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GENERALISED ESTIMATION OF MAGNETOSTRICTION AND FRACTURE TOUGHNESS TEXTURE FACTORS OF IRON, NICKEL, COBALT BASED SUPER ALLOYS BY AN EXPANSION INTO DIRECTION COSINES $\alpha_1,\alpha_2,\alpha_3$ WITH RESPECT TO THE CRYSTAL AXES

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Abstract
In this present article, Magnetostriction Fracture Toughness and Texture Factor of Iron, Nickel, Cobalt based superalloys is expressed by an expansion into Direction Cosines α_1, α2, α3 with respect to the crystal axes. The General Equation Magnetostriction and Texture Factor of Iron, Nickel, Cobalt based superalloys can be used to determine their values at <100>, <110>, <111> directions respectively. In the present article Magnetostriction, Fracture Toughness and Texture Factor of Iron, Nickel, Cobalt based superalloys is determined at <100>, <110>, <111> directions respectively. The Equation can be generalized to include any element or compound with anisotropic property.

Keywords:
Anisotropic, Magnetostriction, Texture Factor, Fracture Toughness, superalloys, Direction Cosines

Introduction
Anisotropic Properties are those properties which vary with crystal direction Magnetostriction, Fracture Toughness and Texture Factor of Iron, Nickel, Cobalt based superalloys is different at <100>,<110>,<111> directions. Magnetostriction, Fracture Toughness, Texture Factor superalloys can be expressed as an expansion into direction cosines $\alpha_1,\alpha_2,\alpha_3$ with respect to the crystal axes.In the present article, consideration is made up to three terms.

1.1 Standard Equation:
$G^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha22)3+ K6 (\prod\alpha21)2$
Considered Equation:
$G^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)\ ) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha22)3+ K6 (\prod\alpha21)2$

[uvw]	A	B	c	α_1	α_2	α_3	Y
<100>	0	90^0	90^0	1	0	0	K_0
<110>	45^0	45^0	90^0	1/√2	1/√2	0	K0 + K1 /4
<111>	54.7^0	54.7^0	54.7^0	1/√3	1/√3	1/√3	$K_0 + K_1/3 + K_2/27$

From Ref[6] [TABLE I]

S.No	Magnetostriction Iron-Based Superalloy (*10-6)	Magnetostriction Nickel - Based Superalloy ($*10^{-6}$)	Magnetostriction Cobalt - Based Superalloy ($*10^{-6}$)
1.	20	-40	30
2.	10	-20	15
3.	5	-10	7

I. Calculation Of Iron,Nickel,Cobalt based Superalloys By An Expansion Into

Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

$G^* = K_0 + K_1 (\sum\alpha^2_1 \alpha^2_2) + K_2 (\prod\alpha^2_1)) + K_3 (\sum\alpha^2_1 \alpha^2_2)^2 + K_4 (\sum\alpha^2_1 \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1 \alpha 22)3+ K6 (\prod\alpha 21)2$

For <100> directions, $\alpha_1 =1, \alpha 2 =0, \alpha_3=0$ ….[I]

For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha 2 =1/\sqrt{2}, \alpha_3=0$….[II]

For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha 2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III]

1. Calculation Of Iron based Superalloys By An Expansion Into

Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

From Ref[5] , We have For <100> directions, $\alpha_1 =1, \alpha 2 =1, \alpha_3=0$ ….[I], in Standard Equation

$VR^* = K_0 + K_1 (\sum\alpha^2_1 \alpha^2_2) + K_2 (\prod\alpha^2_1)) + K_3 (\sum\alpha^2_1 \alpha^2_2)^2 + K_4 (\sum\alpha^2_1 \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1 \alpha 22)3+ K6 (\prod\alpha 21)2$

We have

$VR^*[100] = K0 = 20;$

For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha 2 =1/\sqrt{2}, \alpha_3=0$

Using [II] , in Standard Equation

$10= K_0 + K_1 (\sum\alpha^2_1 \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1 \alpha^2_2)^2 + K_4 (\sum\alpha^2_1 \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1 \alpha^2_2)^3+ K6 (\prod\alpha 21)2$

- **$10 = 20+ K_1/4 + K_3/16 + K_5/64$**
- **$[16K_1+ 4K_3 + K_5] = -640$ ……..[IV];**

For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha 2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III];

Using [III] , in Standard Equation

$VR^* = K_0 + K_1 (\sum\alpha^2_1 \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1 \alpha^2_2)^2 + K_4 (\sum\alpha^2_1 \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1 \alpha 22)3+ K6 (\prod\alpha 21)2$

$5= 20+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =2.345 + K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ K_6 /729$ [re-arraging K_2, K_5

- **$27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= -15* 729 = -10935$**
- **$27 *-400 - 135 = -10935$**
- **$[9K_1+ 3K_3 + K_5] = -400$….[V];**
- **$[9K_4+ 27K_2 + K_6]= -135$**
- **$-2*9+27*-4 -9= -135$**
- **$K_4 = -2; K_2= -4; K_6= -9$**
- **From[IV] - [V];We have**
- **$16K_1+ 4K_3 + K_5= -640$**

  (-)

  $9K_1+ 3K_3 + K_5 = -400$

  ____

  $7K_1+ K_3 = -240$
- **$7*-30 -30 = -240$**
- **$K_1= -30;K_3 = -30;$**
- **$K_5 = -400 + 3*30 +9*30$**
- **$K_5 = -40$**

Substituting , $K_0, K_1, K_2\ K_3, K_4, K_5, K_6$,in standard equation, we have **$Y^* = K_0 + K_1 (\sum\alpha^2_1 \alpha 22) + K2 (\prod\alpha 21) + K3 (\sum\alpha 21\ \alpha 22)2 + K4 (\sum\alpha 21\ \alpha 22)(\prod\alpha 21) + K5 (\sum\alpha 21\ \alpha 22)3+ K6 (\prod\alpha 21)2$**

$K_0=20, K_1= -30, K_2= -4; K3= -30; K_4= -2; K_5= -40 ; K_6 = -9$

$VR^*= 20 -30(\sum\alpha^2_1 \alpha^2_2) -4(\prod\alpha^2_1) -30 (\sum\alpha^2_1 \alpha^2_2)^2 -2(\sum\alpha^2_1 \alpha^2_2)(\prod\alpha^2_1) -40(\sum\alpha^2_1 \alpha^2_2)^3- 9(\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] Above Is Generalised Estimation of iron based super alloy By An Expansion Into Direction Cosines A_1,A_2,A_3 With Respect To The Crystal Axes**
-
- **FOR <100> Directions, $VR^* = 20$**

