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DESIGN AND ANALYSIS OF SYMMETRIC ANDASYMMETRIC SPUR GEARS

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ABSTRACT— Transmission of power from one part of a machine to another is impossible without gears. Variations in power transmission parameters, such as velocity, torque, and input direction, are all possible. They are exposed to a variety of loads throughout the power transmission process. These loads cause significant strains in the gears. Failures occur if the stresses are greater than the surface of the gear can withstand. The design of gears is modified somewhat to alleviate these strains. Among these alterations to the design is the use of asymmetric pressure angles between the driving and coast sides. The term "asymmetric spur gear" describes this kind of gear. These asymmetric spur gears' large roots teeth allow them to withstand less stress than conventional spur gears.

In this study, we analyse and validate spur gear designs using Hertz theory and compare designs that use different module numbers and pressure angles.

The analytical representation of asymmetric spur gears with various module combinations is also included in this study, along with their design and analysis. In the end, a comparison is made between each gear, graphs are produced, and conclusions are offered.

Hertz theory, modules, pressure angles, and stresses in asymmetric spur gears are discussed.

INTRODUCTION

Gears are the most important and common instruments in present mechanical world for transmitting power. They vary in many sizes starting from smallest gears used in watches to large and huge gears used in heavy machines. They are very vital in any mechanical machines. These are used mainly for varying speeds, power, and also direction of input and output. For the different kinds of use there are different kinds of gears i.e., Bevel gears, helical gears, spur gears, worm gears. Among all these gears spur gears are simple gears. Their design is very simple compared to other gears. These gears while they are operating, they are subjected to different kinds of loads, thus resulting in lots of stresses in gears.

These stresses are of two types bending stresses and contact stresses. Bending stresses are calculated theoretically by using Lewis theory and contact stresses are calculated theoretically by Hertz theory. The gears are defined by many factors like module, pressure angle, pitch circle and many more factors. This paper mainly deals with module and pressure angle. Gears with different modules and pressure angles are designed and analyzed. Conventional gears have similar design on both sides of gears i.e., drive and coast side. They are subjected to many stresses. To reduce these stresses to some extent, we need to alter the design. The design alteration includes different pressure angles on drive and coast side. They are called asymmetric spur gears. Coming to point of discovery of gears it is dated at the time of 4th century BC in China which has been preserved at the Luoyang Museum of Henan Province, China. The earliest preserved gears in Europe were found in the Antikythera mechanism, it is an example of very early and complex design of a gear to calculate the astronomical positions. The time of construction Antikythera mechanism is now estimated between 150 and 100 BC. These gears were greatly developed by the then Greek polymath Archimedes (287–212 BC).

I. DESIGN OF SYMMETRIC SPUR GEARS

Design of symmetric spur gears is done in Catia V5 software. The dimensions required for the design of spur gears are obtained from the formulae [1]. This paper depicts the affects of module and pressure angle on the spur gear. Different modules are used to design a spur gear. Each module is again designed by two kinds of pressure angles i.e., 14.5 and 20. Modules are selected through the table given below [1]. Four modules are taken from choice1 and one module is taken from choice-2.

TABLE I
DIFFERENT KINDS OF MODULES

Choice	1.0	1.25	1.5	2.0	2.5	3.0	4.0
1	5.0	6.0	8.0	10	12	16	20
Choice	1.125	1.375	1.75	2.25	2.75	3.5	4.5
2	5.5	7	9	11	14	18	

The modules used in this paper are highlighted i.e., 2.5, 3.5, 3.0, 4.0, 5.0.

Calculations:

a. Number of teeth on pinion = Np = 50

b. Number of teeth on gear = Ng = 50

c. Pressure angle = α = 20

d. Module = m = 3.5mm

e. Pitch circle diameter = Dp = m×N = 175mm

f. Base circle diameter = Db = Dp×cos(α)=164.45mm

g. Addendum circle diameter = Da= Dp+2m= 182mm

h. Dedendum circle diameter = Dd= Dp-(2+π/N)×m = 167.78mm

i. Face width = b= 10×m = 35mm

j. Fillet radius = rp= 0.4×m = 1.4mm

These are the formulae for obtaining dimensional values of a symmetric spur gear with module 3.5 and 20°pressure angle. In the similar way, different modules and pressure angles are taken and calculated to obtain the dimensional values. These values are used for the design of different symmetric spur gears in Catia V5.

