Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 2, No. 3, February : 2024

VIBRATION ANALYSIS OF PIEZOELECTRIC SMART BAR AND CANTILEVER BEAM STRUCTURE

Amit Kumar singh Assistant Professor,Department of Mechanical Engineering, Rashtrakavi Ramdhari Singh Dinkar College of Engineering, Begusarai, Bihar, India-851134
E-Mails: amitsingh051@gmail.com,
Shashi Kumar Assistant Professor, Department of Civil Engineering, Sitamarhi Institute of Technology, Sitamarhi, Bihar, India-843302,shashi351@gmail.com
Sanjeet Kumar Assistant Professor,Department of Civil Engineering, Rashtrakavi Ramdhari Singh Dinkar College of Engineering, Begusarai, Bihar, India-851134 sanjeetmit08@gmail.com;
Lakshmi Kant Assistant Professor,Department of Civil Engineering, Rashtrakavi Ramdhari Singh Dinkar College of Engineering, Begusarai, Bihar, India-851134 lakshmikantseth@gmail.com;

Abstract
This article presents an active vibration control technique applied to a smart bar and beam. The beams that are used in the craft vessels in more numbers are the frequent victims of the vibrations. The vibrations tend to make the beam deformation – very risky for lives if they are left as simple. The paper provides the deformation of the cantilever beam using the Finite Element Method which makes the approach so efficient. The smart beam consists of an aluminum beam modeled in a cantilevered configuration with surface-bonded piezoelectric (PZT) patches. The effectiveness of piezoelectric material in the development of new actuators as elements of smart structures has been theoretically investigated. A simple cantilever beam bonded with a piezoelectric layer is analyzed for vibration damping. Simple finite element modeling is done with two nodded bar and beam elements. The governing equation is obtained by Hamilton's principle. Vibration is controlled by changing the gain of the controller to provide the required voltage to the piezoelectric actuator. The controlled response is illustrated through plots of time response and frequency functions. The results indicate that this new piezoelectric material may be a superior material for use in developing lightweight smart structures.
Keywords: Cantilever beam, Vibration analysis, piezoelectric material, Finite Element Method

Introduction:

Aerospace, one of the emerging fields is affected greatly because of some unpreventable vibrations. Crude vibrations are generated at the time of craft operations. Many engineering applications use structures that can be considered to be flexible. Flexible structures are distributed parameter systems. Therefore, the vibration of each point is dynamically related to the vibrations of every other point over the structure. It is important to design a controller to minimize structural vibrations of the entire structure, rather than a limited number of points. This would ensure that structural vibrations of the entire structure are suppressed.The developments in piezoelectric materials have a very important role for many researchers working in the field of smart structures. A smart structure can be defined as "A system or material which has a built-in or intrinsic sensor(s), actuator(s) and control mechanism(s) whereby it is capable of sensing a stimulus, responding to it in a predetermined manner and extent, in a short/ appropriate time, and reverting to its original state as soon as the stimulus is removed". Smart structures consist of highly distributed active devices which are primarily sensors and actuators either embedded or attached to an existing passive structure with integrated processor networks. Therefore, our work mainly considers the application of Piezoelectric crystals (PZT)patches to smart beam-like and smart plate-like structures for active vibration control.
The motivation for this work stems from the possibility of using induced strain actuation for vibration suppression, stability augmentation, and noise reduction in beam-like aerodynamic surfaces. These beams are used in such applications as helicopter and airplane wings, turbo-machine

blades, missiles, space structures, and civil structures. Several theories apply to the control of vibration. The best way to optimize a single mode, as proposed in this research, is to optimize the performance metric corresponding to the mode of interest. This methodology is ideal for the design of low-order controllers. A smart structure involves distributed actuators and sensors along the structure and some type of processor that can analyze the response from the sensor and use control theory to output commands to the actuator. The actuator applies local stresses/strains to alter the behavior of the system. Therefore, a smart structure has four major components: the structure, sensor, actuator, and controller. Actuators and sensors are widely used in various applications and are generally integrated with main structures via surface bonding or embedding.

