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# DESIGN CRITERIA FOR MINIMUM BUILDING LIFE-CYCLE COSTS.

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**ABSTRACT:** Building design requirements are created in response to the significant damage caused by previous hurricanes and earthquakes in order to safeguard life and keep losses to a manageable level. Minimizing the anticipated total life-cycle cost while taking into account the design load and resistance is the issue. Include occurrence time, intensity, and duration in the load uncertainty. Cost discounting over time is taken into account, along with the expenses of building, maintenance, and failure-related effects including fatalities and injuries. For single and multiple time-varying loads, optimal values of design variables and target dependability are attained. The vulnerability of the ideal design to significant loads and structural factors is also examined. It is discovered that the structural life has a minor impact on the optimal design and that the limit state consequences (costs) have a greater influence. When there are many dangers, the hazard with the highest degree of uncertainty and the worst possible failure effects controls the best design. Applications for the method to design multistory office buildings against earthquakes and winds are provided in a companion paper.

## INTRODUCTION

Design procedures for buildings and structures have grad- ually changed from deterministic, based on judgment and ex- perience, to probabilistic, based on consideration of the load and resistance uncertainty. However, the treatment of uncer- tainty, has been mostly limited to selection of the return period for the design load. The long-term risk versus benefit impli- cations of such designs cannot be quantified to form a basis for rational design decisions. Such consideration is equally im- portant in design of new structures and retrofit of existing structures. For example, in design of structures of short life (under repair and retrofit or temporary structures), reduction of the design load seems justified because of the short expo- sure time. On the other hand, lower design loads result in higher risks of failure and possibly grave consequences, such as large losses, including human lives. Obviously, under such circumstances, designs based on probability alone would not be sufficient to resolve the problem.

The inadequacy of the current approach became apparent after the recent Northridge and Kobe earthquakes and Hurri- cane Andrew caused inordinately large damage and economic losses. As a result, new multilevel performance-based design concepts and methods have been proposed as possible replace- ment for the current procedures. For example, in the United States, the SEAOC 1997 Vision 2000 document (1995) pro- posed four levels of performance checks, each corresponding to a seismic hazard of a given return period (43, 72, 475, and 2,475 years). A

structure is evaluated and designed to meet the performance criteria specified at each level, from imme- diate occupancy to collapse prevention. A similar target per- formance-based procedure was proposed in Japan (Hiraishi et al. 1998). The selection of the hazards, or more specifically, the associated return periods and the corresponding perfor- mance levels, however, has been based on professional expe- rience and judgment. While collective professional wisdom may be the only recourse at present, there is no assurance that
it can achieve the ultimate objective of all engineering design
— satisfactory performance at a reasonable cost. To achieve such an objective, the life-cycle cost and the uncertainty in the loadings (caused by natural hazards and structural system ca- pacity) need to be carefully considered.

**BACKGROUND AND PREVIOUS WORK**

Probabilistic methods and reliability analysis have been used in developing codes and standards for design, e.g., in the United States, the Minimum Design Loads for Buildings and Structures of ASCE-7 (1998), LRFD procedures for steel structures (AISC- 1994), offshore structures (API RP 1990), and bridges (Kulicki et al. 1995). In these procedures, the tar- get reliability is inferred from what is used in current practice and acceptable to the profession. All procedures are intended for design of new and regular structures with normal design life. However, the target reliability may be different for dif- ferent load combinations and may have caused some concern among researchers and design professionals.

The major objective is protection of life in designing against natural hazards, such as earthquakes and extreme winds. The design load is based on a given probability of exceedance (or return period). The design load is then multiplied by a series of factors accounting for soil condition, structural dynamic characteristics, ductility capacity, and importance, etc. The re- liability of the structure is undefined and unknown (probability or return period) against limit states of concern to the owners or users such as serviceability, damage, and collapse under future earthquakes. In search for a new concept and method- ology to deal with the large uncertainty in both seismic de- mand and structural capacity, Wen and Foutch (1997) pro- posed and reviewed the critical issues in developing a reliability framework. They emphasized the need for stating structural performance goals in terms of limit state probability and rational procedure for determining target performance goal based on consideration of risks and costs. The need for incor- poration of reliability and consideration of life-cycle cost in future codes has been also emphasized in the recently pro- posed performance-based designs (Hiraishi 1998).

The design procedure, based on optimization considering benefit and cost, is generally referred to as level IV reliability- based design. For example, Rosenblueth (1976a,b) and Liu et al. (1976) made strong and convincing arguments for the pro- fession to move from a semiprobabilistic, second moment, or full distribution design format to one based on optimization. However, in these studies the expected loss was taken over an infinite time period. The effect of finite life span of the facility was not considered. Recent developments in reliability-based

optimization and applications to design of structural systems can be found in Frangapol and Corotis (1994). Several Federal Emergency Management Agency (FEMA) studies (FEMA 1992a,b) dealt with decision making in rehabilitation of exist- ing buildings. A standard benefit/cost model was developed for seismic rehabilitation of existing buildings. Field data in nine cities were collected to support the study. Ang and Leon (1996) studied optimal, cost-effective earthquake-resistance design criteria for reinforced concrete buildings in Mexico City and Tokyo. The damage, injuries and fatality, and dis- count of cost over time were considered. The target reliability was obtained and expected building damage cost was found to contribute the most in the total cost and the optimal design. Kanda and Shah (1997) emphasized the importance of failure cost evaluation, as a key parameter for safety, and the role played by engineers in arriving at optimal decisions in earth- quake resistance design. Kanda and Ellingwood (1991) inves- tigated optimal reliability-based design loads and load factors for possible implementation in a code format.