- **FOR <110> Directions, VR*\ = 20 -30/4 -30/16 -40/64 = 10**
  8
- **FOR <111> Directions, VL*=20 -30/3 -4/27 -30/9 - 2/81 -40/27 -9/729 = 5**

II. Calculation Of Magnetostriction of Nickel Based Super Alloy By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

$VN^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1)) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha22)3+ K6 (\prod\alpha21)2$

For <100> directions, $\alpha_1 =1, \alpha2 =0, \alpha_3=0$ ….[I]
For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha2 =1/\sqrt{2}, \alpha_3=0$….[II]
For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III]

2 Calculation Of Magnetostriction of Nickel Based Super Alloy By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

For <100> directions, $\alpha_1 =1, \alpha2 =1, \alpha_3=0$ ….[I], in Standard Equation

$VN^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1)) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha22)3+ K6 (\prod\alpha21)2$

We have

$VT^*_{[100]} = K_0 = -40$;

For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha2 =1/\sqrt{2}, \alpha_3=0$

Using [II] , in Standard Equation

$-20= K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha22)3+ K6 (\prod\alpha21)2$

- **$-20 = -40 + K_1/4 + K_3/16 + K_5/64$**
- **$[16K_1+ 4K_3 + K_5] = 20*64 = 1280$……..[IV];**

For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III];

Using [2 III] , in Standard Equation

$VN^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ 2\alpha22)3+ K6 (\prod\alpha21)2$

$-10 = -408+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =2.345 + K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ K_6 /729$ [re-arraging K_2, K_5

- **$27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= 30* 729 = 21870$**
- **27 *800+270 = 21870**
- **$[9K_1+ 3K_3 + K_5] =- 800$….[V];**
- **$[9K_4+ 27K_2 + K_6]= 270$**
- **2*9+27*9 +9= 270**
- **$K_4 = 2; K_2=9; K_6= 9$**
- **From[IV] - [V];We have**
- **$16K_1+ 4K_3 + K_5= 1280$**
  (-)
  $9K_1+ 3K_3 + K_5 = 800$
  2__________________________
  $7K_1+ K_3 = 480$
- **7* 70 -10= 480;**
- **$K_1= 70;K_3 = -10$;**
- **$K_5 = 800 -9*70+30$**
- **$K_5 = 200$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$VT^* = K_0 + K_1 (\sum\alpha21\ \alpha22) + K2 (\prod\alpha21) + K3 (\sum\alpha21\ \alpha22)2 + K4 (\sum\alpha21\ \alpha22)(\prod\alpha21) + K5 (\sum\alpha21\ \alpha22)3+ K6 (\prod\alpha21)2$ $K_0= -40$, $K_1= 70$, $K_2= 9$; K3= -10; $K_4= 2$; $K_5= 200$; $K_6 = 9$**

VN^*_{NICKEL} =-40 +70($\sum\alpha^2_1\ \alpha^2_2$) +9($\prod\alpha^2_1$) -10($\sum\alpha^2_1\ \alpha^2_2)^2$+2($\sum\alpha^2_1\ \alpha^2_2$)($\prod\alpha^2_1$) +200($\sum\alpha^2_1\ \alpha^2_2)^3$ +9($\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] Above Is Generalised Estimation Of Magnetostriction of Nickel based super alloysBy An Expansion Into Direction Cosines A_1,A_2,A_3 With Respect To The Crystal Axes**
- **FOR <100> Directions, VT* = -40**
  8
- **FOR <110> Directions, VT*\ = -40 +70/4 -10/16 +200/64 = -20**
- **FOR <111> Directions, VT*= -40 +70/3 + 9/27 -10/9 +2/81 +200//27 +9/729= 10**

III Calculation Of Magnetostrictionof cobalt based superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

VC* = $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)$)+ $K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5$ (∑α21 α22)3+ K6 (∏α21)2

For <100> directions, α_1 =1, α2 =0, α_3=0 ….[I]
For <110> directions, $\alpha_1 =1/\sqrt{2}$, α2 =1/√2, α_3=0….[II]
For <111> directions, $\alpha_1 =1/\sqrt{3}$, α2 =1/√3, $\alpha_3=1/\sqrt{3}$….[III]

3. Calculation Of Magnetostriction of cobalt based superalloyBy An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

For <100> directions, α_1 =1, α2 =1, α_3=0 ….[I], in Standard Equation

VC* = $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)$) + $K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1$ α22)3+ K6 (∏α21)2

We have

VC*[100] = K0 =30 ;

For <110> directions, $\alpha_1 =1/\sqrt{2}$, α2 =1/√2, α_3=0
Using [II] , in Standard Equation

15= $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha^2_2)^3$+ K6 (∏α21)2

- **15 = 30+ K_1/4 + K_3/16 + K_5/64**
- **[16K_1+ 4K_3 + K_5] = -15*64 = -960……..[IV];**

For <111> directions, $\alpha_1 =1/\sqrt{3}$, α2 =1/√3, $\alpha_3=1/\sqrt{3}$….[III];
Using [III] , in Standard Equation

VC* = $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1$ α22)3+ K6 (∏α21)2

7 = 30+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =30+ K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ 8K_6 /729 [re-arraging K_2, K_5

- **27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= -23* 729 = -16767**
- **27 *-611-270= -16767**
- **[9K_1+ 3K_3 + K_5] = -611….[V];**
- **[9K_4+ 27K_2 + K_6]= -270**
- **-2*9+27*-9 -9= -270**
- **K_4 = -2; K_2= -9; K_6= -9**
- **From[IV] - [V];We have**
- **16K_1+ 4K_3 + K_5= -960**
  (-)
  9K_1+ 3K_3 + K_5 = -611

  7K_1+ K_3 = -349
- **7*-50 +1= -349;**
- **K_1= -50;K_3 = 1;**

- K_5 = -611 +9*50 -3
- K_5 = -164

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$VC^* = K_0 + K_1 (\sum \alpha^2_1\, \alpha 22) + K2 (\prod \alpha 21) + K3 (\sum \alpha 21\, \alpha 22)2 + K4 (\sum \alpha 21\, \alpha 22)(\prod \alpha 21) + K5 (\sum \alpha 21\, \alpha 22)3 + K6 (\prod \alpha 21)2$**