The validation of the spur gears is done using Hertz theory. Those calculations are depicted below [1], [2].

Earle Buckingham used the hertz theory to determine the contact stress between a pair of teeth while transmitting power by treating the pair of teeth in contact as cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point. According to hertz theory [1], when two cylinders are in contact with each other, the contact stress is given by,

$$\sigma c = 2P/\pi BL \qquad \text{(i)}$$

$$\text{And } B = \sqrt{\left[\frac{2P\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}{\pi L\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]} \qquad \text{(ii)}$$

σc = Maximum contact stress (N/mm2)

P = Force acting between both the cylinders (N)

B = Half width of deformation (mm)

L = Axial length of cylinders (mm)

d1, d2 = diameters of two cylinders (mm) E1, E2 = modulus of elasticity of two cylinder materials (N/mm2) μ1, μ2 = Poisson's ratio of two cylinder materials.

TABLE II

TOLERANCES ON THE ADJACENT PITCH

Grade	***e*(microns)**
1	0.80+0.06Ø
2	1.25+0.10Ø
3	2.00+0.16Ø
4	3.20+0.25Ø
5	5.00+0.40Ø
6	8.00+0.63Ø
7	11.00+0.90Ø
8	16.00+1.25Ø
9	22.00+1.80Ø
10	32.00+2.50Ø
11	45.00+3.55Ø
12	63.00+5.00Ø

From [1] tolerance factor φ is given by,

$$\emptyset = m + 0.25\sqrt{d_p}$$

Where,
m = module of gear (mm)
dp = pitch circle diameter gear (mm)

In our case we have,
m = 3.5mm and dp = 175mm
Therefore we have,
φ= 3.5+0.25 175 (xvi)
From [1] the error e is given by,

$$e = e_p + e_g \qquad \text{(xvii)}$$

Where, e_p = error for pinion , e_g = error for gear,
Since, in our case the pinion and gear are of equal geometry, therefore tolerance factor φ is same for both gear and pinion. Also the grades listed in table 2 from grade 1 to grade 12 are arranged in decreasing order of precision. Considering the gear and pinion to be of grade 1 which is of top precision, we have $e_p = e_g = 0.80 + 0.06\emptyset$ (xviii)
From equations (xvii) & (xviii) we have,
e = 2ep = 2eg = 2(0.80 + 0.06φ) (xix)
From equation (xvi) we have

$$\emptyset = 3.5 + 0.25\sqrt{175}$$

Substituting this value of Ø in equation (xix) we have,

$$e = 2[0.80 + 0.06(3.5 + 0.25\sqrt{175})]$$

$$\Rightarrow e = 2.417\mu m = 2.417\times 10^{-3}\ mm$$

Now from equation 3.15 we have, the dynamic load,

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{Ceb + P_t}}$$

Substituting the values of e, P_t, C, v and b in the above the equation we have,

$$P_d = \frac{21\times54.978\times\left[\left(10711.5\times2.417\times10^{-3}\times35\right)+508.571\right]}{21\times54.978+\sqrt{\left(10711.5\times2.417\times10^{-3}\times35\right)+508.571}}$$

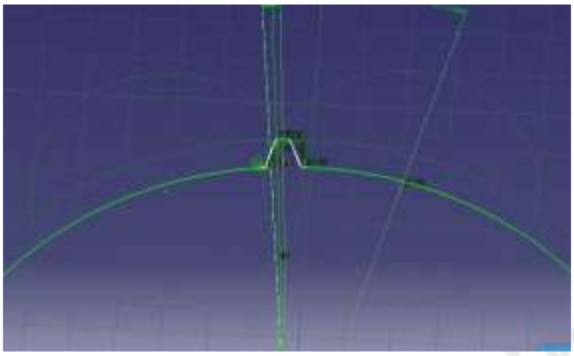
$$\Rightarrow P_d = \frac{21\times54.978\times1414.710}{21\times54.978+\sqrt{1414.710}}$$

$\Rightarrow P_d = 1370.0756\ N$ (xx)