The advantage of incorporating these special types of materials into the structure is that the sensing and actuating mechanism becomes part of the structure by sensing and actuating directly Song et al. (2002).Baillargeon (2003) used piezoelectric materials as distributed sensors and actuators to attain a great deal of importance in active control of vibrations of beams, plates, and shells and achieved a significant amount of active damping in smart structures.Waghulde and Kumar (2011) explained the application of Piezoelectric crystals (PZT) ceramics for reconstruction filters and used a single actuator to suppress vibrations by using acceleration feedback controllers.Waghulde and Kumar (2012) studied the surface bonding for piezoelectric actuators that there is better access for fabrication, easier access for inspection, and less maintenance cost.Bailey and Hubbard (1985) presented engineering applications technology for activevibration suppression in the experimental study.Crawley and de Luis (1987) studied adaptive structures using piezoelectric materials usually employing lead zirconium titanate (PZT) ceramic sensors and actuators to detect and mechanically deform a structure.Ray and Mallik (2005)obtained exact solutions of simply supported rectangular piezoelectric laminates for the static generalized plane strain deformation of the piezoelectric plates.These devices are possibly subjected to relatively large impact forces which can be outside their designed specifications when incorporated into vehicles and portable products. Due to the generation of relatively large forces at points of contact for relatively short periods, the impact is characterized by shock and vibration (Patil and Kumar (2009).Schoeftner and Irschik (2011) carried out an eigenfrequency analysis on beams to study flexural vibrations with various mechanical boundary conditions.Kandasamy et al. (2011) proposed the active vibration control of isotropic and laminated composite box-type structures under a thermal environment and investigated the effect of temperature and control gain on natural frequency and damping of the system for different boundary conditions by using active feedback control.

This paper presents the dynamic analysis of the cantilever beam and bar with vibration control of the smart cantilever beam and bar by using a finite element approach. In this study,
finite element formulationof bar and beam are done forlongitudinal vibration and transverse vibration, respectively formulation is done for both longitudinal and transverse vibration. Finite element analysis results are compared with theoretical for deflection and frequency of vibration.

2. Finite Element Method and methodology

For determining inexact solutions of partial differential equations (PDE) and of integral equations, the finite element method (FEM) is a finite element analysis numerical technique. The solution approach is based either on eradicating the differential equation (steady state problems), or rendering the PDE into an approximating system of ordinary differential equations, which are subsequently solved using standard techniques for instance Euler's method, Runge- Kutta. A profound survey was held on finite element modeling and the advancements in its formulations and applications for the finite element modeling of adaptive structural elements namely,solids, shells, plates, and beams. Moreover, the model was also applied to the optimal design ofpiezoelectric actuators.The cantilever beam bonded with the piezoelectric surface is induced by an electric field as shown in Fig. 1. Properties of the aluminum bar or beam and piezoelectric material (PZT) are shown in T**able 1.**

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 2, No. 3, February : 2024

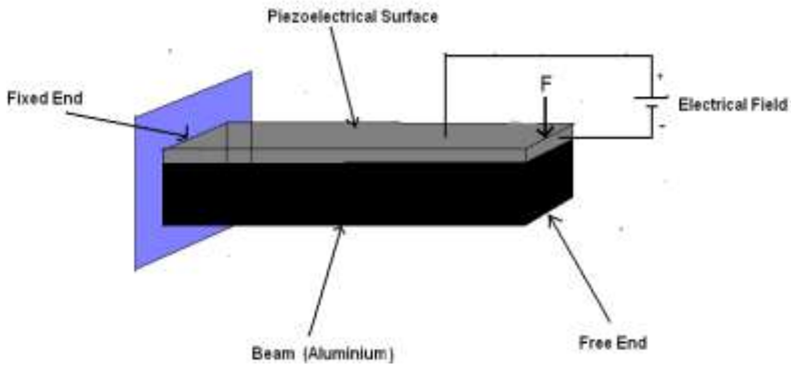

Fig. 1: Cantilever beam bonded with piezoelectric surface and induced by an electric field

Table 1. Properties of aluminum bar or beam and piezoelectric material (PZT).