K_0=30, K_1= -50, K_2= -9; K3= 1; K_4= -2; K_5= -164; K_6 = -9

$VC^* = 30 - 50 (\sum \alpha^2_1\, \alpha^2_2) - 9 (\prod \alpha^2_1) + 1(\sum \alpha^2_1\, \alpha^2_2)^2 - 2(\sum \alpha^2_1\, \alpha^2_2)(\prod \alpha^2_1) - 164(\sum \alpha^2_1\, \alpha^2_2)^3 - 9(\prod \alpha^2_1)^2$ ……….[VI];

- **[VI] ABOVE IS GENERALISED ESTIMATION OF BY AN EXPANSION INTO DIRECTION COSINES $\alpha_1, \alpha_2, \alpha_3$ WITH RESPECT TO THE CRYSTAL AXES**
- **FOR <100> Directions, VC* = 30**
- **FOR <110> Directions, VC*\ = 30-50/4 +1/16 -164/64 = 15**
- **FOR <111> Directions,VC*=30 -50/3 -9/27 +1/9 - 2/81 -164/27 -9/729 = 7**

III Evaluation of Texture Factors Based on Magnetostriction of Iron,Nickel,Cobalt based superalloys

S.N 0	Crystallographic Direction	RelationShip between Magnetostriction and Texture Factor for Iron-Based Superalloy	RelationShip between Magnetostriction and Texture Factor for Iron-Based Superalloy	RelationShip between Magnetostriction and Texture Factor for Iron-Based Superalloy
21.	<100>	λFe = 0.85F+1.2	λCo = 1.1F+0.9	λNi = 0.65F+1.4
2.	<110>	λFe = 0.8F+1.1	λCo= 1.05F+0.85	λNi= 0.6F+1.3
3.	<111>	λFe = 0.75F+1.0	λCo = 1.0F+0.8	λNi = 0..55F+1.2
S.N 0	Crystallograp hic Direction	Texture Factor for Iron-Based Superalloy	Texture Factor for Cobalt-Based Superalloy	Texture Factor for Nickel-Based Superalloy
1.	<100>	22.12	26.45	-63.69
2.	<110>	11.13	13.48	-35.5
3.	<111>	5.33	6.2	-20.36

By Expressing Texture Factor(F) = a λ [Magneto-Striction] + b, λ values taking from TABLE I , we can express F* as following for Iron,Co-balt,Nickel Super Alloys and K_0, K_1, K_2, K_3, K_4, K_5, K_6 can be computed.

$FR^* = K_0 + K_1 (\sum \alpha^2_1\, \alpha^2_2) + K_2 (\prod \alpha^2_1)) + K_3 (\sum \alpha^2_1\, \alpha^2_2)^2 + K_4 (\sum \alpha^2_1\, \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha^2_1\, \alpha 22)3 + K6 (\prod \alpha 21)2$

IV.Calculation Of Texture Factor of iron based superalloys By An Expansion Into Direction Cosines $\alpha_1, \alpha_2, \alpha_3$ With Respect To The Crystal Axes

$FR^* = K_0 + K_1 (\sum \alpha^2_1\, \alpha^2_2) + K_2 (\prod \alpha^2_1)) + K_3 (\sum \alpha^2_1\, \alpha^2_2)^2 + K_4 (\sum \alpha^2_1\, \alpha^2_2)(\prod \alpha^2_1) + K_5 (\sum \alpha 21\, \alpha 22)3 + K6 (\prod \alpha 21)2$

For <100> directions, $\alpha_1 = 1$, $\alpha 2 = 0$, $\alpha_3 = 0$ ….[I]

For <110> directions, $\alpha_1 = 1/\sqrt{2}$, $\alpha 2 = 1/\sqrt{2}$, $\alpha_3 = 0$….[II]

For <111> directions, $\alpha_1 = 1/\sqrt{3}$, $\alpha 2 = 1/\sqrt{3}$, $\alpha_3 = 1/\sqrt{3}$….[III]

2.1 Calculation Of Texture Factor of iron based superalloys By An Expansion Into Direction Cosines $\alpha_1, \alpha_2, \alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

For <100> directions, $\alpha_1 = 1$, $\alpha 2 = 1$, $\alpha_3 = 0$ ….[I], in Standard Equation

FR* = $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)\ ) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1$ α22)3+ K6 (∏α21)2

We have

$FR^*_{[100]} = K_0$ =22.12;

For <110> directions, $\alpha_1 = 1/\sqrt{2}$, α2 =1/√2, $\alpha_3=0$

Using [II] , in Standard Equation

11.13= $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1$ α22)3+ K6 (∏α21)2

- **11.13 = 22.12+ $K_1/4 + K_3/16 + K_5/64$**
- **[$16K_1 + 4K_3 + K_5$] = -703.36……..[IV];**

For <111> directions, $\alpha_1 = 1/\sqrt{3}$, α2 =1/√3, $\alpha_3=1/\sqrt{3}$….[III];

Using [III] , in Standard Equation

FR* = $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1$ α22)3+ K6 (∏α21)2

5.33 = 22.12+ $K_1/3 + K_2/27 + K_3/9 + K_4/81 + K_5/27 + K_6/729$ =30+ $K_1/3 + K_5/27 + K_3/9 + K_4/81 + K_2/27 + K_6/729$ [re-arraging K_2, K_5

- **27[$9K_1 + 3K_3 + K_5$] +[$9K_4 + 27K_2 + K_6$]= -12239.91**
- **27 *-458.33 +135= -12239.91**
- **[$9K_1 + 3K_3 + K_5$] = -458.33….[V];**

[$9K_4 + 27K_2 + K_6$]= 135

- **2*9+27*4 +9= 135**
- **$K_4 = 2$; $K_2 = 4$; $K_6 = 9$**
- **From[IV] - [V];We have**
- **$16K_1 + 4K_3 + K_5 = -703.36$**
  (-)
  $9K_1 + 3K_3 + K_5 = -458.33$
  ____
  $7K_1 + K_3 = -245.03$
- **7*-35 -0.03= -245.03;**
- **$K_1 = -35$; $K_3 = -0.03$;**
- **K_5 = -458.33+9*35+0.03*3**
- **K_5 = -143.24**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **Y* = $K_0 + K_1 (\sum\alpha^2_1$ α22) + K2 (∏α21) + K3 (∑α21 α22)2 + K4 (∑α21 α22)(∏α21) + K5 (∑α21 α22)3+ K6 (∏α21)2**

