EFFECT OF SOIL NON LINEARITY, PILE DIAMETER AND PILE SPACING ON RESPONSE OF 3 PILE IN SERIES AND 3 PILES IN PARALLEL ARRANGEMENT

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ABSTRACT:

The analysis of laterally loaded pile group is complex three-dimensional soil structure interaction problem. The response in terms of lateral displacement and bending moment depends on many factors including pile shape and pile spacing. The non linear behavior of soil needs to be considered. The effect of these parameters is considered in present study. Circular and square piles are considered with spacing from 2 times the diameter to 7 times the diameter. The group of piles with 3 piles in series arrangement and 3 piles in parallel arrangement are used in the study. Complete code is developed in FORTRAN 90 to carry out three-dimensional finite element analysis. Pile is modeled using 20 node elements and soil is modeled using 8 node elements. Soil-pile interface is modeled by 16 node elements of zero thickness. The pile is assumed to remain elastic throughout the analysis. The soil behavior is modelled using basic Von Mises yield criterion. The plastic behavior of soil is considered and associative flow rule is considered. The analysis output clearly indicates the influence of soil non linearity, pile shape and pile spacing on response of laterally loaded pile group.

Keywords –
pile group, pile spacing, Von Mises yield criterion, non-linearity.

INTRODUCTION:

The vertical pile or pile group subjected to lateral load can be analyzed by various methods. They are elastic continuum approach, elastic subgrade reaction approach, p-y curve approach and finite element method. Poulus [1] presented the solutions of displacement and rotation at the ground surface in terms of dimensionless influence factors using elastic continuum approach. Both the free head as well as the fixed head piles were considered in the analyses. Pise [2, 3] presented lateral response of fixed-head and free head pile respectively in two-layer soil system by treating soil as an elastic continuum. Hsiung and Ya-ling Chen [4] developed a simplified method for the analysis and design of long piles under lateral load in uniform clays. The method was based on the concept of the coefficient of subgrade reaction with consideration of the soil properties being extended to include elasto-plastic behaviour. Zhang [5] developed a method for nonlinear analysis of laterally loaded rigid piles in cohesionless soil. The method assumes that both the ultimate soil resistance and the modulus of horizontal subgrade reaction increase linearly with depth. By considering the force and moment equilibrium, the system equations are derived for a rigid pile under a lateral eccentric load. The degradation of the modulus of horizontal subgrade reaction with pile displacement at ground surface is also considered.

Different procedures were suggested to construct *p-y* curves for various soil types. Matlock [6] developed such procedures for soft clays, Reese et al. [7] for sand and Reese et al. [8] for stiff clays. All of them were based on data from field load tests on instrumented piles. Later on, Sullivan et al. [9] presented a unified method, which could be used for both soft and stiff clays. This method has the ability of treating the piles of any geometry, stiffness and head fixities and is much suited to analyze a single pile but not well adapted for group of piles. Al-Obaid [10] presented an automated analysis of laterally loaded piles using subgrade reaction theory and the *p*-*y* curves governing the soil properties. The finite difference method is applied in establishing the governing equations. Tahghighi and Konagi [11] proposed a convenient method to study the soil-pile-structure interaction using nonlinear Winkler foundation model. The results obtained from this Winkler model were evaluated against two sets of physical experimental data. Zhu et al. [12] tested four large diameter concrete piles. Piles were 15 m long with one meter diameter. Piles were instrumented to measure displacement and bending moment. Bending moments predicted from other Winkler models were compared with measured response. Numerical study was performed using FEM. Predicted response using FEM were in better agreement as compared to Winkler approach. The general solution of governing differential equation of beams on elastic foundation was adapted by Chatterjee et al. [13] to model displacement variation within the element. Stiffness matrix of pile element was derived for assumed displacement variation in FE formulation. Proposed a FE model was applied to single pile for predicting pile response. Mukherjee and Dey [14] analyzed the fixed headed single pile subjected to lateral load in layered soil using *p*-*y* approach. Layered soil was considered as the alternate layer of clay and sand.