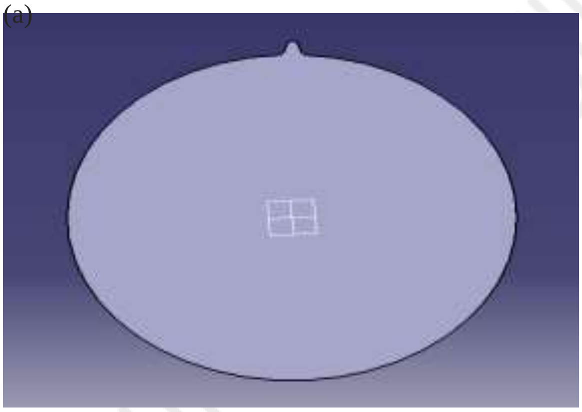
From equation (xiv) and (xx) we have,

$$P_{ts} > P_d$$

Therefore the design is safe from surface durability considerations. In the same above procedure, calculations for other modules and pressure angles are carried out. The results satisfy the above conditions. The final design and assembly of gear is shown in below figures



(a)



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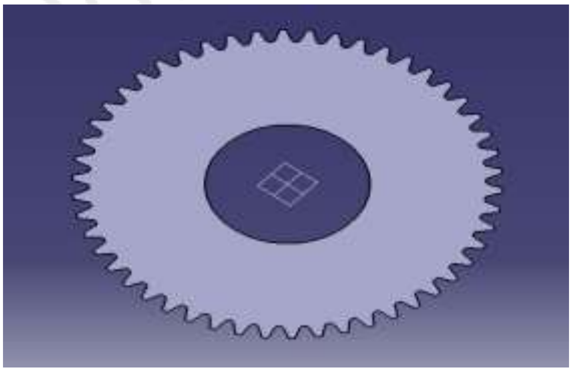

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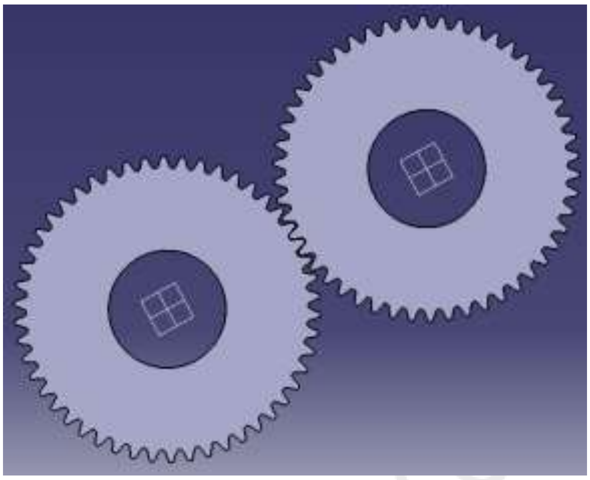


(d)

Fig. 1 (a)showing tooth profile in catia; (b)tooth profile after pad option; (c)tooth profile with 50 teeth; (d)assembly of spur gears

II. DESIGN OF ASYMMETRIC SPUR GEARS

The design of asymmetric spur gears [7] is slightly different from symmetric spur gears. This is due to the reason that these gears have different pressure angles of the drive and coast side.

Due to different pressure angles, these gears have different base circle diameters. These base circle diameters form the main aspect for the design of asymmetric spur gears. Though the base circles are different the centrodes (pitch circles) of both side profiles are similar when meshing of gear and pinion is considered. The centrodes are in tangency to point P which is the instantaneous center of rotation. The line of action is the common tangent to pair of base circles. The pressure angle is the angle between the line of action and the line which is tangent to both the centrodes.

The relation between pinion-gear tooth thicknesses: In case of standard gear drive both the thicknesses of pinion and gear teeth are similar i.e., s_p and s_g respectively. In these asymmetric spur gears we have similar pinion and gear teeth, so we can take s_p and s_g as equal. We have a relation between s_p and s_g . It is represented as

$$\lambda_t = \frac{s_p}{s_g} \qquad \text{(i)}$$

Whereas, λ_t is ratio of tooth thickness of pinion to the gear measured on the meshing centrodes of the gears.

In this case the asymmetric gears have similar s_p and s_g.

Due to rolling of centrodes, we have

$$s_g = w_p \qquad \text{(ii)}$$

$$s_p = w_g \qquad \text{(iii)}$$

Where w_p and w_g are width of space of pinion and gear measured on centrodes.