## ANALYTICAL FORMULATION

The major considerations in a life-cycle cost analysis of a constructed facility are proper treatment of uncertainties of the demand and capacity of the structure and costs incurred due to unsatisfactory performance. In this study, the random oc- currence and the intensity variation in time of the hazards are described by simple random process models by Wen (1990). According to the time scale of the intensity variation, the haz- ard fluctuation can be modeled in time by either a pulse pro-

over time $t$; h = constant discount rate/year; $P_{ij}$ = probability of $j$ th limit states being exceeded given the $i$ th occurrence of a single hazard or joint occurrence of different hazards; $k$ = total number of limit states under consideration; and $C_m$ = operation and maintenance costs per year. Note that the above formulation properly accounts for the loading and capacity un- certainty and the costs over the structural life. Implicit in the formula is the assumption that the structure will be restored to its original condition after each hazard occurrence. The dis- count factor $e^{-h\,t}$ converts cost due to hazard that occurs in the future into present dollar value. For example, at a discount rate of 5%, \$1 million damage cost due to an earthquake 20 years from now is converted into present dollar value of 1 million × $e^{-1}$ = 0.36 million.

Under the assumption that hazard occurrences can be mod- eled by a simple Poisson process with occurrence rate of v/year and for resistance that is time-invariant, (1) can be eval- uated in closed form.

*For Single Hazard*

Assuming that the limit state probability $P_{ij}$ does not change with time (i.e., ignoring the deterioration of the structural ca- pacity with time), one can show that the lifetime total expected cost can be obtained as follows (see Appendix I for details of derivation):

captured using the pulse process defined by the mean occur- rence rate v, mean duration $\mu_d$, and a random variable for intensity variation. The microscale fluctuation within each pulse can be modeled by a continuous random process. It is an efficient tool to model and evaluate structural performance under random loads. Costs considered include those of con- struction, maintenance and operation, repair, damage, and fail- ure consequence (loss of revenue, deaths, and injuries, etc.). Discounting of cost over time is also considered. It is reason- able to assume that the number of structural limit states of concern is small and the demands that can cause the limit states are due to severe hazards w

**Expected Life-Cycle Cost**

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Over a time period ($t$) which may be the design life of a new structure or the remaining life of a retrofitted structure, the expected total cost can be expressed as a function of $t$ and the design variable vector **X** as follows:

$$+ \quad C_m(X)e^{-hr}\, dr$$
$$0 \qquad\qquad (1)$$

in which $E[\cdot]$ = expected value; $C_0$ = initial cost for new or retrofitted facility; **X** = design variable vector (design loads and resistance, or load and resistance factors associated with nominal design loads and resistance); $i$ = severe loading oc- currence number, including joint occurrence of different haz- ards such as live, wind, and seismic loads; $t_i$ = loading occur- rence time, a random variable; $N(t)$ = total number of severe loading occurrences in $t$, a random variable; $C_j$ = cost in pres- ent dollar value of $j$th limit state being reached at time of the loading occurrence, including costs of damage, repair, loss of service, and deaths and injuries; $e^{-ht}$ = discounted factor of where $v_i$ = mean occurrence rate of hazard $i$; $v_{ij}$ = $v_i v_j (\mu d_i + \mu d_j)$, the coincidence rate of hazards $i$ and $j$; $v_{ijk} = v_i v_j v_k (\mu d_i\, \mu d_j + \mu d_j\, \mu d_k + \mu d_i \mu d_k)$, the coincidence rate of hazards $i$, $j$, and $k$; $p^i$ = probability of limit state 1 given the occurrence of hazard $i$ ; $P^{ij}$ = probability of limit-state 1 given the coincidence of hazards $i$ and $j$; $P^{ijk}$ = probability of limit-state 1 given the joint occurrence of hazards $i$, $j$, and $k$ ; and $\mu_d$ = mean duration of hazard $i$. It is assumed that the structure is restored to its original condition if damaged during a hazard occurrence. The design decision is then made based on the criterion that the expected total life-cycle cost should be min- imized with respect to the design variable vector **X**. The im- plications of the independence assumptions in the occurrence

and intensity in the Poisson pulse process will be examined in the following.

**Design Optimization**

The objective of design is to minimize the total expected life-cycle cost, i.e., primarily balance between initial cost and expected failure (limit state) costs as formulated in (1)–(4). For building structures, the maintenance costs, such as heating and cooling, may be a significant item in life-cycle cost con- sideration but do not depend on the design variables (structural strength) under consideration in this study. Note that for other structures, such as bridges and offshore platforms, the situation may be different and the maintenance (including inspection) costs could be dependent on the design variables. Similarly, a general formulation may allow consideration of the benefit as- sociated with the construction of the structure. In most struc- tural designs, the benefit does not depend on the design vari- ables. These items certainly can be included if there is evidence to show the dependence of these costs on the design variables. The optimization problem, as formulated above, is an unconstrained minimization. Since the expected total life- cycle cost can be evaluated in closed form based on the Pois- son occurrence model, the unconstrained minimization can be solved analytically. It facilitates the parametric and sensitivity analyses of the optimal design. Proper constraints may be in- troduced in the above minimization problem; the constraints may be limits of design variables or minimum acceptable re- liability levels for limit states, or both. The designer may want to impose a limit on minimum strength or annual probability of death and injury. These constraints can be added to the minimization problem.