The elasticity approach [15], which takes into account the soil non-linearity, is well established. But in this approach, soil modulus is assumed to be elastic and does not account for soil yielding. An elastic subgrade reaction solution can be used for small working loads but the pile geometry can be considered only indirectly. Although non linear p-y curve method is widely used due to its simplicity, the accuracy depends upon the correctness of the spatially distributed properties of soil in long piles. The finite element method is rigorous method which can account for soil continuity, soil nonlinearity, pile-soil interaction and 3 D boundary conditions.

METHODOLOY:

Finite Element Model:

A complete 3 D finite element program is developed in FORTRAN 90 to model the soil pile system. The pile and pile cap is modeled using 20 node isoperimetric continuum elements as these elements are suitable for material with bending dominated deformation. Eight node elements, which are suitable to model medium with shear dominated deformation, are used to model the soil. The interface between pile and pile cap and soil is modeled using 16 node element with zero thickness, to stimulate the stress transfer. The stiffnesses of this element in normal and tangential direction are assigned to permit the relative vertical displacement but to prevent horizontal displacement between pile and soil [16].

Von-Mises Yield Criterion:

In the present study, Von-Mises yield criterion is used to model the soil behavior. It is elastic perfectly plastic model. It is an improvement over Tresca yield criterion which plots a regular hexagon with corners in deviatoric plane. In finite element formulation, additional efforts are required to deal with singularities and numerical ill conditioning caused by corners in three-dimensional analysis. Von–Mises yield criterion, when represented in principal stress space, plots a circular cylinder with central axis coinciding with space diagonal, as shown in Figure 1. It can be represented in terms of stress invariants, effective stress, ρ', deviatoric stress, *J*, and Lade's angle, Θ. They are related to principal stresses as below [17]

$$p' = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad J = \frac{1}{\sqrt{6}}\sqrt{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2}$$

$$\text{and } \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_2 - \sigma_3)}{(\sigma_1 - \sigma_3)} - 1\right)\right) \tag{1}$$

Von-Mises criterion is represented as,

$$J - \alpha = 0 \tag{2}$$

where, α is material parameter representing the shear strength of soil [17].Throughout the analysis, pile is assumed to behave elastically and the soil is modeled using Von-Mises criterion which ensures elasto-plastic behavior. The associate flow rule (yield function and plastic potential function is same) is used which yields symmetrical constitutive matrix and stiffness matrix. It dramatically improves efficiency of computation and storage space.

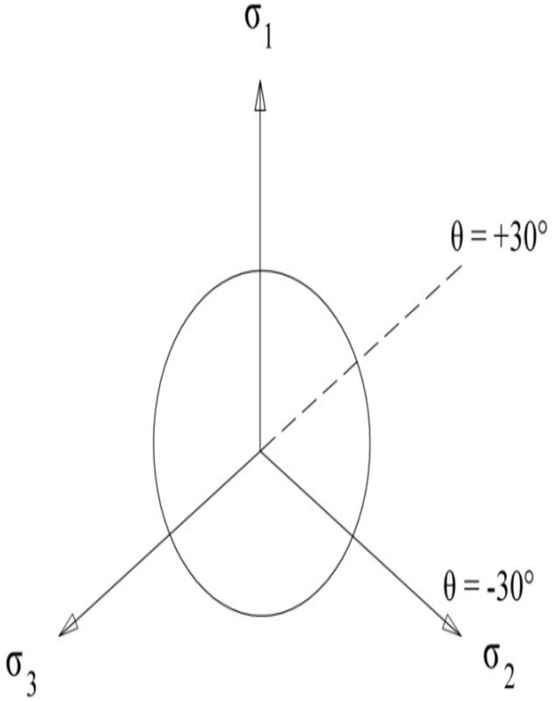


Figure 1: Representation of Von-Mises Yield Criteria in deviatoric plane

ELASTO-PLASTIC CONSTITUTIVE MATRIX:

In elasto-plastic state, incremental stresses, $\{\Delta\sigma\}$ and incremental strains $\{\Delta\varepsilon\}$ are related with each other as given in equation below.

$$\{\Delta\sigma\} = [D]_{ep}\{\Delta\varepsilon\} \tag{3}$$

where, $[D_{ep}]$ is elasto-plastic constitutive matrix.