Bar and Beam (aluminum)	Values
Mass density, ρ (kg/m^3)	2700
Young's modulus, C (Gpa)	69
Length of the structure, l(m)	0.3
Width of the structure, w(m)	0.03
Height of the structure, H(m)	0.003
Relaxation modulus	30
Piezoelectric material (PZT)	
Mass density, ρ_P (kg/m^3)	7500
Young's modulus, C_P (Gpa)	61
Length of the structure, l_P(m)	0.3
Width of the structure, w_P(m)	0.03
Height of the structure, h_P(m)	$2.5*10^{-6}$
Electrical constant, e (coulomb/m^2)	-3.1

3. Results and Discussion:

Numerical results are analyzed by an aluminum bar or beam on the top surface piezoelectric material patch (PZT) and applying an electrical field with the help of an electrode.

3.1 Calculation:

For Bar element

Case I: - static analysis for bar

By strength of material approach, we know that longitudinal displacement is

$Q = \frac{F_m l}{CA} = \frac{I*0.3}{68.2*10^9*9.0075*10^{-5}} = 4.8835*10^{-8}$ m

By the finite element method do not supply any voltage that is G=0.

$Q = K^{-1} F_m = 4.8273*10^{-8}$

Case II: - dynamic analysis for bar

By the strength of the material approach, we know that the longitudinal frequency of a cantilever bar in which one end is fixed and another end is free where the axial load is applied is $f_r = \frac{n\Pi}{2l}\sqrt{\frac{C}{\rho}}$
where n= 1,2, 3...
By the finite element method do not supply any voltage that is G=0. And we use the equation of motion that is,

$$M\ \ddot{Q} + (f_v^s - f_v^a)\dot{Q} + QK = F_m$$

$$S^2MQ(S) + S(f_v^s - f_v^a)Q(S) + KQ(S) = F_m$$

$$T.F = \frac{OUTPUT}{INPUT} = Q(S) = \frac{F_m}{MS^2 + (f_v^s - f_v^a)S + K}$$

Transfer function=

4.468e067 s^6 + 2.345e078 s^4 + 2.836e088 s^2 + 6.433e097

5.076e065 s^8 + 3.305e076 s^6 + 5.721e086 s^4 + 2.546e096 s^2 + 1.333e105
and to find out the Longitudinal frequency for different mode shapes we use and get results.

Case I: - static analysis

Table 2. Resultsof SOM approach and FEM approach for the longitudinal displacement of the bar

Bar element	Strength of material (SOM) approach	Finite element method approach			
		Single element	Double element	Triple element	Four elements
Longitudinal Displacement(m)	$4.8835*10^{-8}$	$4.8273*10^{-8}$	$4.8273*10^{-8}$	$4.8273*10^{-8}$	$4.8273*10^{-8}$

Case II: - dynamic analysis

Table 3. Resultsof SOM approach and FEM approach for longitudinal frequency of bar for different modes.

Longitudinal Frequency(rad/sec)	Strength of material approach	Finite element method approach			
		Single element	Double element	Triple element	Four elements
I mode	24500	29170	27100	27000	24000
II mode	49000	55632	57000	56006	50000
III mode	73500	92000	94000	95000	75600

Fig. 2 shows thetime response curve of the bar without structural damping for different values of G. It is observed that amplitude decreases with increasing time response for the bar without structural damping. Fig. 3 showsthe frequency response curve of the bar without structural damping for different values of G. Resultsof the SOM approach and FEM approach is illustrated in Table 2 and Table 3 for the longitudinal displacement of the bar and longitudinal frequency of bar for different modes respectively. The time response curve and frequency response curve of the bar with structural dampingfor different values of G are shown in Fig. 4 and Fig. 5 respectively.