Parametric studies of optimal design under a single or multiple hazards are carried out in the following.

## OPTIMAL DESIGN UNDER SINGLE HAZARD

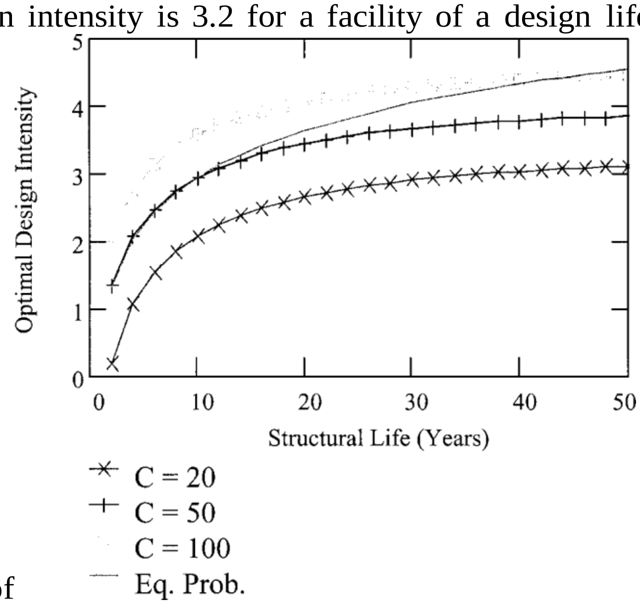
Under a single hazard modeled by a Poisson process, with an occurrence rate of v/year and for a resistance that is time- invariant, a closed form analytical solution of the expected life-cycle cost given by (2) can be obtained. It greatly facili- tates the determination of the optimal solution as a function of design life and other important parameters. A parametric study has been carried out for the optimal design intensity against seismic hazard. To illustrate the concept, the following assumptions are made:

1. Hazard intensity is modeled by an exponential distribu-tion with a mean value of 1.0.
2. Resistance is deterministic and a single limit state of de- sign intensity being exceeded is considered.
3. Initial cost is proportional to the design intensity **X**, and the maintenance cost is not considered.

The expected total cost as a function of the lifetime $t$ can be shown to be according to (2) (Appendix I)

$$E[C(t, X)] = (aX + C)e^{-X} v \frac{1 - e^{-ht}}{h} + aX \quad (5)$$

The optimal (minimum expected cost) solution can be deter- mined from (5) in closed form. The solution allows sensitivity studies of the optimal design intensity to the load parameters, structural life, and failure consequence. Fig. 1 shows the op- timal design intensity (arbitrary unit) as a function of design life and cost of limit-state being reached (arbitrary unit). Under the condition that the failure cost $C$ = 20, which is of the same order of the construction cost (considering only repair and re- placement costs), the design intensity is 3.2 for a facility of a design life of 50




years. Compared with a design intensity of

**FIG. 1. Optimal Design Intensity as Function of Design Life for t = 0.05/Year; v = 0.2/Year; and (Initial Cost/Unit Design Inten- sity) = 5**

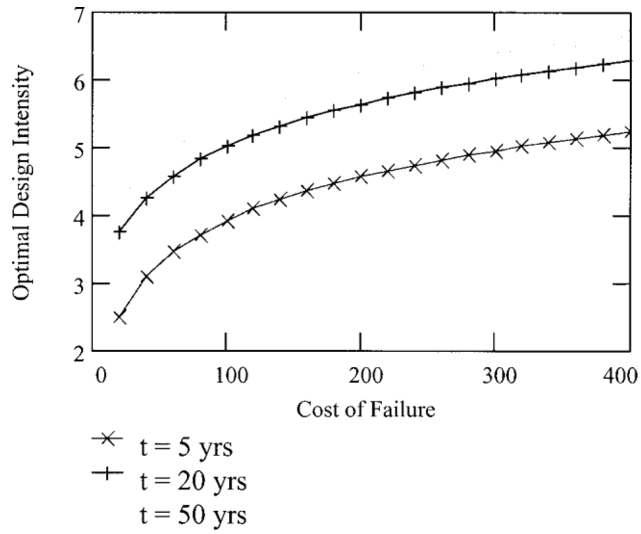



**FIG. 2. Optimal Design Intensity as Function of Cost of Failure for t = 0.05/Year; v = 0.5/Year; and (Initial Cost/Unit Design Inten-sity) = 5**

1.2 for a life of 5 years, the reduction is almost a factor of three. On the other hand, when the failure cost *C* = 100, which is about five times the construction cost (considering revenue loss, deaths, and injuries), the design intensity reduces only from 4.4 to 3.0. The design intensity, based on a criterion of equal lifetime probability of exeedance (10%), is also shown in Fig. 1. It would lead to underdesign for a system of short life and high failure consequence and overdesign for a system of long life and low failure consequences. Fig. 2 shows the dependence of design intensity on failure consequence. When the failure consequence is large, high design intensity is needed, even for a facility with a short design life. In this case, the additional initial cost ensures much less failure cost and saving in the long run. An equal (10%) lifetime probability of exceedance criterion would lead to design intensity of 2.25, 3.63, and 4.55 for *t* = 5, 20, and 50 years, respectively, in- dependent of the failure consequence. The results show that a rational, quantitative design decision can be made based on results of such a minimum life-cycle cost analysis and cannot be obtained based on judgment and experience or considera- tion of probability alone.