$$[D]_{ep} = [D] - \frac{[D]\left\{\frac{\partial p}{\partial\sigma}\right\}\left\{\frac{\partial F}{\partial\sigma}\right\}^T[D]}{A + \left\{\frac{\partial F}{\partial\sigma}\right\}^T[D]\left\{\frac{\partial p}{\partial\sigma}\right\}} \tag{4}$$

Where, [D] = elastic constitutive matrix, $\{\partial p/\partial\sigma\}$ and $\{\partial F/\partial\sigma\}$ are plastic potential function and yield function respectively. For material with perfect plasticity, $A = 0$. [17]

Implementation of Yield Criterion:

Using chain rule, the flow vector, a, can be written as:

$$a = \frac{\partial F}{\partial p'}\frac{\partial p'}{\partial\sigma} + \frac{\partial F}{\partial J}\frac{\partial J}{\partial\sigma} + \frac{\partial F}{\partial\theta}\frac{\partial\theta}{\partial\sigma} \tag{5}$$

It can also be expressed as explained by Nayak and Zienkiwicz [18]:and and Viladkar et al. [19].

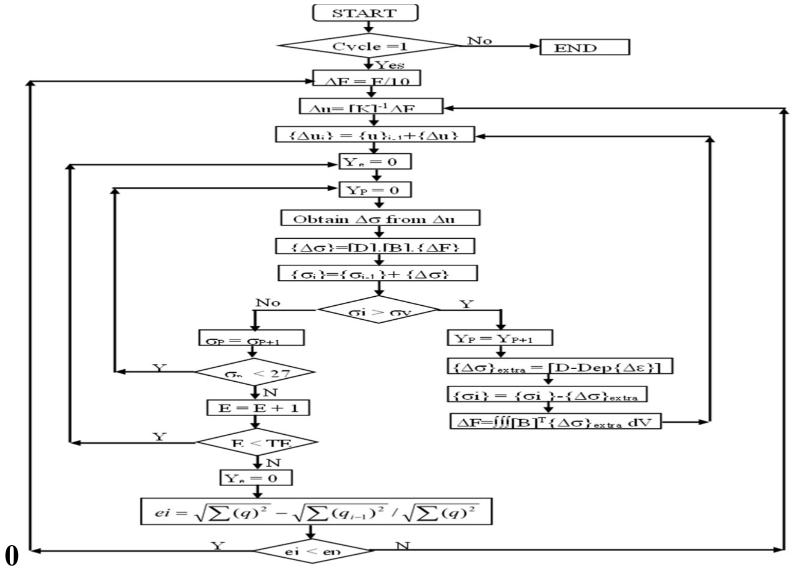
$$a = C_1 a_1 + C_2 a_2 + C_3 a_3$$

where, $C_1 = \dfrac{\partial F}{\partial p'}; C_2 = \dfrac{\partial F}{\partial J}; C_3 = \dfrac{\partial F}{\partial \theta}$ and $a_1 = \dfrac{\partial p'}{\partial \sigma}; a_2 = \dfrac{\partial J}{\partial \sigma}; a_3 = \dfrac{\partial \theta}{\partial \sigma}$ (6)

$$a_1 = \frac{\partial p'}{\partial \sigma} = \frac{1}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}; \quad a_2 = \frac{\partial J}{\partial \sigma} = \frac{1}{2J}\begin{Bmatrix} \sigma_x - p' \\ \sigma_y - p' \\ \sigma_z - p' \\ 2\tau_{xy} \\ 2\tau_{yz} \\ 2\tau_{zx} \end{Bmatrix}; \quad a_3 = \frac{\partial \theta}{\partial \sigma} = \frac{\sqrt{3}}{2J^3 \cos 3\theta}\left(3\frac{J_3}{J}\frac{\partial J}{\partial \sigma} - \frac{\partial J_3}{\partial \sigma} \right) \tag{7}$$

where, $J_3 = \begin{vmatrix} \sigma_x - p' & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y - p' & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z - p' \end{vmatrix}$

FLOW CHART FOR FEM ANALYSIS:



VALIDATION:

Results obtained by Ismael and Klym (1978) during pile load test are back calculated using present program and are presented in non dimensional form of P/CuDL versus δ/D. (P = applied lateral load, Cu = cohesion of soil, D = diameter of pile, L = length of piles, δ = lateral displacement of pile). The pile load test was conducted on a pile of length 12 meters with 0.3 meters overhang above the ground level. The diameter and flexural rigidity of the pile was 1.52 meters and 2.675 x 10^6 kNm^2. It was embedded in clay with undrained cohesion of 96.0 kPa. The comparison between the results shows a good agreement (Figure 2).