It is easy to verify that

$$s_p + w_p = s_g + w_g = p_c = \frac{\pi}{P_d} \qquad \text{(iv)}$$

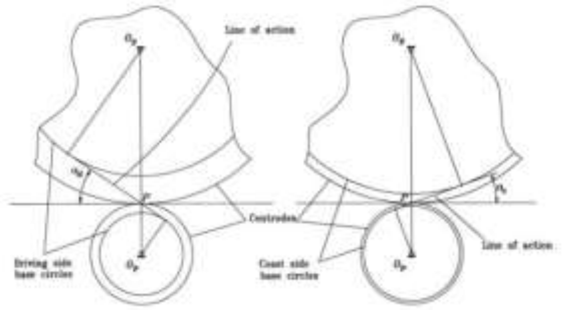


Fig. 2 Representation of line of action, centrodes and base circles in an asymmetric gear.

Where, p_c and P_d are circular and diametral pitches respectively.
Then we have the following expressions for s_p and s

$$s_p = \frac{\lambda_t \times p_c}{1+\lambda_t} = \frac{\lambda_t \pi}{(1+\lambda_t)P_d} \qquad \text{(v)}$$

$$s_g = \frac{s_p}{\lambda_t} = \frac{p_c}{1+\lambda_t} = \frac{\pi}{(1+\lambda_t)P_d} \qquad \text{(vi)}$$

Since we have λ_t =1; P_d = 1/m; $p_c = \pi \times m$; in this case, we obtain

$$s_p = s_g = \frac{\pi}{2 \times P_d} = \frac{\pi \times m}{2} = \frac{\pi \times 3.5}{2} = 5.498$$

Radii of base circles: The radius of the base circle and the radius of the pitch circle are related by below equation.

$$r_{bd} = r_p \cos\alpha_d = \frac{N}{2P} \cos\alpha_d \qquad \text{(vii)}$$

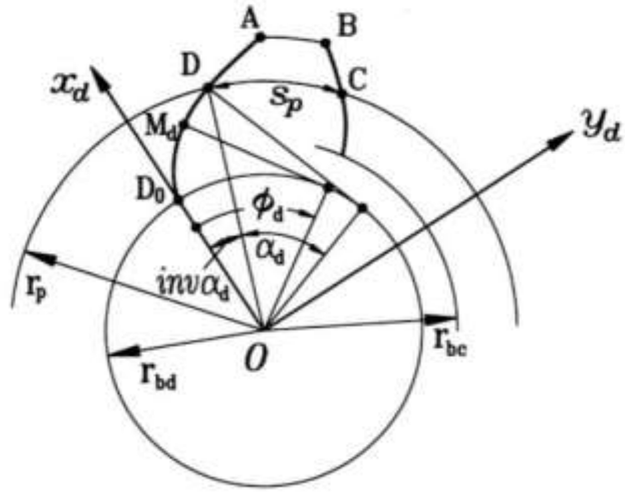
$$r_{bc} = r_p \cos\alpha_c = \frac{N}{2P} \cos\alpha_c \qquad \text{(viii)}$$

From above formulae we have r_{bd} and $r_{bc}r_{bd}$ as 75.777 and 82.223mm respectively.

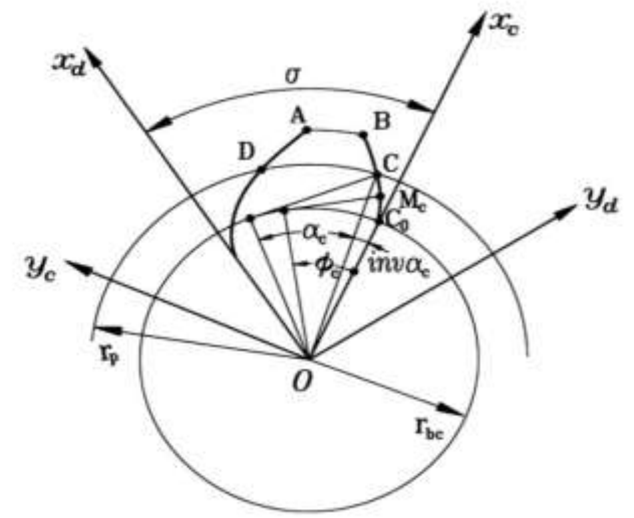
A. Analytical presentation of involute profiles

The figure 3 represents the involute profiles of an asymmetric tooth. The drawings can be either referred to pinion or gear.