K_0=22.12, K_1= -35, K_2= 4; K3= -0.03; K_4= 2; K_5= -143.24; K_6 = 9

FR* = 22.12 -35 $(\sum\alpha^2_1\ \alpha^2_2)$+4 $(\prod\alpha^2_1)$ -0.03$(\sum\alpha^2_1\ \alpha^2_2)^2$ +2$(\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1)$ -143.24$(\sum\alpha^2_1\ \alpha^2_2)^3$ +9$(\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] ABOVE IS GENERALISED ESTIMATION OF BY AN EXPANSION INTO DIRECTION COSINES $\alpha_1,\alpha_2,\alpha_3$ WITH RESPECT TO THE CRYSTAL AXES**
  FOR <100> Directions, FR* = 22.12
- **FOR <110> Directions, FR*\= 22.12-35/4 -0.03/16 -143.24/64 = 11.13**
- **FOR <111> Directions,FR*=22.12 -35/3 +4/27 -0.03/9 -+2/81 -143.24/27 +9/729 = 5.33**

V.Calculation Of Texture Factor of cobalt based superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

FC* = $K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)\ ) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5$ (∑α21 α22)3+ K6 (∏α21)2

For <100> directions, $\alpha_1 = 1$, α2 =0, $\alpha_3=0$ ….[I]

For <110> directions, $\alpha_1 = 1/\sqrt{2}$, α2 =1/√2, $\alpha_3=0$….[II]

For <111> directions, $\alpha_1 = 1/\sqrt{3}$, $\alpha 2 = 1/\sqrt{3}$, $\alpha_3 = 1/\sqrt{3}$….[III]

2.1 Calculation of Texture Factor of cobalt based superalloyBy An Expansion Into Direction Cosines $\alpha_1, \alpha_2, \alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

For <100> directions, $\alpha_1 = 1$, $\alpha 2 = 1$, $\alpha_3 = 0$ ….[I], in Standard Equation

$$FC^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)\ ) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha 22)3 + K6 (\prod\alpha 21)2$$

We have

$$FC^*_{[100]} = K_0 = 26.45\ ;$$

For <110> directions, $\alpha_1 = 1/\sqrt{2}$, $\alpha 2 = 1/\sqrt{2}$, $\alpha_3 = 0$

Using [II] , in Standard Equation

$$13.48 = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha 22)3 + K6 (\prod\alpha 21)2$$

- **$13.48 = 26.45 + K_1/4 + K_3/16 + K_5/64$**
  $[16K_1 + 4K_3 + K_5] = -830.08$……..[IV];

For <111> directions, $\alpha_1 = 1/\sqrt{3}$, $\alpha 2 = 1/\sqrt{3}$, $\alpha_3 = 1/\sqrt{3}$….[III];

Using [I2II] , in Standard Equation

$$FC^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha 22)3 + K6 (\prod\alpha 21)2$$

$6.2 = 26.5 + K_1/3 + K_2/27 + K_3/9 + K_4/81 + K_5/27 + K_6/729 = 30 + K_1/3 + K_5/27 + K_3/9 + K_4/81 + K_2/27 + K_6/729$ [re-arraging K_2, K_5

- **$27[9K_1 + 3K_3 + K_5] + [9K_4 + 27K_2 + K_6] = -23 * 729 = --14762.25$**
- **$27 * -551.75 + 135 = -14762.25$**
- **$[9K_1 + 3K_3 + K_5] = -551.75$….[V];**
- **$[9K_4 + 27K_2 + K_6] = 135$**
- **$2*9 + 27*4 + 9 = 135$**
- **$K_4 = 2$; $K_2 = 4$; $K_6 = 9$**

- **F8rom[IV] - [V];We have**
- **$16K_1 + 4K_3 + K_5 = -830.08$**
  (-)
  $9K_1 + 3K_3 + K_5 = -551.75$

  $7K_1 + K_3 = -278.33$
- **$7 * -39 - 5.33 = -349$;**
- **$K_1 = -39$; $K_3 = -5.33$;**
- **$K_5 = -551.75 + 9*39 + 3*5.33$**
- **$K_5 = -184.76$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$Y^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha 22) + K2 (\prod\alpha 21) + K3 (\sum\alpha 21\ \alpha 22)2 + K4 (\sum\alpha 21\ \alpha 22)(\prod\alpha 21) + K5 (\sum\alpha 21\ \alpha 22)3 + K6 (\prod\alpha 21)2$**

$K_0 = 26.45$, $K_1 = -39$, $K_2 = 4$; $K3 = -5.33$; $K_4 = 2$; $K_5 = -184.76$; $K_6 = 9$

$FC^* = 26.45 - 39 (\sum\alpha^2_1\ \alpha^2_2) + 4 (\prod\alpha^2_1) - 5.33(\sum\alpha^2_1\ \alpha^2_2)^2 + 2(\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) - 184.76(\sum\alpha^2_1\ \alpha^2_2)^3 + 9(\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] ABOVE IS GENERALISED ESTIMATION OF BY AN EXPANSION INTO DIRECTION COSINES $\alpha_1, \alpha_2, \alpha_3$ WITH RESPECT TO THE CRYSTAL AXES**
- **FOR <100> Directions, $FC^* = 26.45$**
- **FOR <110> Directions, $FC^*\backslash = 26.45 - 39/4 - 5.33/16 - 184.76/64 = 13.48$**
- **FOR <111> Directions, $FC^* = 26 - 39/3 + 4/27 - 5.33/9 + 2/81 - 184.76/27 + 9/729 = 6.2$**

VI. Calculation Of Texture Factor of nickel based superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

$FN^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1)) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha 21\ \alpha 22)3 + K6 (\prod\alpha 21)2$

For <100> directions, $\alpha_1 =1, \alpha 2 =0, \alpha_3=0$ ….[I]

For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha 2 =1/\sqrt{2}, \alpha_3=0$….[II]

For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha 2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III]

3.2 Calculation Of Texture Factor of nickel based superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

For <100> directions, $\alpha_1 =1, \alpha 2 =1, \alpha_3=0$ ….[I], in Standard Equation

$FN^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1)) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha 22)3 + K6 (\prod\alpha 21)2$

We have

$FN^*_{[100]} = K_0 = -63.69$;