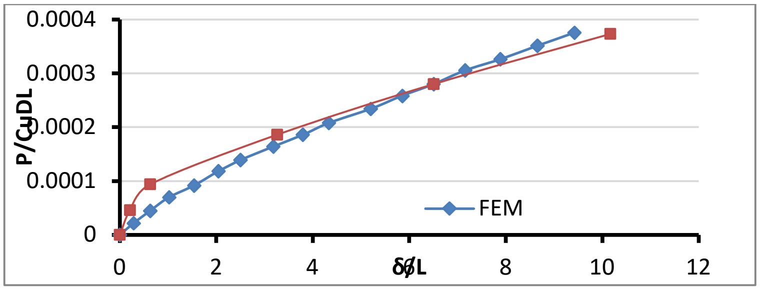


Figure 2: Validation with experimental results (Ismael and Klym 1978)

PARAMETRIC STUDY:

Two pile configurations i.e. 3 piles in series (direction of load parallel to line joining centers of three piles) and 3 pile in parallel (direction of load perpendicular to line joining centers of three piles) are used for parametric study. Each pile dimension, D is 1meter (D = diameter in circular piles and width in square piles) with 20 meters length. Properties of reinforced concrete and soil are as mentioned in Table 1. The lateral load of 3000 kN is applied in ten equal load steps. Spacing between the piles is varied from 1D to 7D. Lateral displacement at the center of gravity of pile group and maximum bending moment amongst all piles are calculated at each load level. The results are depicted in Figure 3 to Figure 6 for 3 piles in series and in Figure 7 to Figure 10 for 3 piles in parallel. The lateral displacement and maximum bending moment at highest load (3000 kN) for different spacing are presented in Table 2 for 3 piles in series and Table 3 for 3 piles in parallel. The results of the linear analysis are also presented for comparison.

Table 1 Properties of pile and soil for parametric study

Properties of soil		Properties of pile		Properties of interface element	
Elasticity Modulus, E_s	20000 kPa	Elasticity Modulus, E_p	25 GPa	Normal stiffness, K_n	1.0×10^6 kN/m^3
Unit Weight, γ_s	18 kN/m^3	Pile diameter, D	1.0 m	Tangential stiffness, K_s	1000 kN/m^3
Poisson's ratio, μ_s	0.35	Poisson's ratio, μ_p	0.25	-	-
Yield stress, σ_y	100 kPa	L/D ratio	20	-	-
Cohesion, c	100 kPa	Unit Weight, γ_p	25 kN/m^3	-	-
Angle of internal friction, ϕ	0	s/D ratio	2, 3, 4, 5, 6 and 7	-	-
-	-	Thickness of Pile cap, t_p	0.5 m	-	-

Table 2 Results for 3 piles in series

Pile spacing (s/D)	Circular Piles				Rectangular Piles			
	Max. displacement (mm)		Max. Moment (kNm)		Max. displacement (mm)		Max. Moment (kNm)	
	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear
2	25.024	159.81	970.4	3549.6	22.018	112.44	700.5	2470.11

3	21.566	104.99	829	2894.63	18.956	74.319	589.2	1984.74
4	19.632	78.974	765.5	2511.7	17.279	53.86	545.3	1631.44
5	18.37	64.09	733.7	2236.92	16.241	43.256	529.4	1496.3
6	17.439	55.004	714.6	2025.98	15.515	37.238	524	1354.83
7	16.694	48.661	700.6	1911.32	14.956	33.372	522.1	1237.81

