$U(r) = N_n r$$

$$\sum_{h=1}$$

$$r_0^h$$

$$-\sqrt{A_0-\lambda} \quad +2\sum_{h=1}[$$

$$e$$

$$r_0^{h+2} \ ] \ r$$

$$L_n$$

$$2 \quad h=1$$

$$r_0^h$$

$$4$$

$$(r)$$

For the potential in equation (2)

$$() \quad \alpha \quad A \quad B \quad C$$

$$V \ r \ = \ \frac{}{r} + \frac{}{r^2} + \frac{}{r^3} + \frac{}{r^4}$$

Where $A_{-1} = \alpha, A_{-2} = A, A_{-3} = B, A_{-4} = C$ and the rest of the summation parameters in $_1(r)$ i equal to zero The energy eigenvalues become

$$\lambda^2$$

$$= [\ _0^{-3}$$

$$+ 3Cr_0$$

$$[3Br_0^{-2} + 8Cr_0^{-3} - \alpha]^2$$

$$^{-4}] - \quad \frac{}{[(2n+1) \pm 2\sqrt{(A + [3Br_0^{-1} + 6Cr_0^{-2}] + \frac{1}{})}]} \quad 2 \qquad 4$$

And, the state eigenfunctions are given as

$$\frac{1}{}+\sqrt{A+[3Br^{-1}+6Cr^{-2}]} \quad \frac{1}{}$$

$$-\sqrt{[Br^{-3}+3Cr^{-4}]-\lambda 2} \, r$$

$$\sqrt{A+[3Br0^{-1}+6Cr0^{-2}]+\frac{1}{}}$$
$$(r) = N_n \, r^2$$
$$0 \quad 0 \quad +4 \, e \quad 0 \quad 0$$

$$L_n$$

$$^4(r)$$

5. Conclusions

In the present paper, we discussed some special cases of the two-body potentials which have an oscillatory shape. The energy eigenvalues and eigenfunctions for bound states of such potentials have been obtained by Nikiforov-Uvarov method. The obtained results are useful in nuclear physics and quantum mechanics for magnetic interactions.

REFERENCES

[1] A. O. Barut and J. Kraus, Resonances in e+,e− system due to anomalous magnetic moment interactions, Physics Letters B59,175, 1975.

[2] A. O. Barut and J. Kraus, Solution of the Dirac equation with Coulomb and magnetic moment interactions, Journal of Mathematical Physics 17, 506, 1976.

[3] A. O. Barut and J. Kraus, Form-factor corrections to superpositronium and short-distance behavior of the magnetic moment of the electron., physical review D 16, 161, 1977.

[4] Alberto Sadum and Masoud Asadi-Zeydabadi, A Study of the Two-body Problem with Magnetic Interactions, Journal of physics, astrophysics and physical cosmology, 3, Issue 1, 2009.

[5] Bruno Gruber, two body problems with magnetic interaction "Symmetries in Science VIII", Springer Science, 1994.

[6] M. Naghdi, Nucleon-Nucleon Interaction: A Typical/Concise Review, arXiv:nucl-th/0702078.

[7] Bernard Schaeffer, Electromagnetic Theory of the Nuclear Interaction, World Journal of Nuclear Science and Technology, 6, no.4, 199, 2016.

[8] F.BLOCH, Le moment magnétique du neutron, Annales de l'institut Henri Poincaré, 8, no. 1, 63, 1938.

[9] Richard P Feynman; Robert B Leighton, Matthew Linzee Sands, the Feynman lectures on physics, San Francisco, Pearson addition Wesley, 2006.

[10] Rajmund Krivec, Mitja Rosina, Bojan Golli and Simon Širca, Few-Body Problems in Physics, Springer Wein New York, 2003; W.T. Grandy, book: Relativistic Quantum Mechanics of Leptons and Fields, Springer Science & Business Media, 1990.

[11] A. O. Barut and G.strobel, Large Magnetic Moment Effects at High Energies and Composite Particle Models, Field Theory in Elementary Particles. Studies in the Natural Sciences (A Series from the Center for Theoretical Studies), Springer,Boston, 19, 323, 1983.

[12] C. Prion, F. Reuse, Relativistic dynamics for the spin 1/2 particle, Helvetica Physica Act, 51, 146, 1978.

[13] F. Reuse, A new relativistic model for the hydrogen atom, Helvetica Physica Acta, 51, 157, 1978.

[14] A. O. Barut, lectures on magnetic interaction of stable particles and magnetic resonances, mathematical reviews, American mathematical society, P 4933, 1982.

[15] A. O. Barut, Magnetic resonances between massive and massless spin - 1/2 particles with magnetic moments, Journal of Mathematical Physics 21, 568, 1980.

[16] A. O. Barut, radial equations for Dirac particle with anomalous magnetic moment in a dipole field, quantum theory groups, fields and particles, mathematical physics studies, 1983.

[17] G. Aad et al. (ATLAS Collaboration), Search for magnetic monopoles and stable particles with high electric charges in 8 TeV pp collisions with the ATLAS detector, Phys. Rev. D 93, 052009, 2016.

[18] The MoEDAL collaboration, Search for magnetic monopoles with the MoEDAL prototype trapping detector in 8 TeV proton-proton collisions at the LHC, Journal of High Energy Physics, 8, 67, 2016.

