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ANTIMAGIC LABELING OF CERTAIN FAMILIES OF DIGRAPHS USING HOOKED SKOLEM SEQUENCE

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Abstract: The vertex sum at each given vertex u is equal to the total of all the edges directed into it minus the sum of all the edges directed out of it. This is known as an antimagic labeling of a directed graph G with v vertices and q edges. An antimagic digraph is a digraph that permits an antimagic labeling. In this paper, we analyze a few antimagic digraph features and use hooked Skolem sequences to demonstrate the anti magicness of specific digraph families.

.***Keywords:*** *Digraphs,antimagic,hooked Skolem sequence,graceful,vertex sum,edge sum,labeling.*

I. INTRODUCTION :

Subject to certain restrictions, a graph labeling is the assignment of integers to the vertices, edges, or both[1]. There are many different ways to label a graph. Harmonious labeling, magic labeling, antimagic labeling, and other types of graph labeling have all been researched thus far. There has been a significant lot of research on labeling graphs, but little on labeling digraphs. A labeling of a digraph D with m arcs is a one-one and onto function from the set of arcs of D to 1,...,m . The total of labels for all arcs entering u less than the sum of labels for all arcs coming out of u makes up the vertex-sum of a vertex u. If no two vertices in D have the same vertex-sum, a labeling of D is antimagic.

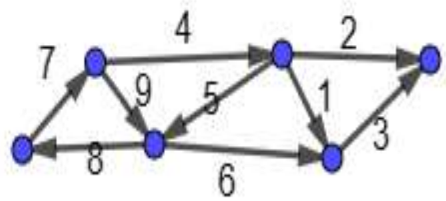

Fig1.1 Antimagic labeling

By giving each vertex v in a digraph D with p vertices and q arcs a unique integer value g(v) from 0 to q, the digraph is labeled. Each arc (u, v) is given a value g(u, v) by the vertex values, where g(u, v) = (g(v) g(u)) (mod q + 1). The labeling is referred to as a graceful labeling of digraph if the arc values are all distinct[2].

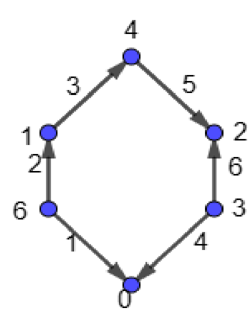

Fig1.2 Graceful labeling

A hooked Skolem sequence of order n is a sequence $hS_n = (s_1, s_2, \dots \dots s_{2n+1})$ of 2n+1 integers containing each of the integers 1,2,....n exactly twice such that two occurrences of the integers j∈ $\{1,2.\dots\dots n\}$ are separated by exactly $j-1$ integers and $S_{2n} = 0$[3]

$\vec{C}_k^r$ is the directed graph obtained from r copies of $\vec{C}_k$ in which any two consecutive directed cycles have a common edge

$\vec{C}$(n,n-3) is an orientation of the shell graph C(n,n-3) such that each shell is a unicycle of length 3[4].

Every connected 2d-regular graph has an antimagic orientation.

II.LITERATURE SURVEY

In literature it is observed that the existence of antimagic labelings of some families of digraphs is investigated using the concept of hooked Skolem sequences,existence of antimagic labelings of symmetric digraphs using Skolem sequences etc. Also the existence of graceful labeling is investigated using subset sum problems,(v,k,λ) difference set etc.. Antimagic labeling of digraphs and graceful labeling of digraphs are observed to be faces of the same coin as all the labeling techniques which hold for graceful digraphs also hold good for antimagic digraphs.. Every connected 2d-regular graph has an antimagic orientation.Hartsfield and Ringel proved that paths,2-regular graphs and complete graphs are antimagic.Antimagic labeling of digraphs can also be proved using using subset sum problems,(v,k,λ) difference set,skolem sequence.In this paper we are going to use hooked Skolem sequence to show the antimagic labeling of directed graph DC_n .Also we use subset sum problems to show that $\vec{C}_k^r$ and $\vec{C}$(n,n-3) is antimagic.

Let DC_n be a digraph with n vertices and 2n-3 edges.Let v_1, v_2,…… v_n be the vertices and e_1, e_2,…… $e_n$be the edges of DC_n. Let the hooked Skolem sequence be $(a_1, b_1), (a_2, b_2), \dots (a_n, b_n)$, By assigning the ordered pairs as labels to each edges of DC_n we can get an antimagic labeling of DC_n.

For $DC_6, n = 6 \equiv 2 mod 4$ .The hooked Skolem sequence is given as (9,10),(1,3),(2,5),(4,8),(6,11),(7,13),(0,12).We can use this sequence to get an antimagic labeling of DC_6.

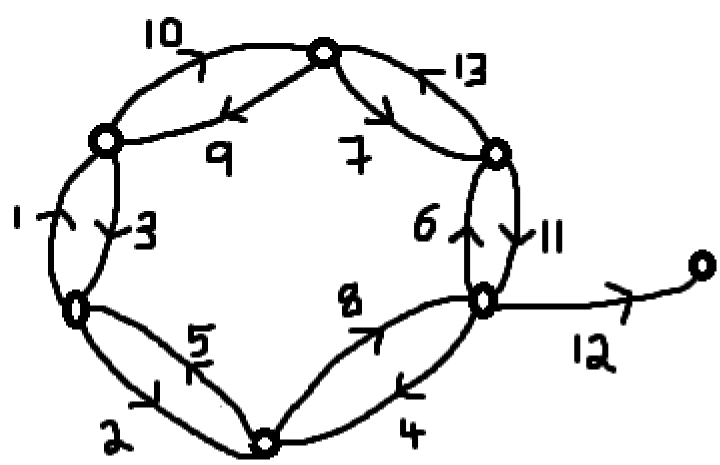

Fig 1.3 Antimagic labelling of DC_6

For $\vec{C}$(8,5) the antimagic labeling is shown in the figure 1.4

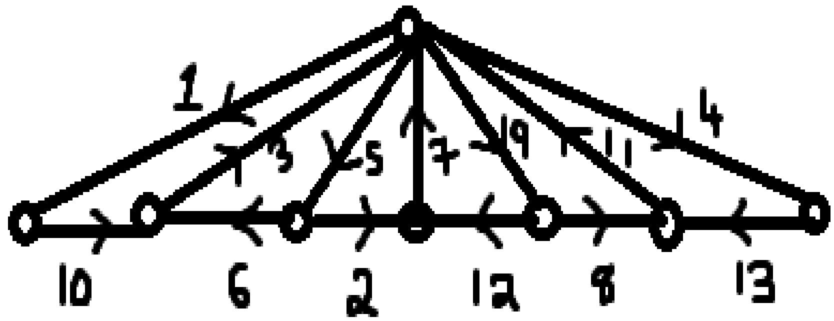

Fig 1.4 Antimagic labelling of $\vec{C}(8,5)$

CONCLUSION

In this paper we have stated that antimagic labeling for DC_n can be done using hooked Skolem sequences.Similarly antimagic labeling of $\vec{C}$(n,n-3) and $\vec{C}_k^r$ can be done using subset sum problems.It can be further proved that some other classes of digraph antimagic using this concept.

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