For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha 2 =1/\sqrt{2}, \alpha_3=0$

Using [II] , in Standard Equation

$-35.5= K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha 22)3 + K6 (\prod\alpha 21)2$

- **$-35.5 = -63.69+ K_1/4 + K_3/16 + K_5/64$**
- **$[16K_1+ 4K_3 + K_5] = 1804.16$……..[IV];**

For <111> directions, $\alpha 9_1 =1/\sqrt{3}, \alpha 2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III];

Using [III] , in Standard Equation

$FN^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha 22)3 + K6 (\prod\alpha 21)2$

$-20.36= -63.69+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =30+ K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ K_6 /729$ [re-arraging K_2, K_5

- **$27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= 31587.57$**
- **$27 *1164.91+1335= 31587.57$**
  $[9K_1+ 3K_3 + K_5] = 1164.91$….[V];
- **$[9K_4+ 27K_2 + K_6]= 135$**
- **$2*9+27*4 +9= 135$**
- **$K_4 = 2; K_2= 4; K_6= 9$**
- **From[IV] - [V];We have**
-
- **$16K_1+ 4K_3 + K_5= 1804.16$**
  (-)
  $9K_1+ 3K_3 + K_5 = 1164.91$
  ____
  $7K_1+ K_3 = 639.25$
- **$7*91 +2.25= 639.25$;**
- **$K_1= 91;K_3 = 2.25$;**
- **$K_5 = 1164.91 -9*91-3*2.25$**
- **$K_5 = 339.16$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$FN^* = K_0 + K_1 (\sum\alpha 21\ \alpha 22) + K2 (\prod\alpha 21) + K3 (\sum\alpha 21\ \alpha 22)2 + K4 (\sum\alpha 21\ \alpha 22)(\prod\alpha 21) + K5 (\sum\alpha 21\ \alpha 22)3 + K6 (\prod\alpha 21)2$**

$K_0= -63.69$, $K_1= 91$, $K_2= 4$; $K3= 2.25$; $K_4= 2$; $K_5= 339.16$; $K_6 = 9$

$FN^* = 63.69 +91(\sum\alpha^2{}_1\ \alpha^2{}_2)+4 (\prod\alpha^2{}_1) +2.25(\sum\alpha^2{}_1\ \alpha^2{}_2)^2 +2(\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1)+339.16(\sum\alpha^2{}_1\ \alpha^2{}_2)^3 +9(\prod\alpha^2{}_1)^2$ ……….[VI];

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- **[VI] ABOVE IS GENERALISED ESTIMATION OF BY AN EXPANSION INTO DIRECTION COSINES $\alpha_1,\alpha_2,\alpha_3$ WITH RESPECT TO THE CRYSTAL AXES**
- **FOR <100> Directions, FN* = -63.69**
- **FOR <110> Directions, FN*\ = -63.69+91/4 +2.2516 +339.16/64 = -35.5**
- **FOR <111> Directions,FN*= -63.69+91/3 +4/27 +2.25/9 + 2/81 +339.16/27 +9/729 = -20.386**

IV Calculation Of Fracture Toughness Iron, Nickel, Cobalt based Superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

S.No	Fracture Toughness Iron-Based Superalloys	Fracture Toughness Nickel -Based Superalloys	Fracture Toughness Cobalt -Based Superalloys
1.	25	30	25
2.	35	40	40
3.	45	55	60

1.1 Calculation Of Fracture ToughnessIron based Superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

For <100> directions, $\alpha_1 =1, \alpha2 =1, \alpha_3=0$ ….[I], in Standard Equation

$VR^* = K_0 + K_1 (\sum\alpha^2{}_1\, \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1)) + K_3 (\sum\alpha^2{}_1\, \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\, \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\, \alpha22)3+ K6 (\prod\alpha21)2$

We have

$VR^*[100] = K0 = 25;$

For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha2 =1/\sqrt{2}, \alpha_3=0$

Using [II] , in Standard Equation

$35= K_0 + K_1 (\sum\alpha^2{}_1\, \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\, \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\, \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\, \alpha^2{}_2)^3+ K6 (\prod\alpha21)2$

- **$35= 25+ K_1/4 + K_3/16 + K_5/64$**
- **$[16K_1+ 4K_3 + K_5] = 640$ ……..[IV];**

For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III];

Using [III] , in Standard Equation

$VR^* = K_0 + K_1 (\sum\alpha^2{}_1\, \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\, \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\, \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\, \alpha22)3+ K6 (\prod\alpha21)2$

$45= 25+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =2.345 + K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ K_6 /729$ [re-arraging K_2, K_5

- **$27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= 20* 729 =14580$**
- **27 *535 +135 = 14580**
- **$[9K_1+ 3K_3 + K_5] = 535$….[V];**
- **$[9K_4+ 27K_2 + K_6]= 135$**
- **2*9+27*4 +9= 135**
- **$K_4 = 2; K_2= 4; K_6= 9$**
- **From[IV] - [V];We have**
- **$16K_1+ 4K_3 + K_5= 640$**

  (-)

  $9K_1+ 3K_3 + K_5 = 535$

  $7K_1+ K_3 = 105$
- **7*(14)+7 =105**
- **$K_1= 14;K_3 = 7;$**
- **$K_5 = 535 -3*7 -9*14$**

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- **$K_5 = 388$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$Y^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha22) + K2 (\prod\alpha21) + K3 (\sum\alpha21\ \alpha22)2 + K4 (\sum\alpha21\ \alpha22)(\prod\alpha21) + K5 (\sum\alpha21\ \alpha22)3+ K6 (\prod\alpha21)2$**

K_0=25, K_1= 14 K_2= 4; K3= 7; K_4= 2; K_5= 388 ; K_6 = 9

$VR^*= 25 +14(\sum\alpha^2{}_1\ \alpha^2{}_2) +4(\prod\alpha^2{}_1) +7 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 +2(\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1)+388(\sum\alpha^2{}_1\ \alpha^2{}_2)^3- 9(\prod\alpha^2{}_1)^2$ ……….[VI];

- **[VI] Above Is Generalised Estimation of iron based super alloy By An Expansion Into Direction Cosines A_1,A_2,A_3 With Respect To The Crystal Axes**
- **FOR <100> Directions, VR* = 25**
- **FOR <110> Directions, VR*\ = 25+14/4 +7/16 +388/64 = 35**
- **FOR <111> Directions, VR*=25 +14/3 +4/27 +7/9 +2/81+388/27 +9/729 =45**