$E[C(t, X^d, X^d)] = C(X^d, X^d) + [v\ \exp[—(c\ X^d + c\ X^d/c\ \mu)]$ 1 2 1 1 1 2 2 1 $X_1$

v1 v2 ($\mu d$ + $\mu d$ )

**OPTIMAL DESIGN UNDER TWO HAZARDS**

$+ v\ \exp[—(c\ X^d + c\ X^d/c\ \mu)] + \frac{1\ 2}{}$
2 1 1 2 2 2 $X2$ $c\ \mu$ — $c\ \mu$

**Question of Uniform Reliability**

Most civil systems are subjected to more than one load. When designing for more than one load, the difficult question

× ($c2$ μ$X$

— $c_1$μ$_X$

2 $X$ 2

exp[—($c1\, X^d$ + $c2\, X^d$/$c2$ μ$X$ )]

2 1 2 2

]$\underline{C}$

1 $X1$

exp[—($c_1X^d$ + $c_2X^d/c_1$μ$_X$ )]) (1 — $e^{-hr}$)

becomes what level of reliability should be aimed for each

1 1 2

1 h (9)

load. Is uniform reliability the logical choice? While such cri- teria have been suggested, it is not obvious that they are cost- effective, since it may be prohibitively costly to maintain high reliability against a hazard of high intensity and large uncer- tainty, such as earthquakes. Ellingwood et al. (1982) showed that the 50-year reliability index in current design for seismic loads is approximately 1.75, which is much lower than those for winds (around 2.5) and dead plus live loads (3.5 or higher). It does not necessarily mean that current code seismic load

in which $C0$ = initial cost; and $C$ = cost of the limit state being reached. For the reasons stated for buildings, the maintenance cost is not considered. The first two terms in the square brack- ets are the contribution from the occurrence of the individual loads and the third term is that from the simultaneous occur- rence of both loads. The initial cost function is assumed to be a simple power function of the design load intensities as fol- lows:

provisions are inadequate. Following, a parametric study of optimal design against two time varying loads is carried out

$C0(X^d, X^d) = d\,(X^d)^{k1} + d\,(X^d)^{k2}$

(10)

by considering the random occurrences in time and the random intensity and duration of the loads and combination of load effects.

**Modeling of Load, Resistance, and Cost Function**

Two time-varying loads, $S1(t)$ and $S2(t)$, are treated as ran- dom processes and (2) is used to evaluate the expected life- cycle cost. To facilitate the parametric study, simple load pro- cesses models and a minimum number of load and resistance parameters are used. The essential features of the time-varying characteristics of the loads are captured by the simple models. The Poisson process is used for both loads, and the intensities given the occurrence are modeled by exponential random var- iables $X1$ and $X2$. The duration of each load is also a random variable. The two load processes are defined by their respec- tive mean occurrence rates (v1 , v2), mean

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duration ($\mu d_1$, $\mu d_2$), and mean load intensities ($\mu X$ , $\mu X$ ). Assume that the load ef- fect can be given by linear combination of load intensity

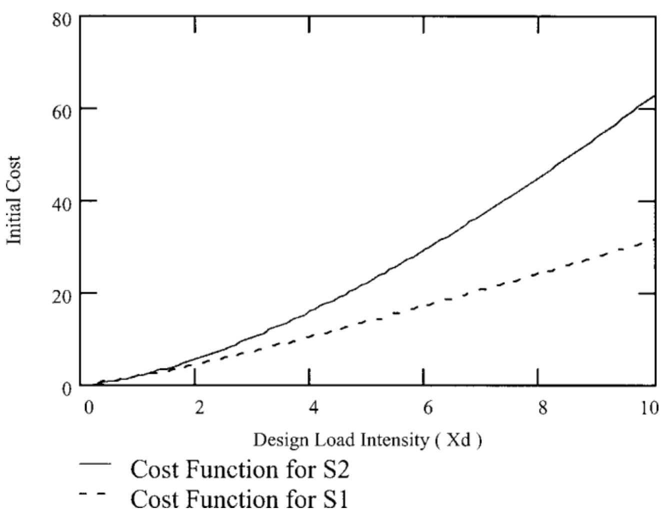
$$Y(t) = c_2 S_1(t) + c_2 S_2(t) \qquad (6)$$

Note that when only one load is present, the above formula reduces to one term proportional to the load intensity $X_1$ or $X_2$. Only one limit state is considered. The capacity of the system against the limit state, $R$, is assumed to be deterministic and given by a linear combination of the design load intensi- ties $X^d_1$ and $X^d_2$
where $d_1$ and $d_2$ = initial cost multipliers for the two load intensities. Past experience shows that initial cost increases slightly faster than a linear function under a single load. No functional forms have been studied for two or more loads. Eq.
(10) is a simple but flexible model for the purpose of the parametric study. The optimal solution is determined by searching for the minimum point of the expected life-cyclecost given by (9).