[19] M. W. Ray, E. Ruokokoski, S. Kandel, M. Möttönen & D. S. Hall, Observation of Dirac monopoles in a synthetic magnetic field, Nature, 505, 657, 2014.

[20] C. Castelnovo, R. Moessner & S. L. Sondhi, Magnetic monopoles in spin ice, Nature 451, 42, 2008.

[21] A.O. Barut, R. Rączka, Two-Fermion Equation with Form-Factors and Electromagnetic High Mass Resonances, acta physica polonica B10, 687, 1979.

[22] M.M. Bakri, H.M.M. Mansour, The Relativistic Two-Fermion Equations (I), acta physica polonica B11, 413, 1980.

[23] M.M. Bakri, H.M.M. Mansour, The Relativistic Two-Fermion Equations (II), acta physica polonica B11, 543, 1980.

[24] E. E. Salpeter and H. A. Bethe, A Relativistic Equation for Bound-State Problems, Physical Review, 84, 1232, 1951.

[25] W. Królikowski, J. Rzewuski, Covariant one-time formulation of the many-body problem in quantum theory, IlNuovo Cimento, 2, Issue 2, 203, 1955.

[26] W. Królikowski, J. Rzewuski, On " potentials " in the theory of quantized fields, Il Nuovo Cimento, 3, Issue 2, 260, 1956.

[27] W. Królikowski, J. Rzewuski, One-time formulation of the relativistic two-body problem. Separation of angular variables, Il Nuovo Cimento, 4, Issue 5, 975, 1956.

[28] W. Królikowski, J. Rzewuski, Remarks on composite and unstable objects in quantum field theory, Il Nuovo Cimento B, 25, Issue 2, 739, 1975.

[29] M. Hossein Partovi, New treatment of the bound-state problem in quantum field theory, physical review D 12, 3887, 1975.

[30] H. Suura, Relativistic Two-Body Wave Equation, and Meson Spectrum, Physical Review Letters, 38, 636, 1977.

[31] G. Preparata, N.S .Craigie, A space-time description of quarks and hadrons, Nuclear Physics B 102, Issue 3, 478, 1976.

[32] A. O. Barut, M. Berrondo, and G. García -Calderón, Narrow resonances as an eigenvalue problem and applications to high energy magnetic resonances: An exactly soluble model, Journal of Mathematical Physics 21, 1851, 1980.

[33] H. M. M. Mansour, and H. J. W. Müller - Kirsten, Perturbative technique as an alternative to the WKB method applied to the double - well potential, Journal of Mathematical Physics 23, 1835, 1982.

[34] H. M. M. Mansour and H. J. W. Müller-Kirsten, Matching of WKB solutions to other solutions, physical review, D 24, 1033, 1981.

[35] Andrzej A.Skorupski, Reports on Mathematical Physics, Improved higher order phase-integral approximations of the JWKB type in the vicinity of zeros and singularities of the "wave number", 17, Issue 2, 161, 1980.

[36] Nouredine Zettili, Quantum Mechanics Concepts and Applications, John Wiley, 2009.

[37] Cüneyt Berkdemir, Theoretical concept of quantum mechanics, In Tech, 2012.

[38] F. Nikiforov and V. B. Uvarov, Special Functions of Mathematical Physics, Birkhäauser, Basel, Switzerland, 1988.

[39] E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T. M. Yan, "Spectrum of charmed quark-antiquark bound states," Physical Review Letters, 34, no. 6, 369, 1975.

[40] S. M. Kuchin and N. V. Maksimenko, "Theoretical Estimations of the Spin – Averaged Mass Spectra of Heavy Quarkonia and Bc Mesons," Universal Journal of Physics and Application, 7, 295, 2013.

[41] S. Laachir and A. Laaribi, "Exact Solutions of the Helmholtz equation via the Nikiforov-Uvarov Method," International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, 7, no. 1, 2013.

[42] H. Ciftci, R. L. Hall, and N. Saad, "Iterative solutions to the Dirac equation," Physical Review A: Atomic, Molecular and Optical Physics, 72, no. 2, part A, 022101, 2005.

[43] M. Alberg and L. Wilets, "Exact solutions to the Schrödinger equation for potentials with Coulomb and harmonic oscillator terms," Physics Letters A, 286, no. 1, 7, 2001.

[44] B. Ita, P. Tchoua, E. Siryabe, and G. E. Ntamack, "Solutions of the Klein-Gordon Equation with the Hulthen Potential Using the Frobenius Method," International Journal of Theoretical and Mathematical Physics, 4, no. 5, 173, 2014.

[45] H. Goudarzi and V. Vahidi, "Supersymmetric approach for Eckart potential using the {NU} method," Advanced Studies in Theoretical Physics, 5, no. 9, 469, 2011.

[46] N. V. Masksimenko and S. M. Kuchin, "Determination of the mass spectrum of quarkonia by the Nikiforov–Uvarov method," Russian Physics Journal, 54, no. 57, 54, 2011.

[47] Hesham Mansour and Ahmed Gamal, Bound State of Heavy Quarks Using a General Polynomial Potential, Advances in High Energy Physics, 1, Article ID 7269657, 2018.