Table 3 Results for 3 piles in parallel

Pile spacing (s/D)	Circular Piles				Rectangular Piles			
	Max. displacement (mm)		Max. Moment (kNm)		Max. displacement (mm)		Max. Moment (kNm)	
	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear
2	26.993	133.9	1500.3	4898.48	23.663	95.409	1099.4	3250.87
3	23.8	94.35	1376.7	3984.98	20.799	65.086	1012.6	2677.26
4	21.705	73.364	1285.9	3388.6	18.942	50.032	950.7	2254.88
5	20.222	60.343	1211.4	2938.04	17.668	40.613	900.7	1902.78
6	19.136	50.069	1146.1	2520.12	16.765	34.039	856.6	1626.02
7	18.327	42.383	1086.6	2187.19	16.119	30.296	815.9	1457.02

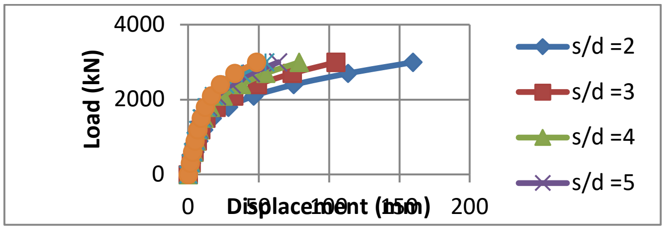


Figure 3: Load - displacement for 3 circular piles in series

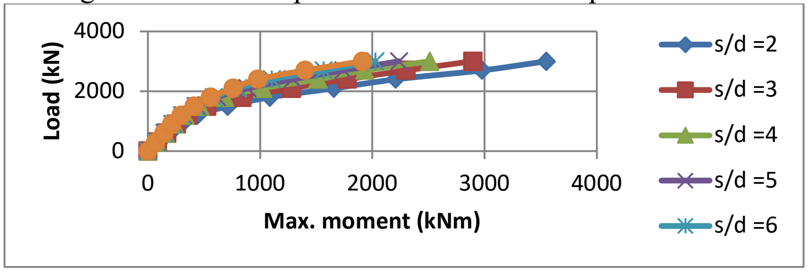


Figure 4: Load - maximum moment for 3 circular piles in series

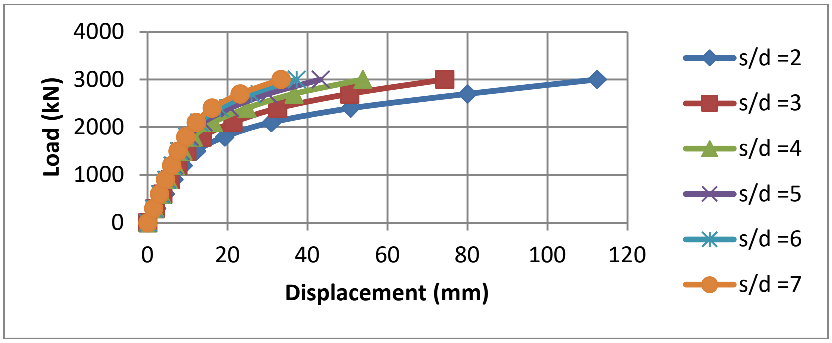


Figure 5: Load - displacement for 3 square piles in series

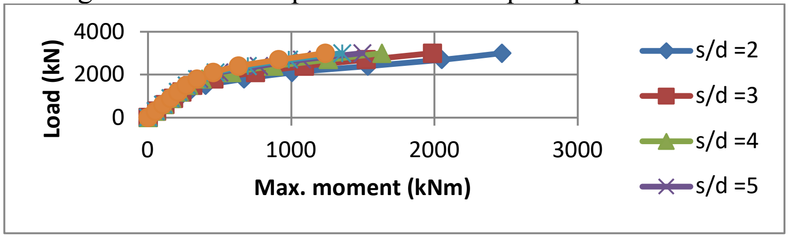


Figure 6: Load - maximum moment for 3 square piles in series

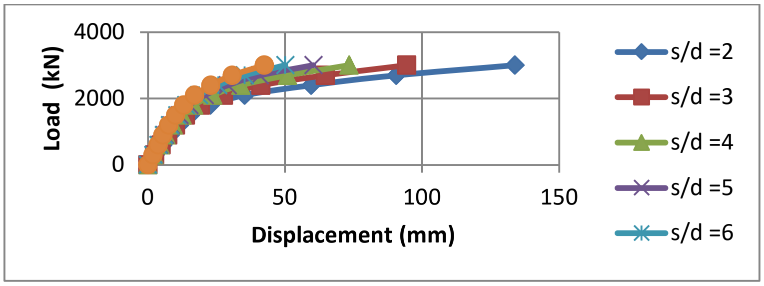


Figure 7: Load - displacement for 3 circular piles in parallel

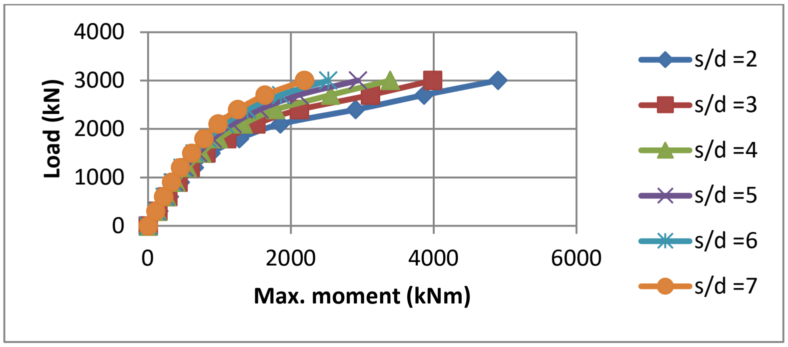


Figure 8: Load - maximum moment for 3 circular piles in parallel

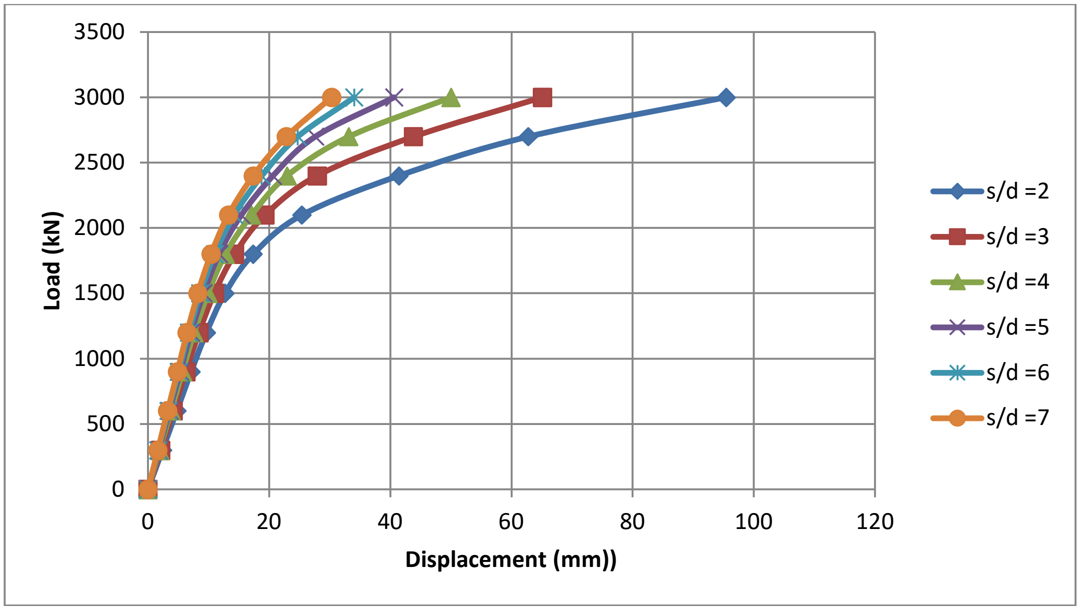


Figure 9: Load - displacement for 3 square piles in parallel

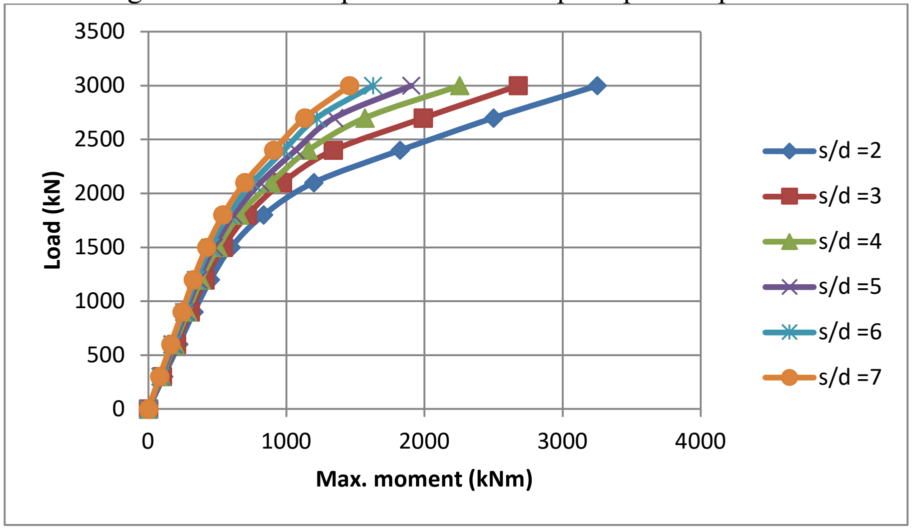


Figure 10: Load - maximum moment for 3 square piles in parallel

RESULTS AND DISCUSSION :

A large increase in displacement and maximum bending moment is found in non linear analysis as compared to linear analysis. After yielding, the soil undergoes a large deformation due to plasticity characteristics. The maximum increase in displacement in 3 circular piles in series