**Numerical Results**

Numerical examples are carried out. The system parameters are given in Table 1. As can be seen by comparison of the parameters, $S_1(t)$ occurs much more frequently and has longer duration, whereas $S_2(t)$ is more intense and variable with a mean and a standard deviation twice those of $S_1(t)$. The $k_1$ and $k_2$ values are chosen to be 1.2 and 1.5, respectively, such that the initial cost will increase slightly faster than a linear func-

$$R = c_1 X^d_1 + c X^d_2 \qquad (7)$$



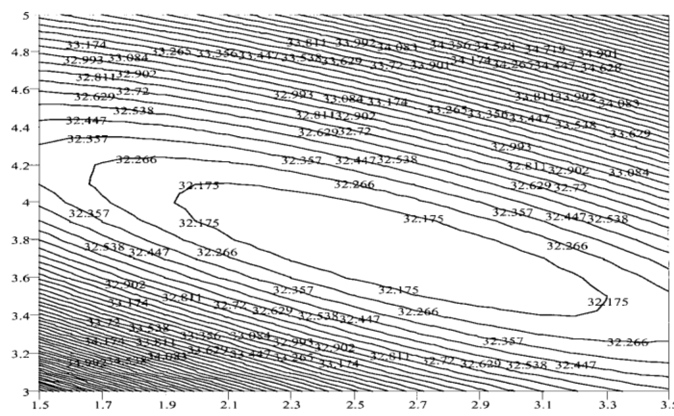




**FIG. 3. Initial Costs as Functions of Design Load Intensity**

or, in many design situations, the capacity is ‘‘controlled’’ or‘‘governed’’ by the larger design load and is given by

$$R = \max(c_1 X^d , c X^d ) \qquad (8)$$

According to (3) and (7), one can show that the expected totalcost over the lifetime of the structure is given by
tion with the design load intensity, and more so for $S2(t)$ as shown in Fig. 3. In other words, design for $S2(t)$ is more ex- pensive. The discount rate is assumed to be 5%/year. It is similar to the situation of design for both winds and earth- quakes. Consider the case of the system capacity given by (7). A typical contour plot of the total expected life-cycle cost as a function of the two design variables is shown in Fig. 4, from which the optimal values of the design loads, which minimize the total, can be determined. Because of the extremely small coincidence rate of the two loads, the contribution of the si- multaneous occurrence of the two loads is negligible in this example.

The optimal design intensities for both loads, as a function of the structural life, are shown in Fig. 5 for three different values of cost of failure (limit state reached). The range of $C$, from 20 to 50, represents the case of considering only cost of

**TABLE 1. Load and Cost Parameters**

| Parameter (1) | Mean occurrence rate (v) (2) | Load intensity distribution (3) | Mean intensity ($\mu X$) (4) | Mean duration ($\mu D$) (5) | Load effect coefficient ($C$) (6) | Cost multiplier ($d$) (7) | Cost power ($k$) (8) |
|---|---|---|---|---|---|---|---|
| $S1(t)$ | 5/year | exponential | 1.0 | 0.001/year | 1.0 | 2.0 | 1.2 |
| $S2(t)$ | 0.2/year | exponential | 2.0 | 0.00005/year | 2.0 | 2.0 | 1.5 |

FIG. 4. Contour Plot of Expected Total Life-Cycle Cost as Function of Design Variables

replacement of the structure, whereas the range from 100 to 500 represents the case where costs of loss of revenue, injuries, and deaths are also included. The solid lines are design loads of $S1(t)$ and the dotted lines are those of $S2(t)$ for different costs of failure consequence. The resultant annual limit state prob- abilities of the optimal design under each load, as a function of the structural life, are calculated and shown in Fig. 6.

Because of the dominance of $S2(t)$, the overall (target) limit state probability is almost the same as that under $S2(t)$ only. The reciprocal of the probability is the return period for the optimal design in terms of one load only. If only structure damage is considered ($C$ = 20) and the intended useful life of the structure is 50 years, the optimal return period of $S2(t)$ for design is 43 years. The corresponding return period increases to 690 years if revenue loss, injury, and death are also consid- ered ($C$ = 500). The initial costs (solid lines) and the mini- mized expected life-cycle costs (dotted lines), as functions of structural life for different values of cost of failure, are shown in Fig. 7. For the case of system capacity given by the dom- inant design load (8), it was found that the expected life-cycle cost generally converges to two local minima, corresponding to considering the two loads separately. As expected, the op- timal solution with respect to $S2(t)$ is the global minimum. The optimal system design capacity and the target failure proba- bilities for this case were also obtained and show only small differences from those given in Figs. 6 and 7.

The salient features of the results can be summarized as follows:

1. In the case of a single hazard, the optimal design loads increase with the structural life, but the increase is small for structural life longer than 50 years.
2. The failure probability of the optimal design considering $S2(t)$ only is consistently and considerably higher than that considering $S1(t)$ only, due to the fact that the former has a higher intensity, and is more variable and expensive to design against. Because of the widely different char- acteristics of the two hazards, uniform reliability is not necessary and would not be cost-effective.
3. When cost of failure consequence is large (500) and ex-

**FIG. 5. Optimal Values of Design Variable $X^d$ and $X^d$ as Functions of Structural Life—$X^d$ (Solid Line); $X^d$ (Dashed Line); $C$ = 20 (×);**
**50 (▫); 100 (◇); and 500 (o)**