1.2 Calculation Of Texture factor of based on Fracture Toughness Iron based Superalloys By An Expansion Into Direction Cosines α_1,α_2,α_3 With Respect To The Crystal Axes

$K_{IC100} = 20 + 5 *(T_{100}) + 2 (T100)^2$;$K_{IC110} = 20 + 3*(T_{110}) + 1.5 (T_{110})^2$;$K_{IC111} = 20 + 7*(T_{100}) + 1.5 (T_{111})^{2\backslash}$

$25= 20 + 5 *(T_{100}) + 2 (T100)^2$;$35 = 20 + 3*(T_{110}) + 1.5 (T_{110})^2$;$45= 20 + 7*(T_{100}) + 1.5 (T_{111})^2$

$T_{100} = 0.7655$;$T_{100} = 2.316$;$T_{100} = 2.37$ Along 100,110,111 directions

For <100> directions, α_1 =1, α2 =1, α_3=0 ….[I], in Standard Equation

$TR^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1)) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha22)3+ K6 (\prod\alpha21)2$

We have

TR*[100] = K0 = 0.7655;

For <110> directions, α_1 =1/√2, α2 =1/√2, α_3=0

Using [II] , in Standard Equation

$2.316= K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha^2{}_2)^3+ K6 (\prod\alpha21)2$

- **$2.316= 0.7655+ K_1/4 + K_3/16 + K_5/64$**
- **[$16K_1+ 4K_3 + K_5$] = 99.232 ……..[IV];**

For <111> directions, α_1 =1/√3, α2 =1/√3, α_3=1/√3….[III];

Using [III] , in Standard Equation

$TR^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha^2{}_2) + K_2 (\prod\alpha^2{}_1) + K_3 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + K_4 (\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) + K_5 (\sum\alpha^2{}_1\ \alpha22)3+ K6 (\prod\alpha21)2$

$2.316=0.7655+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =2.345 + K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ K_6 /729$ [re-arraging K_2, K_5

- **27[$9K_1+ 3K_3 + K_5$] +[$9K_4+ 27K_2 + K_6$]= 39.366**
- **27 *1 + 12.366= 39.366**
- **[$9K_1+ 3K_3 + K_5$] = 1….[V];**
- **[$9K_4+ 27K_2 + K_6$]= 12.366**
- **-3*9+27*1 +12.366= 12.366**
- **K_4 = -3; K_2= 1; K_6=12.366**
- **From[IV] - [V];We have**
- **$16K_1+ 4K_3 + K_5$= 99.232**

  (-)

  $9K_1+ 3K_3 + K_5 = 1$

  $7K_1+ K_3 = 98.232$
- **7*(14)+0.232 =98.232**
- **K_1= 14;K_3 = 0.232;**
- **$K_5 = 1 -3*0.232-9*14$**

- **K_5 = -125.696**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **Y* = K_0 + K_1 ($\sum\alpha^2_1$ α22) + K2 (∏α21) + K3 (∑α21 α22)2 + K4 (∑α21 α22)(∏α21) + K5 (∑α21 α22)3+ K6 (∏α21)2**
K_0=0.7655, K_1= 14 K_2= -3; K3= 0.232; K_4= 1; K_5= -125.696 ; K_6 = 12.366
TR*= 0.7655 +14($\sum\alpha^2_1 \alpha^2_2$) -3($\prod\alpha^2_1$) +0.232($\sum\alpha^2_1 \alpha^2_2)^2$ +1($\sum\alpha^2_1 \alpha^2_2$)($\prod\alpha^2_1$)+ -125.696 ($\sum\alpha^2_1 \alpha^2_2)^3$+12.366($\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] Above Is Generalised Estimation of iron based super alloy By An Expansion Into Direction Cosines A_1,A_2,A_3 With Respect To The Crystal Axes**
- **FOR <100> Directions, VR* = 25**
- **FOR <110> Directions, VR*\ = 25+14/4 +7/16 +388/64 = 35**
- **FOR <111> Directions, VR*=25 +14/3 +4/27 +7/9 +2/81+388/27 +9/729 =45**
-

II. Calculation Of Fracture Toughness of Nickel Based Super Alloy By An Expansion Into Direction Cosines α_1,α_2,α_3 With Respect To The Crystal Axes
VN* = K_0 + K_1 ($\sum\alpha^2_1 \alpha^2_2$) + K_2 ($\prod\alpha^2_1$))+ K_3 ($\sum\alpha^2_1 \alpha^2_2)^2$ + K_4 ($\sum\alpha^2_1 \alpha^2_2$)($\prod\alpha^2_1$) + K_5 ($\sum\alpha^2_1$ α22)3+ K6 (∏α21)2
For <100> directions, α_1 =1, α2 =0, α_3=0 ….[I]
For <110> directions, α_1 =1/√2, α2 =1/√2, α_3=0….[II]
For <111> directions, α_1 =1/√3, α2 =1/√3, α_3=1/√3….[III]

2.1 Calculation Of Fracture Toughness of Nickel Based Super Alloy By An Expansion Into Direction Cosines α_1,α_2,α_3 With Respect To The Crystal Axes
From Ref [5] , We have
For <100> directions, α_1 =1, α2 =1, α_3=0 ….[I], in Standard Equation
VN* = K_0 + K_1 ($\sum\alpha^2_1 \alpha^2_2$) + K_2 ($\prod\alpha^2_1$)) + K_3 ($\sum\alpha^2_1 \alpha^2_2)^2$ + K_4 ($\sum\alpha^2_1 \alpha^2_2$)($\prod\alpha^2_1$) + K_5 ($\sum\alpha^2_1$ α22)3+ K6 (∏α21)2
We have
VT*$_{[100]}$ = K_0 = 30;
For <110> directions, α_1 =1/√2, α2 =1/√2, α_3=0
Using [II] , in Standard Equation
40= K_0 + K_1 ($\sum\alpha^2_1 \alpha^2_2$) + K_2 ($\prod\alpha^2_1$) + K_3 ($\sum\alpha^2_1 \alpha^2_2)^2$ + K_4 ($\sum\alpha^2_1 \alpha^2_2$)($\prod\alpha^2_1$) + K_5 ($\sum\alpha^2_1$ α22)3+ K6 (∏α21)2