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 2, No. 3, February : 2024

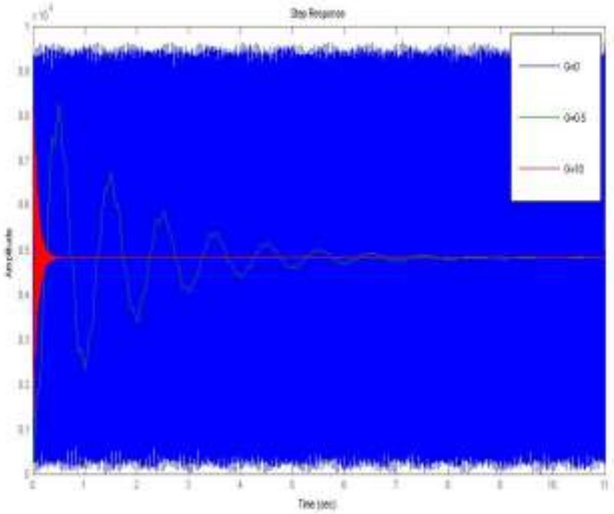

Fig. 2: Time response curve of bar without structural damping for different values of G.

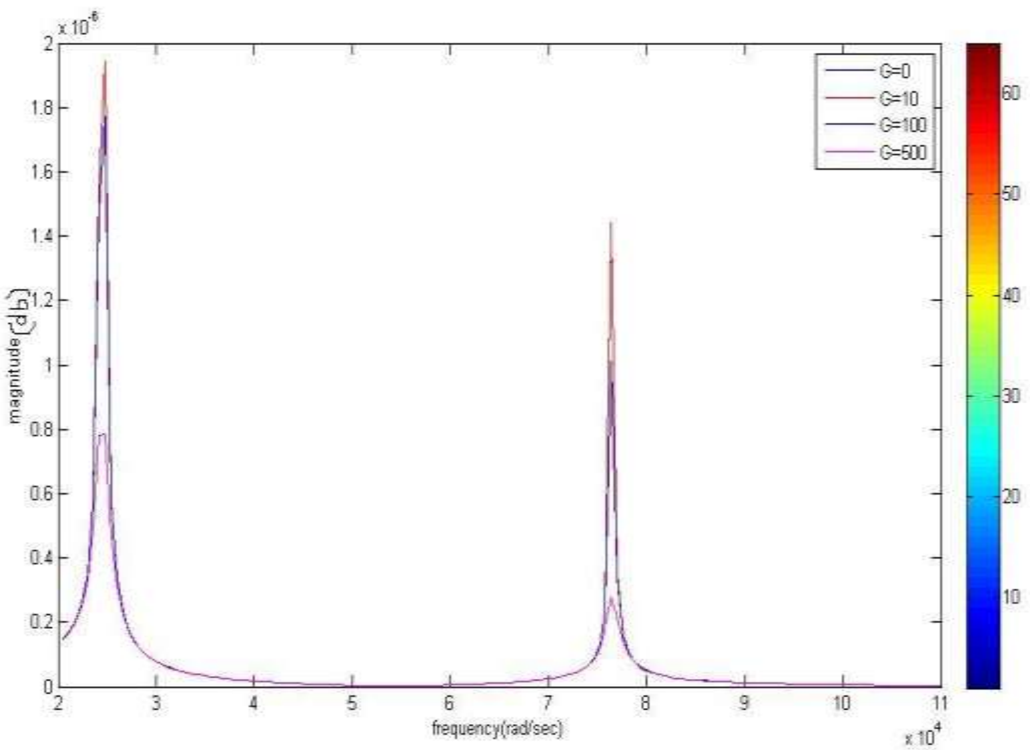

Fig. 3: Frequency response curve of bar without structural damping for different values of G.