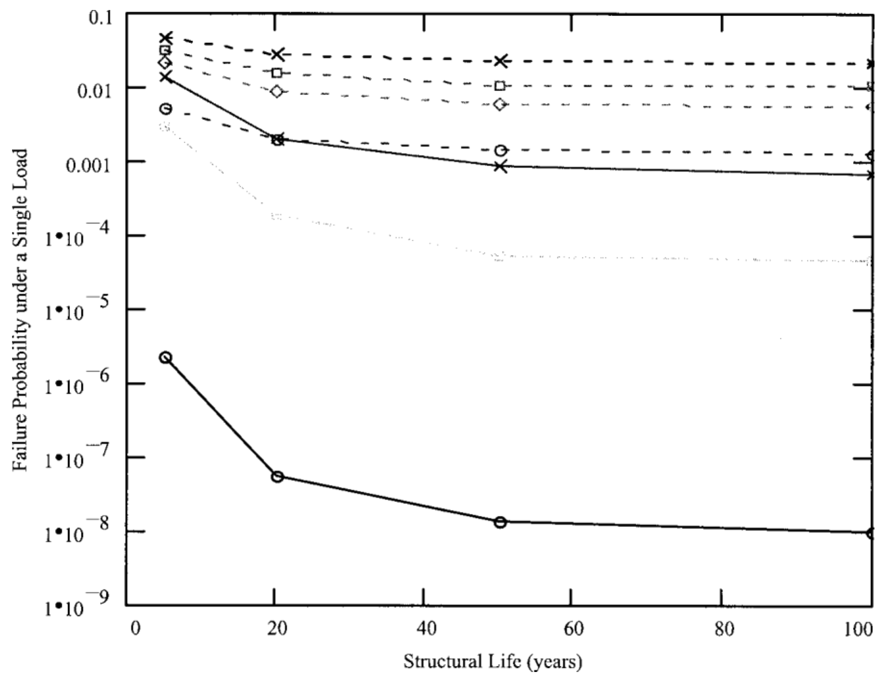



**FIG. 6. Annual Limit State Probability under One Load Only as Function of Structural Life—*S*1(*t*) [Solid Line], *S*2(*t*) [Dashed Line]; *C***

**= 20 (×); 50 (▫); 100 (◇); and 500** (o)

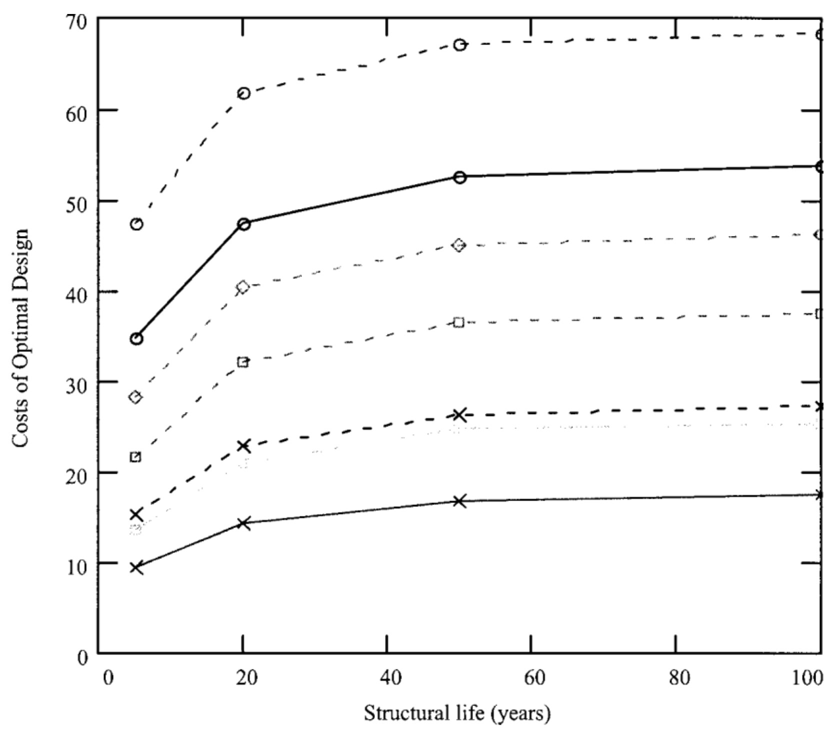



**FIG. 7. Initial and Life-Cycle Costs of Optimal Design as Functions of Structural Life—Initial Cost (Solid Line); Life-Cycle Cost (Dashed Line); *C* = 20 (×); 50 (▫); 100 (◇); and 500 (o)**

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posure time is long (100 years), large initial cost is jus- tified by the fact that it keeps the expected lifetime fail- ure cost small (<20).

**Implication of Load Independence**

The Poisson pulse processes used in the preceding analysis are based on assumptions of independence of occurrence time and hazard intensity within each hazard and between different hazards. Many natural and man-made hazards may have cor- related occurrence times, load intensity, and duration depen- dencies within each hazard, as well as between hazards. The dependencies may be due to the common physical mechanism that generates demands on the system. For example, severe storms may produce large wind, wave, current, surge, snow, and thermal loads. These loads may have strong correlation in occurrence time, duration, and intensity. Similarly, many of the man-made, operational and accidental loads, such as in nuclear structures, may also be correlated. The extent to which these dependencies affect the optimal design, as compared with that based on the idealized Poisson pulse assumption, needs to be examined. The issues related to the affect of hazard dependencies on the structural safety and performance evalu- ation have been comprehensively investigated by Wen (1990).