- **40 = 30+ K_1/4 + K_3/16 + K_5/64**
- **[16K_1+ 4K_3 + K_5] = 10*64 = 640……..[IV];**

For <111> directions, α_1 =1/√3, α2 =1/√3, α_3=1/√3….[III];
Using [2 III] , in Standard Equation
VN* = K_0 + K_1 ($\sum\alpha^2_1 \alpha^2_2$) + K_2 ($\prod\alpha^2_1$) + K_3 ($\sum\alpha^2_1 \alpha^2_2)^2$ + K_4 ($\sum\alpha^2_1 \alpha^2_2$)($\prod\alpha^2_1$) + K_5 ($\sum\alpha^2_1$ 2α22)3+ K6 (∏α21)2
55= 30+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =2.345 + K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ K_6 /729 [re-arraging K_2, K_5

- **27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= 25* 729 = 18225**
- **27 *670+135= 18225**
- **[9K_1+ 3K_3 + K_5] =- 800….[V];**
- **[9K_4+ 27K_2 + K_6]= 670**
- **2*9+27*4 +9= 135**
- **K_4 = 2; K_2=4; K_6= 9**
- **From[IV] - [V];We have**
- **16K_1+ 4K_3 + K_5= 640**

(-)
9K_1+ 3K_3 + K_5 = 670

$7K_1 + K_3 = -30$

- **7*-3 -9= -30;**
- **$K_1 = -3; K_3 = -9$;**
- **$K_5 = 670 + 9*3+27$**
- **$K_5 = 724$**

Substituting , $K_0, K_1, K_2\ K_3, K_4, K_5, K_6$,in standard equation, we have **VT* = $K_0 + K_1$ (∑α21 α22) + K2 (∏α21) + K3 (∑α21 α22)2 + K4 (∑α21 α22)(∏α21) + K5 (∑α21 α22)3+ K6 (∏α21)2 K_0= 30, K_1= -3, K_2= 4; K3= -9; K_4= 2; K_5= 729; K_6 = 9**

$VN^*_{NICKEL} = 30 -3(\sum\alpha^2_1\ \alpha^2_2) +4(\prod\alpha^2_1) -9(\sum\alpha^2_1\ \alpha^2_2)^2+2(\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) +729(\sum\alpha^2_1\ \alpha^2_2)^3 +9(\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] Above Is Generalised Estimation Of Magnetostriction of Nickel based super alloysBy An Expansion Into Direction Cosines A_1, A_2, A_3 With Respect To The Crystal Axes**
- **FOR <100> Directions, VN* = 30**

8

- **FOR <110> Directions, VN*\ = 30-3/4 -9/16 +729/64 = 40**

FOR <111> Directions, VN*= 30-3/3 + 4/27 -9/9 +2/81 +729//27 +9/729=55

2.2Calculation Of Texture Factor based on Fracture Toughness of Nickel based Superalloys By An Expansion Into Direction Cosines $\alpha_1, \alpha_2, \alpha_3$ With Respect To The Crystal Axes

$30= 20 + 5 *(T_{100}) + 2\ (T100\ )^2; 40= 20 + 3*(T_{110}) + 1.5\ (T_{110}\ )^2; 55= 20 + 7*(\ T_{100}) + 1.5\ (T_{111}\ )^2$

T_{100} = 1.312; T_{110} = 2.786; T_{111} = 4.146;

For <100> directions, α_1 =1, α2 =1, α_3=0 ….[I], in Standard Equation

$TN^* = K_0 + K_1\ (\sum\alpha^2_1\ \alpha^2_2) + K_2\ (\prod\alpha^2_1)\ ) + K_3\ (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4\ (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5$ (∑α^2_1 α22)3+ K69 (∏α21)2

We have

TN*[100] = K0 = 1.312;

For <110> directions, α_1 =1/√2, α2 =1/√22, α_3=0

Using [II] , in Standard Equation

$2.786= K_0 + K_1\ (\sum\alpha^2_1\ \alpha^2_2) + K_2\ (\prod\alpha^2_1) + K_3\ (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4\ (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5\ (\sum\alpha^2_1\ \alpha^2_2)^3$+ K6 (∏α21)2

- **$2.786= 1.312+ K_1/4 + K_3/16 + K_5/64$**
- **$[16K_1+ 4K_3 + K_5] = 94.336$ ……..[IV];**

For <111> directions, α_1 =1/√3, α2 =1/√3, α_3=1/√3….[III];

Using [III] , in Standard Equation

$TN^* = K_0 + K_1\ (\sum\alpha^2_1\ \alpha^2_2) + K_2\ (\prod\alpha^2_1) + K_3\ (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4\ (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5$ (∑α^2_1 α22)3+ K6 (∏α21)2

$4.146=1.312+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5\ /27+ K_6\ /729 =2.345 + K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2\ /27+ K_6\ /729$ [re-arraging K_2, K_5

- **$27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= 2065.986$**
- **27 *71.158 +135 = 2065.986**
- **$[9K_1+ 3K_3 + K_5] = 71.158$**….[V];
- **$[9K_4+ 27K_2 + K_6]= 135$**
- **2*9+27*4 +9= 135**
- **$K_4 = 2; K_2= 4; K_6= 9$**

From[IV] - [V];We have

$16K_1+ 4K_3 + K_5= 94.336$

(-)

$9K_1+ 3K_3 + K_5 = 71.158$

$7K_1 + K_3 = 23.178$

- **$7*(3)+2.178 = 23.178$**
- **$K_1 = 3; K_3 = 2.178;$**
- **$K_5 = 71.158 - 3*2.178 - 9*3$**
- **$K_5 = 37.624$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$Y^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha22) + K2 (\prod\alpha21) + K3 (\sum\alpha21\ \alpha22)2 + K4 (\sum\alpha21\ \alpha22)(\prod\alpha21) + K5 (\sum\alpha21\ \alpha22)3 + K6 (\prod\alpha21)2$**

$K_0=1.312$, $K_1= 3$ $K_2= 4$; $K3= 2.178$; $K_4= 2$; $K_5= 37.624$; $K_6 = 9$

$TN^* = 1.312+3(\sum\alpha^2_1\ \alpha^2_2) + 4(\prod\alpha^2_1) + 2.178 (\sum\alpha^2_1\ \alpha^2_2)^2 + 2(\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + 37.624(\sum\alpha^2_1\ \alpha^2_2)^3 + 9(\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] Above Is Generalised Estimation of iron based super alloy By An Expansion Into Direction Cosines A_1,A_2,A_3 With Respect To The Crystal Axes**