We require the following as inputs: (a) DC = s_p, the tooth thickness on the pitch circle, (b) radius r_p of the pitch circle, (c) pressure angles αd and αc of drive and coast side profiles, (d) radii of base circles r_{bd} and r_{bc} for the drive and coast side profiles. Analytical representation of the asymmetric profile is to be obtained by these values.



(a)



(b)

Fig.3 (a) Representation of drive profile; (b) Representation of coast profile.

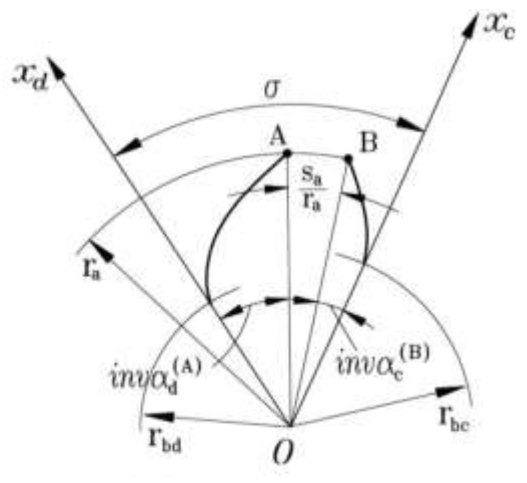


Fig.4 Determination of addendum tooth tip thickness

Step 4: Drawings of fig. yield

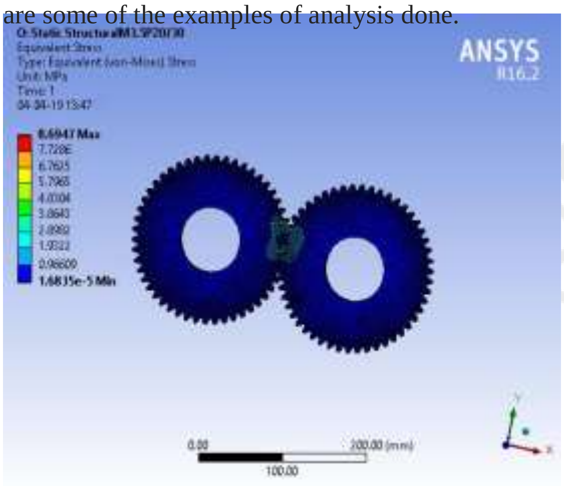
$$S_a = r_a \left(\sigma - \mathrm{Inv}\left(\alpha_d^{(A)}\right) - \mathrm{Inv}\left(\alpha_c^{(B)}\right)\right)$$

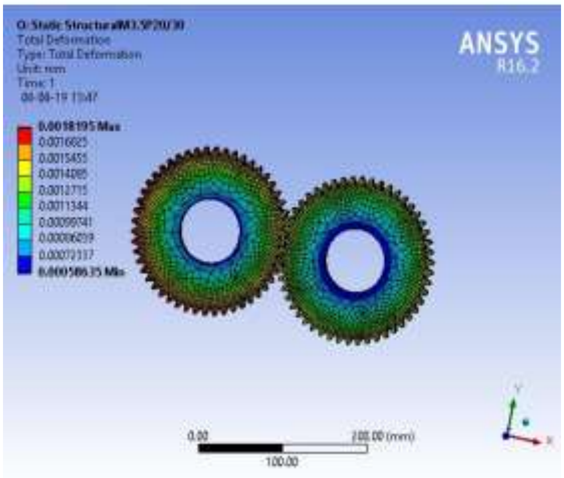
S_a = 91(0.13149 - 0.078136 - 0.03140) = 1.998
Here: parameter σ can be determined by using equation (xiii), $\mathrm{Inv}\left(\alpha_d^{(A)}\right)$ by using equations (xv) and (xvi), and $\mathrm{Inv}\left(\alpha_c^{(B)}\right)$ by using equation (xvii).

Step 5:
The derived equations enable to obtain radius r_a for a pointing tooth taking in equation (xvii) that $s_a = 0$.
In this similar way the asymmetric gears with different modules are calculated and designed using Catia V5.