By relaxing the independence assumptions in the Poisson pulse and intermittent continuous processes, it has been found that:

1. Within-load duration-intensity correlation causes a slight increase in the limit state probability at the high response threshold levels. Within-load intensity and occurrence dependencies cause a moderate increase in the limit state probability at lower response levels but have little affect at the high response levels.
2. Between load occurrence and intensity dependencies are important and their effect is multiplicative and can lead to large increases in the coincidence rates and limit state probability at the high threshold levels.

Therefore, the implications in the life-cycle cost analysis and optimal design are obvious. Since the effects of within- hazards dependencies are moderate or minor, they may be ne- glected in the expected life-cycle cost analysis. If there were evidence to believe that there are significant dependencies in occurrence times as well as intensity between hazards, then, the correlation parameters need to be carefully identified and quantified and the coincidence rates and limit state probabili- ties evaluated with these dependencies taken into considera- tion. Some approximate methods of taking the affects of haz- ard dependencies into consideration have been given in Wen (1990). The expected life cycle and optimization can be cal- culated following the procedure given in (1)–(4). It is an ap- proximate but efficient method.

For example, in the two-load problem, if there were occur- rence dependence, i.e., each load occurs clustering around a common reference point in time with a random time delay. The reference time is the occurrence time of the underlying physical mechanism that generates the loads. It has been shown in Wen (1990) that, compared with the case of inde- pendent occurrence, the coincidence rate of the two loads in- creases by a factor of

$$1 + \frac{1}{p(a_1 + a_2)} \qquad [\qquad] \quad (11)$$

1. The proposed life-cycle cost method accounts for the ma- jor factors in rational design decision making. Conven- tional methods, based on deterministic analysis or prob- abilistic analysis alone, may not be justified from long-term benefit versus cost consideration.
2. The optimal design intensity generally increases with lifetime due to longer hazard exposure time. However, the increase is small for structural life longer than 50 years.
3. The optimal design intensity depends heavily on the con- sequence of failure. This dependence can be the single most important factor in design, when the consequence is large. It may overshadow other factors, such as ex- posure time.
4. When designing for multiple hazards, uniform reliability against each hazard or hazard combination is not re- quired in an optimal design. The design is generally dominated by hazards that have large uncertainty. The resultant optimal design may have large disparity in re- liability against different hazards.

The emphasis of this paper has been on methodology devel- opment and parametric and sensitivity study. Applications of the methodology to realistic design against earthquakes and winds are give in Wen and Kang (2001).

## APPENDIX I. DERIVATION OF EXPECTED FAILURECOST AS FUNCTION OF LIFETIME, HAZARD OCCURRENCE RATE, AND DISCOUNT RATE

In (1), the expected summation value of cost of future fail- ure needs to be evaluated considering the random number of occurrences, random occurrence times of the hazards, and the discounting of cost over time.

First consider the case of a single hazard. Since the occur- rence of the loading is modeled by a simple Poisson process, the number of occurrences $N$ is a random variable. Conditional on $N = n$ in (0, $t$), the occurrence times $t_1$, $t_2$, ..., $t_n$ of the

in which p = mean occurrence rate of the generating process; and $a$ and $a$ = mean occurrence hazard are independent and uniformly distributed in (0, $t$). Therefore, in taking the expectation in (1) with respect to the delay times of $S_1(t)$ and $S_2(t)$.

Assuming p = 5/year and $a_1 = a_2 = 10^{-3}$ years (8 h), the

coincidence rate will increase by a factor of about 100. If the

random hazard occurrence times $t_i$ ($i$ = 1, $n$), which are uni-formly distributed between 0 and $t$, one obtains

[O ] O $\int$ $^n$ $^n$ $^t$ $e^{-\lambda\tau} d\tau$

load intensities are also dependent, the conditional probability of limit state will also increase by a large factor. The net effect of these dependencies is that the contribution of the load co- incidence term in (1) would no longer be small and needs tobe considered in the life-cycle cost analysis.

## CONCLUSIONS

$i = 1$ $i = 1$ 0 $t$

$= \frac{n}{t\lambda} [1 - e^{-\lambda t}]$ (12)

The unconditional expected value considering the randomnumber of occurrences is given by

∞ ∞

O— $n$ —$(\nu t)^{n-1}$ —$(\nu t)^n$

Recent large economic losses suffered in hurricanes and

$(1 — e^{—h\,t})$ $e^{—v\,t}$ = $[1 — e^{—h\,t}]$ $n$ $e^{—v\,t}$

earthquakes have made it increasingly evident that rational de- sign decision needs to be based on consideration of long-term benefit versus cost. The minimum, expected life-cycle cost,

$n$ =0 $t$h $n$! $t$h

$= {}^{1}\ [1 — e^{—h\,t}]vt = {}^{v}\ [1 — e^{—h\,t}]$ _ _

$n$ =0$n$!

design criteria are a viable approach to this problem by prop- erly considering the costs of limit states over the system life- time and the uncertainties associated with the demand and ca- pacity. A method is proposed for modeling the uncertainties and evaluation of the expected life-cycle cost of an engineer- ing system under multiple hazards. Initial cost, costs due to multiple limit states under a single or multiple hazards, and cost discounting over time are properly considered. Parametric studies are carried out under one hazard and two hazards, widely different in occurrence frequency, intensity variability. Based on the numerical results, the following conclusions can be drawn:

$t$h h (13)

which can be substituted back into (1) to obtain results shown in (2). Note that for very long lifetime $t$ }. The aforemen- tioned solution v/h and the expected lifetime cost given in (2) is the same as the classical solution obtained by Rosen- blueth (1976).