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 2, No. 3, February : 2024

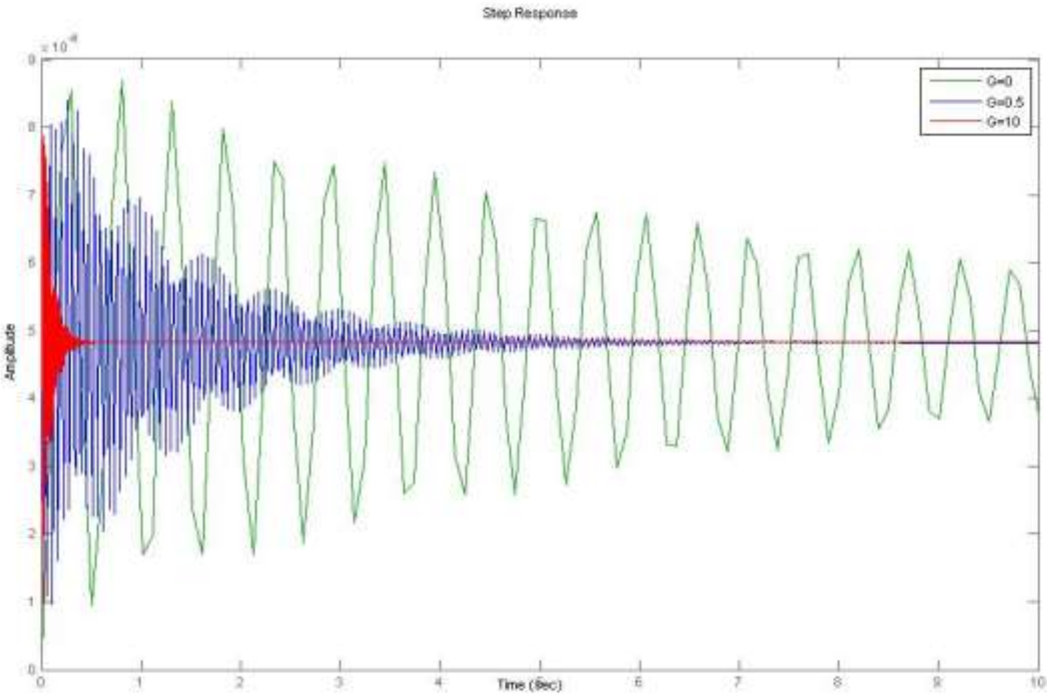

Fig. 4: Time response curve of bar with structural damping for different values of G.