is 538.62 % (spacing = 2D) and minimum increase is 191.49 % (spacing = 7D). The maximum increase in displacement in 3 square piles in series is 410.67 % (spacing = 2D) and minimum increase is 123.13 % (spacing = 7D). It is clearly seen that the effect of non linearity decreases with increase in spacing. This is because with increase in spacing the overlapping of stresses reduced which in turn means lower range of non linearity. Same trend is observed in 3 piles in parallel arrangement for circular and square piles. The maximum increase in maximum bending moment due to non linearity in 3 circular piles in series is 265.78 % (spacing = 2D) and minimum increase is 172.82 % (spacing = 7D). The maximum increase in maximum bending moment in 3 square piles in series is 252.62 % (spacing = 2D) and minimum increase is 137.08 % (spacing = 7D). Same trend is observed in 3 piles in parallel arrangement for circular and square piles.

The lateral displacement of circular piles is higher by 30.89 % to 48.58 % than in square piles. Similarly, maximum bending moment in circular piles is higher by 43.7 % to 54.99 % than the square piles. This may be attributed to higher moment of inertia of square piles than circular piles.

The comparison of 3 piles in series and 3 pile in parallel shows higher displacements in 3 pile in parallel for linear analysis. However, for non linear analysis displacements are more in 3 piles in series arrangement. In three piles in parallel arrangement, the load is applied perpendicular to length of pile cap whereas in other arrangement it is parallel. Due to this, more moment of inertia of pile cap is available in the direction of load in 3 piles in series arrangement. This reduces the displacement. This fact is