Next consider the case of multiple hazards. The total ex- pected cost consists of initial cost $C_0$, expected limit state costs, and maintenance cost. The total expected limit state costs can be attributed to the occurrence of individual hazards and joint (simultaneous) occurrences of different hazards. Ac- cording to Wen (1990), the joint occurrences of Poisson pulse

rocess are again a Poisson process with the joint occurrence rate of two hazards given by

$$v_{ij} = v_i v_j (\mu_{d_i} + \mu_{d_j}) \qquad (14)$$

and joint occurrence rate of three hazards given by

$$v_{ijk} = v_i v_j v_k(\mu_{d_i}\mu_{d_j} + \mu_{d_j}\mu_{d_k} + \mu_{d_i}\mu_{d_k}) \qquad (15)$$

Knowing the occurrence rates of individual hazards and joint occurrence of difference hazards, one can obtain the overall expected lifetime limit-state cost by adding the contributions from all occurrences following the same procedure given above. The result is obtained and given in (3) in the text.

## APPENDIX II. REFERENCES

American Institute of Steel Construction (AISC). (1994). *Manual of steel construction—load and resistance factor design*, 2nd Ed., Chicago.

Ang, A. H. S., and Leon, D. (1997). ''Development of target reliability for design and upgrading of structures.'' *Struct. Safety*, Amsterdam, 14(1).

American Petroleum Institute (API) RP2A-LRFD. (1990). *Recommended practice for planning and construction fixed platforms—load and re- sistance factor design*, 1st Ed., Dallas.

ASCE. (1998). ''Minimum design load for buildings and other struc-tures.'' *ASCE 7-98*.

Cornell, C. A. (1996). ''Reliability-based earthquake-resistant design— The future.'' *Proc., Emilio Rosenblueth Symposium, 11th World Con- ference on Earthquake Engineering*, Acapulco, Mexico.

Ellingwood, B. R., Galambos, T. V., MacGregor, J. G., and Cornell, C. A. (1982). ‘‘Probability-based load criteria: Load factors and load com- binations.’’ *J. Struct. Div.*, ASCE, 108(5), 978–997.
Federal Emergency Management Agency. (1992a). *NEHRP handbook for the seismic evaluation of existing buildings*, 178.
Federal Emergency Management Agency. (1992b). *A benefit-cost model for the seismic rehabilitation of buildings*, Vols. 1 and 2, 227, 228.
Frangopol, D., and Corotis, R. B., eds. (1994). ‘‘Reliability-based struc- tural system assessment, design and optimization.’’ *J. Struct. Safety*, 16.
Hiraishi, H., Midorikawa, M., Teshigawara, M., and Gojo, W. (1998). ‘‘Development of performance-based building code in Japan—Frame- work of seismic and structural provisions.’’ *2nd U.S.-Japan Workshop on Performance-Based Engineering*, Applied Technology Council (ATC), San Francisco.
Kanda, J., and Ellingwood, B. (1991). ‘‘Formulation of load factors based on optimal reliability.’’ *J. Struct. Safety*, 9, 197–210.
Kanda, J., and Shah, H. (1997). ‘‘Engineering role in failure cost evalu- ation for buildings.’’ *J. Struct. Safety*, 19, 79–90.
Kulicki, J. M., Mertz, D. R., and Wassef, W. G. (1995). ‘‘Load and re- sistance factor design for highway bridges.’’ *FHWA HI-95-0017*, Wash-ington, D.C.
Liu, S. C., Neghabat, F., and Dougherty, M. R. (1976). ‘‘Optimal aseismic design of building and equipment.’’ *J. Engrg. Mech.*, ASCE, 102(3), 395–414.
Rosenblueth, E. (1976). ‘‘Optimum design for infrequent disturbances.’’ *J. Struct. Div.*, ASCE, 102(9), 1807–1825.
Rosenblueth, E. (1976). ‘‘Towards optimum design through building codes.’’ *J. Struct. Div.*, ASCE, 102(3), 591–607.
Saito, T. (1998). ‘‘Target performance levels in seismic design.’’ *Proc., 2nd U.S.-Japan Workshop on Performance-Based Engineering*, Ap- plied Technology Council (ATC), San Francisco.
SEAOC. (1995). *Vision 2000—Performance based seismic engineering of buildings*, Structural Engineers Association of California, I, II.
*Second (ACT/ BRI) U.S.-Japan Workshop on Performance-Based Engi-neering*. (1998). Applied Technology Council (ATC), San Francisco. Wen, Y. K. (1990). *Structural load modeling and combination for per- formance and safety evaluation*, Elsevier Science, New York.
Wen, Y. K. (1995). ‘‘Building reliability and code calibration.’’ *Earth- quake Spectra*, 11(2), 269–296.
Wen, Y. K., and Foutch, D. A. (1997). ‘‘Proposed statistical and reliability framework for comparing and evaluating predictive models for evalu- ation, design, and critical issues in developing such framework.’’ *SAC/BD-97/03*, SAC Joint Venture.
Wen, Y. K., and Kang, Y. J. (2001). ‘‘Minimum building life-cycle cost design criteria. II: Applications.’’ *J. Struct. Engrg.*, ASCE, 127(3), 338–346.