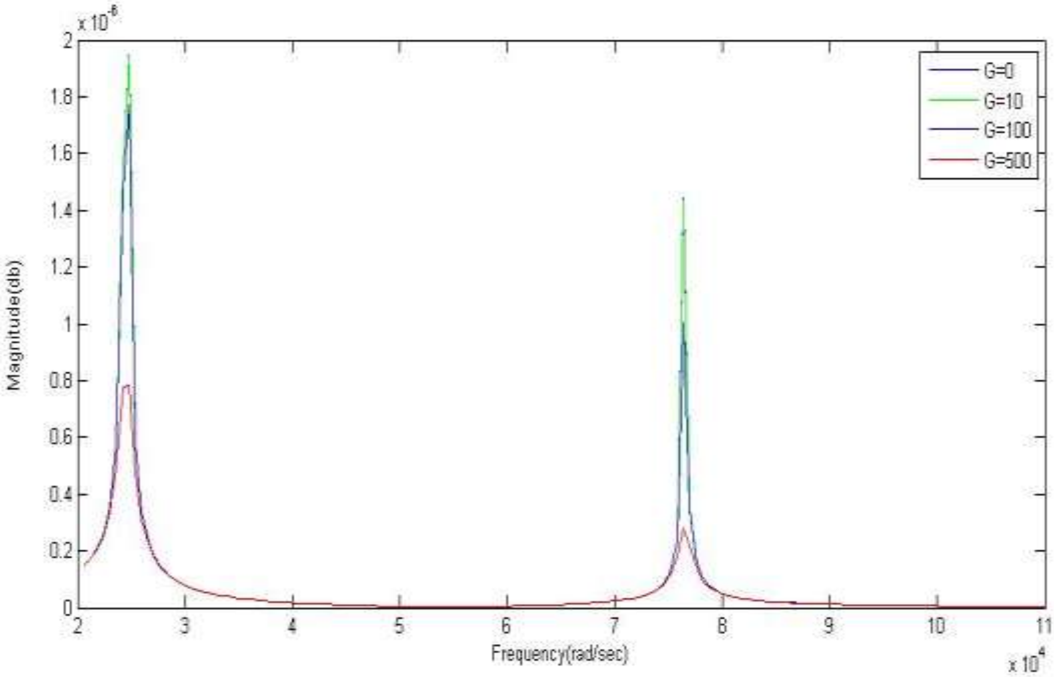

Fig. 5: Frequency response curve of the bar with structural damping for different values of G.

<u>For Beam element</u>: -

Case I: - static analysis for beam

By the strength of the material approach, we know that the deflection of a cantilever beam at the free end is

$y = \frac{F_m l^3}{3CI}$ = 0.001927 m

By the finite element method do not supply any voltage that is G=0 and find out the deflection

$Q = K^{-1} F_m$ =0.001927 m

Case II: - dynamic analysis for beam

By the strength of the material approach, we know that the transverse frequency of a cantilever beam which one end is fixed and another end is free where transverse load is applied is

$\cosh(cl) * \cos(cl) + 1 = 0$ Where c is the transverse frequency of the beam.

By the finite element method do not supply any voltage that is G=0. And we use the equation of motion that is,

$M\ddot{Q} + F_p\dot{Q} + QK = F_m$

$S^2MQ(S) + SF_pQ(S) + KQ(S) = F_m$

$T.F = \frac{OUTPUT}{INPUT} = Q(S) = \frac{F_m}{MS^2 + F_pS + K}$

Transfer function=

5.277e128 s^14 + 1.25e138 s^12 + 8.746e146 s^10 + 2.116e155 s^8 + 1.72e163 s^6 + 3.979e170 s^4 + 1.974e177 s^2 + 9.888e182

6.961e125 s^16 + 2.391e135 s^14 + 2.235e144 s^12 + 7.055e152 s^10 + 7.542e160 s^8 + 2.377e168 s^6 + 1.793e175 s^4 + 1.805e181 s^2 + 5.129e185

and for finding out the transverse frequency for different mode shapes we use Matlab and get a result.

Case I: - static analysis

Table 4. Resultsof SOM approach and FEM approach for transverse deflection of beam.

Beam element	Strength of material approach	Finite element method approach			
		Single element	Double element	Triple element	Four elements
Transverse Deflection(m)	0.001927	0.001927	0.001927	0.001927	0.001927

Case II: - dynamic analysis

Table 5. Resultsof SOM approach and FEM approach for transverse frequency of beamfor different modes.

Transverse Frequency(rad/sec)	Strength of material approach	Finite element method approach			
		Single element	Double element	Triple element	Four elements
I mode	169	171	180	170	170
II mode	609	980	720	620	610
III mode	1048	1650	1100	1050	1050

Now for different values of gain in dynamic case, we plot the time response and frequency response for bar and beam. Here we consider two cases for bar without structural damping and with structural damping. The time response curve of the cantilever beam is shown in Fig. 6for different values of G.Frequency response curve of the cantilever beam is shown in Fig. 7 for different values of G. Resultsof SOM approach and FEM approach are illustrated in Table 4 and the Table 5 for a longitudinal displacement of beam and longitudinal frequency of beam for different mode respectively.