found dominating in linear range. However, at higher loads, the shadowing effect in series arrangement is dominant. This causes increase in stress in piles which in turn increases the displacement. The shadowing effect in series arrangement is far more than the edge effect in parallel arrangement. Also, more soil is mobilized in three piles in parallel arrangement. Hence, in non linear range, the displacements in 3 piles in series arrangement is more.

CONCLUSIONS:

From the analysis of 3 piles in series arrangement and 3 piles in parallel arrangement subjected to lateral load of 3000 kN, following conclusions are made

1. The lateral displacements and bending moments predicted by nonlinear analysis are higher as compared to linear analysis at all load levels. The maximum increase in displacement observed is 538.62 % in 3 circular piles in series at a spacing of 2D whereas the maximum increase in maximum bending moment is 265.78 % for the same case.
2. In both cases i.e. 3 piles in series and 3 piles in parallel, the lateral displacements and bending moments observed in circular piles are higher at compared to square piles at all load levels. At maximum load, the lateral displacement of circular piles is higher by 30.89 % to 48.58 % than in square piles. Similarly, maximum bending moment in circular piles is higher by 43.7 % to 54.99 % than the square piles.
3. The lateral displacements and bending moments are higher in 3 piles in parallel for linear analysis and 3 piles in series for non linear analysis at all load levels.
4. The effect of non linearity reduces with increase in pile spacing in 3 piles in series arrangement as well as 3 piles in parallel arrangement.

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