III. FINNITE ELEMENT ANALYSIS

The finite element analysis is carried out in ANSYS work bench 16.2. The static analysis is carried out in workbench. The input given is moment for the pinion , which is got from the hertz theory calculations. The moment value is 44500 N-mm. This value is taken for every case in this paper .The gear material used in this analysis is stainless steel. The figures depicted below are some of the examples of analysis done.





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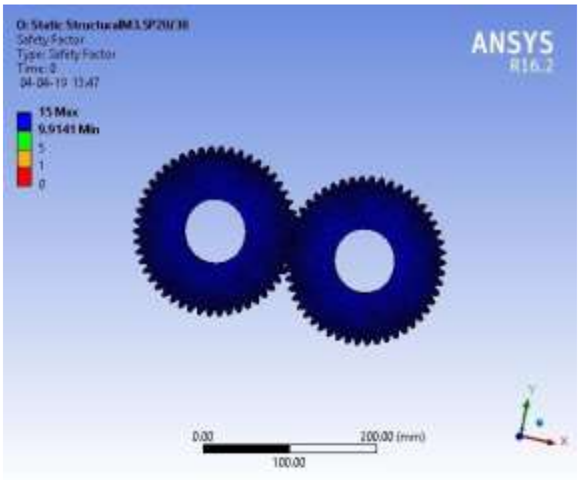


(c)

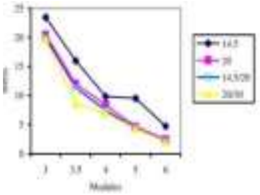
Fig.5 (a) equivalent stress ; (b) Total deformation; (c) safety factor of spur gear with module 3.5 and pressure angle 20

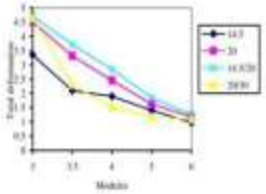
V. RESULTS AND COMPARISONS

The design of the symmetric and asymmetric spur gears is done in Catia, and their analysis is done in ANSYS softwares. The design and analysis are done for five module samples and each module gear is designed with three different pressure angles. Thus we obtain 15 results. The final results in the form of table are as follows.

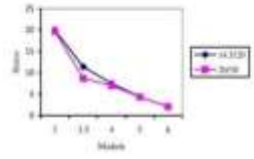
TABLE 2 RESULTS OF ANSYS

Module (mm)	Pressure angle($^\circ$)	Stress (Von-mises) (MPa)	Total Deformation (10^3xmm)	Safety factor
3	14.5	23.451	3.3645	3.6757
	20	20.492	4.511	4.2065
	14.5/20	19.933	4.715	4.3244
	20/30	19.695	4.64	4.3767
3.5	14.5	16.017	2.006	5.3816
	20	12.023	3.323	7.1669
	14.5/20	11.372	3.72	7.5801
	23/30	8.6947	2.2998	9.9141
4	14.5	9.8434	1.8984	8.7572
	20	8.4392	2.4586	9.9141
	14.5/20	7.5026	2.8498	11.489
	20/30	6.9936	1.523	12.325
5	14.5	9.5531	1.4054	9.023
	20	4.635	1.6271	15
	14.5/20	4.3911	1.848	15
	20/30	4.3434	1.1763	15
6	14.5	4.7405	0.9796	15
	20	2.464	1.1842	15
	14.5/20	2.0618	1.262	15
	20/30	2.0411	1.098	15

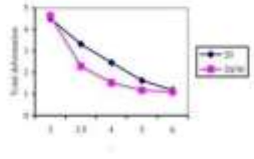




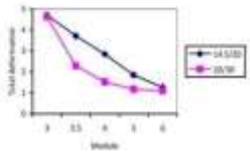
Graph 1 Module v/s Stress with different pressure angled gears   Graph 2 Module v/s Total deformations with different pressure angle gears



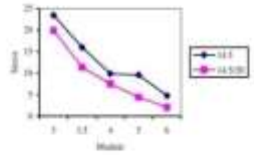
Graph 3 Comparison of equivalent stresses between two different asymmetric spur gears (14.5/20°) (20/30°) with different modules



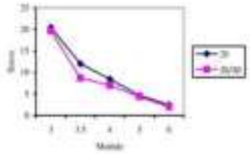
Graph 6 comparison of total deformation between symmetric (20°) and asymmetric (20/30°) spur gears with different modules