Industrial Engineering Journal
ISSN: 0970-2555
Volume : 53, Issue 2, No. 3, February : 2024

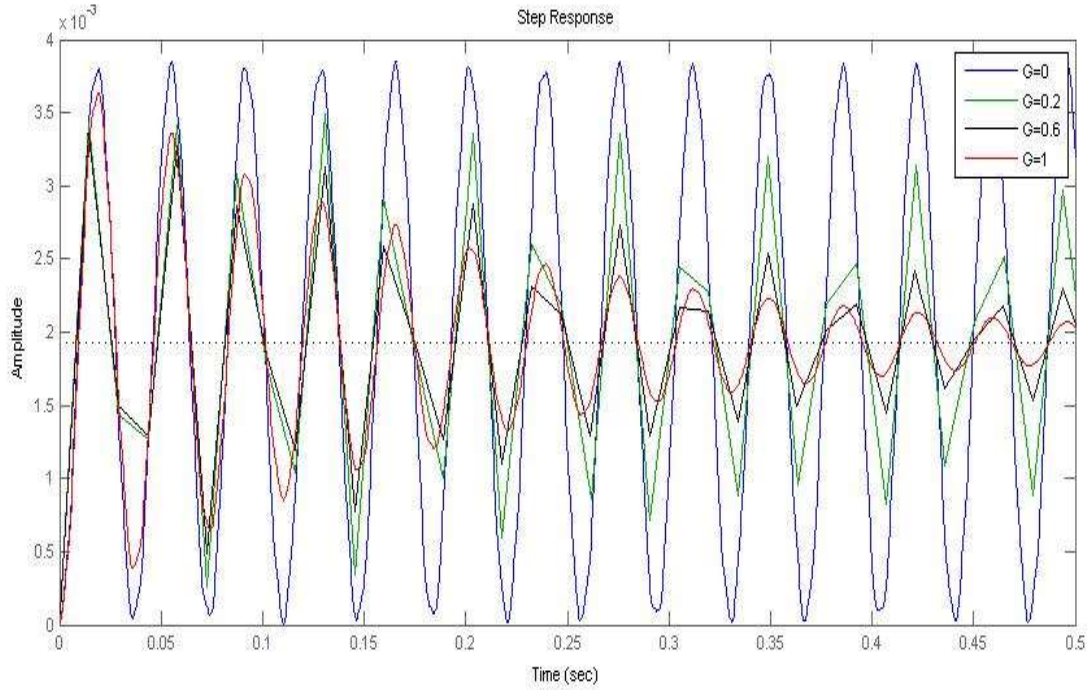

Fig. 6: Time response curve of a cantilever beam for different values of G.

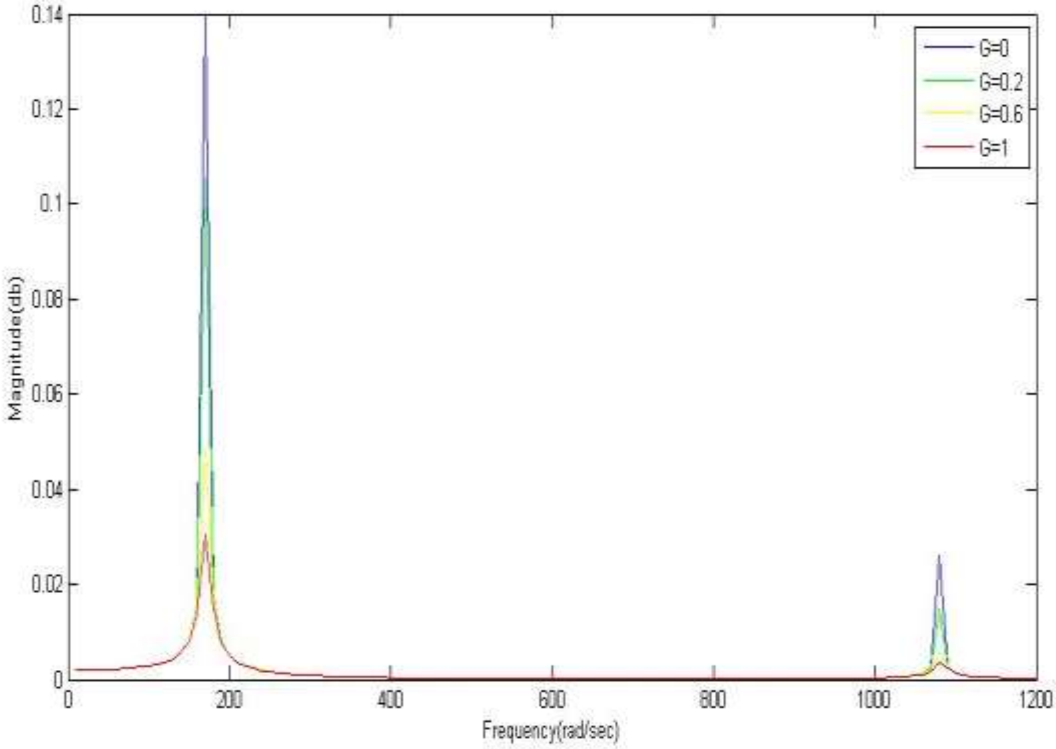

Fig. 7: Frequency response curve of a cantilever beam for different values of G.

4. CONCLUSIONS:

In this paper, a comprehensive study of smart materials and smart structures was done for the effect of the piezoelectric actuator placement on controlling the structural vibrations. Two systems were used and modeled in the finite element method for this study, the first one was a bar and the second one was a beam with a PZT actuator. A comparative study is carried outfrom the finite element mathematical model of bar and beam to the strength of material approach for displacement and deflection in static analysis for 1 to 4 elements. In the case of dynamic analysis, the time response and frequency response are plotted for different values of gain for a tip of the element. In the case of the bar, the structural damping is considered and saw the effect of that. The value of gain increases the amplitude and magnitude decrease continuously and time of elapse also decreases continuously. It was observed that such a controller resulted in the suppression of the longitudinal displacement of the bar and transverse deflection of the beamby controlling the voltage of the closed-loop system. When we take five nodes the result is almost nearer to the accurate result.

References:

[1] Song G, Qiao P.Z, Binienda W.K, Zou Z.P. (2002) ‛‛Active Vibration Damping of Composite”. JOURNAL OF AEROSPACE ENGINEERING,3: 97-103.
[2] Baillargeon, B.P. (2003) "Active Vibration Suppression of Smart Structures Using Piezoelectric Shear Actuators". Electronic Theses and Dissertations. 284. http://digitalcommons.library.umaine.edu/etd/284.
[3] Waghulde K. B. and Kumar B. (2011) “Vibration analysis of cantilever smart structure by using piezoelectric smart material”, International Journal on smart sensing and intelligent systems, (4)3: 1178 – 5608.
[4] Waghulde K.B. and Kumar B. (2012) “Active Vibration control of a cantilevered smart structure by using spatial H∞ control technique”, Journal of Electronic and Electrical Engineering, 3(1) 64-69.
[5] Bailey T. and Hubbard J. E. (1985) “Distributed piezoelectric polymer active vibration control of a cantilever beam” J. Guidance Control Dynam. Vol.8, pp.605–611.
[6] Crawley E. F. and de Luis J. (1987) “Use of piezoelectric actuators as elements of intelligent structures”, J. AIAA25:1373–1385.
[7] Ray M. C. and Mallik N. (2005) “Exact solutions for the analysis of piezoelectric fiber reinforced composites as distributed actuators for smart composite plates”, International Journal of Mechanics and Materials in Design, 5: 510-536.
[8] Patil G. and Kumar S. (2009) "An efficient model for vibration control by piezoelectric smart structure using finite element method", European Journal of Scientific Research, 34: 485-494.
[9] Schoeftner J. and Irschik H. (2011) “A comparative study of smart passive piezoelectric structures interacting with electric networks: Timoshenko beam theory versus finite element plane stress calculations”, Smart materials and structures, 2: 964-1726.
[10] R. Kandasamy, N. Ganesan, N. Shanmugam(2011) “Finite element based active vibration control studies on laminated composite box type structures under thermal environment”, International Journal of Applied Engineering Research,6: 221–234.