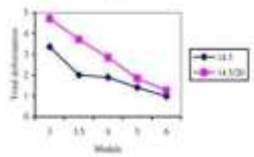
Graph 4 comparisons of total deformation between two different asymmetric spur gears (14.5/20) (20/30) with different modules



Graph 7 comparisons of equivalent stresses between symmetric (14.5°) and asymmetric (14.5/20°) spur gears with different modules



Graph 5 comparisons of equivalent stresses between symmetric (20°) and asymmetric (20/30°) spur gears with different modules



Graph 8 comparisons of total deformation between symmetric (14.5°) and asymmetric (14.5/20°) spur gears with different modules

VI . CONCLUSION

Symmetric spur gear has been designed and verified. Contact stresses are estimated using Hertz theory of contact stresses. Modelling and analysis of symmetric and asymmetric spur gears have been done using CATIA and ANSYS tools. From the results the following conclusions have been made:

1. Increasing the module for same gear ratio reduces the stress induced in the gear teeth.
2. Since the increase in module teeth dimensions proportionally increased, thereby the strength of the teeth has been improved.
3. The more is the pressure angle the less is the stress induced in the gear teeth.
4. By the increase in pressure angle the thickness at the root of teeth is increased. Eventually the strength is also improved.
5. The total deformation is low for high module gear and is also low for low pressure angled gear.
6. Finally by graph it is concluded that asymmetric spur gears have more strength than symmetric spur gears.
7. Among asymmetric spur gears 20/30° pressure angled gear is stronger than 14.5/20° pressure angled gear.
8. The total deformation and stresses induced in the gear ratio 2 spur gears are greater than gear ratio 1.
9. Thus we also conclude that asymmetric spur gears have less stress incited in them.

VII. REFERENCES

[1] V. B. Bhandari, Design of machine elements. Tata McGraw-Hill Education, 2017.

[2] M Jebran Khan et al. “Contact Stress Analysis of Stainless Steel Spur Gears using Finite Element Analysis and Comparison with Theoretical Results using Hertz Theory” in Int. Journal of Engineering Research and Applications ISSN : 2248-9622, Vol. 5, Issue 4, (Part -5) April 2015, pp. 10-18

[3] W.J. Qinn, C.Y. Guan. “An investigation of contact stresses and crack initiation in spur gears based on finite element dynamics analysis in int. journal of mechanical sciences vol. 83, issue 5, page 96-103, june 2014.

[4] Vivek Karaveer et al. “Modeling and Finite Element Analysis of Spur Gear” in International Journal of Current Engineering and Technology ISSN 2277 - 4106 Vol.3, No.5 (December 2013).

[5] P. Marimuthu, G. Muthuveerappan, “Effect of Drive Side Contact Ratio on Direct Design Asymmetric High Contact Ratio Spur Gear Based on Load sharing” Applied Mechanics and Materials Vol 592-594 (July 2014) page 2292-2296

[6] A. Kapelevich, “Geometry and design of involute spur gears with asymmetric teeth” vol 35 (2000) page.117-130.

[7] Faydor L. Litvin et al. “Asymmetric modified spur gear drives: reduction of noise, localization of contact, simulation of meshing and stress” Computational Methods Appl. Mech.Engg.188 (2000) page: 363-390.

[8] M. keerthi, K. sandya, “Static & Dynamic Analysis of Spur Gear using Different Materials” International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056 Volume: 03 Issue: 01 | Jan-2016.

[9] G. Mallesh, Dr. V B Math, Venkatesh, Shankarmurthy H J, Shiva Prasad P, Aravinda K “Parametric analysis of Asymmetric Spur Gear Tooth” 14th National Conference on Machines and Mechanisms (NaCoMM09) NIT, Durgapur, India, December 17-18, 2009.

[10] Ram Kumar Kunjam, Prakash Kumar Sen, “Review on Analysis and Modification of Spur Gear Design” IJIRT, ISSN: 2349-6002, Volume 2, Issue 5, October 2015.

[11] G. Mallesh, Avinash. P, Melvin Kumar R, Sachin G, Zayeem Khan “Finite element modeling and bending stress analysis of non standard spur gear” International Journal of Research in Engineering and Technology eISSN: 2319-1163 ISSN: 2321-7308.