- **FOR <100> Directions, $TN^* = 1.312$**
- **FOR <110> Directions, $TN^*\backslash = 1.312+3/4 +2.178/16 +37.624/64 = 2.786$**
- **FOR <111> Directions, $TN^*=1.312+3/3 +4/27 +2.178/9 +2/81+37.624/27 +9/729 =4.146$**

III Calculation Of Fracture Toughness of cobalt based superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

$VC^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha21\ \alpha22)3 + K6 (\prod\alpha21)2$

For <100> directions, $\alpha_1 =1, \alpha2 =0, \alpha_3=0$ ….[I]
For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha2 =1/\sqrt{2}, \alpha_3=0$….[II]
For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III]

3.1 Calculation Of Fracture Toughness of cobalt based superalloyBy An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes

From Ref [5] , We have

For <100> directions, $\alpha_1 =1, \alpha2 =1, \alpha_3=0$ ….[I], in Standard Equation

$VC^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha22)3 + K6 (\prod\alpha21)2$

We have

$VC^*[100] = K0 = 25$;

For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha2 =1/\sqrt{2}, \alpha_3=0$

Using [II] , in Standard Equation

$40= K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha^2_2)^3 + K6 (\prod\alpha21)2$

- **$40= 15 + K_1/4 + K_3/16 + K_5/64$**
- **$[16K_1+ 4K_3 + K_5] = 15*64 = 960$……..[IV];**

For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III];

Using [III] , in Standard Equation

$VC^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha22)3 + K6 (\prod\alpha21)2$

$60= 25+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =30+ K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ 8K_6 /729$ [re-arraging K_2, K_5

- **$27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= 35* 729 = 25515$**
- **$27 *940-+135= 25515$**
- **$[9K_1+ 3K_3 + K_5] = 940$….[V];**
- **$[9K_4+ 27K_2 + K_6]= 940$**
- **$2*9+27*4 +9= 135$**
- **$K_4 = 2; K_2= 4; K_6= 9$**
- **From[IV] - [V];We have**

- **$16K_1+ 4K_3 + K_5= 960$**
  (-)
  $9K_1+ 3K_3 + K_5 = 940$

  $7K_1+ K_3 = 20$
- **7*3 -1= -349;**
- **$K_1= 3;K_3 = -1;$**
- **$K_5 = 940 -9*3 +3$**
- **$K_5 = 916$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$Y^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha 22) + K2 (\prod\alpha 21) + K3 (\sum\alpha 21\ \alpha 22)2 + K4 (\sum\alpha 21\ \alpha 22)(\prod\alpha 21) + K5 (\sum\alpha 21\ \alpha 22)3+ K6 (\prod\alpha 21)2$**
$K_0=25$, $K_1= 3$, $K_2= 4$; $K3= -1$; $K_4= 2$; $K_5= 916$; $K_6 = 9$
$VC^* = 25 +3(\sum\alpha^2_1\ \alpha^2_2)+4 (\prod\alpha^2_1) -1(\sum\alpha^2_1\ \alpha^2_2)^2 +2(\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) +916(\sum\alpha^2_1\ \alpha^2_2)^3 + 9(\prod\alpha^2_1)^2$ ……….[VI];

- **[VI] ABOVE IS GENERALISED ESTIMATION OF BY AN EXPANSION INTO DIRECTION COSINES $\alpha_1,\alpha_2,\alpha_3$ WITH RESPECT TO THE CRYSTAL AXES**
- **FOR <100> Directions, VC* = 25**
- **FOR <110> Directions, VC*\ = 25+3/4 -1/16 +916/64 = 40**
- **FOR <111> Directions,VC*=25 +3/3 +4/27 -1/9 + 2/81 +916/27 +9/729 = 60**

3.2 Calculation Of Texture Factor based on Fracture Toghness of Cobalt **based Superalloys By An Expansion Into Direction Cosines $\alpha_1,\alpha_2,\alpha_3$ With Respect To The Crystal Axes**

$K_{IC100} = 20 + 5 *(T_{100}) + 2 (T_{100})^2$
$K_{IC110} = 20 + 3*(T_{110}) + 1.5 (T_{110})^2$
$K_{IC111} = 20 + 7*(T_{111}) + 1.5 (T_{111})^2$
$25= 20 + 5 *(T_{100}) + 2 (T_{100})^2$
$40= 20 + 3*(T_{110}) + 1.5 (T_{110})^2$
$60= 20 + 7*(T_{111}) + 1.5 (T_{111})^2$
$(T_{100}) =0.7955;(T_{100}) =2.786;(T_{100}) =3.333$
For <100> directions, $\alpha_1 =1, \alpha 2 =1, \alpha_3=0$ ….[I], in Standard Equation
$TC^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1)) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha 22)3+ K6 (\prod\alpha 21)2$
We have
$TC^*[100] = K0 = 0.7955$;
For <110> directions, $\alpha_1 =1/\sqrt{2}, \alpha 2 =1/\sqrt{2}, \alpha_3=0$
Using [II] , in Standard Equation
$2.786= K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha^2_2)^3+ K6 (\prod\alpha 21)2$

- **$2.786= 0.7955+ K_1/4 + K_3/16 + K_5/64$**
- **$[16K_1+ 4K_3 + K_5] = 129.312$ ……..[IV];**

For <111> directions, $\alpha_1 =1/\sqrt{3}, \alpha 2 =1/\sqrt{3}, \alpha_3=1/\sqrt{3}$….[III];
Using [III] , in Standard Equation
$TC^* = K_0 + K_1 (\sum\alpha^2_1\ \alpha^2_2) + K_2 (\prod\alpha^2_1) + K_3 (\sum\alpha^2_1\ \alpha^2_2)^2 + K_4 (\sum\alpha^2_1\ \alpha^2_2)(\prod\alpha^2_1) + K_5 (\sum\alpha^2_1\ \alpha 22)3+ K6 (\prod\alpha 21)2$
$3.333= 0.7655+ K_1/3 + K_2/27+ K_3/9+ K_4/81+ K_5 /27+ K_6 /729 =2.345 + K_1/3 + K_5/27+ K_3/9+ K_4/81+ K_2 /27+ K_6 /729$ [re-arraging K_2, K_5

- **$27[9K_1+ 3K_3 + K_5] +[9K_4+ 27K_2 + K_6]= 1871.7075$**
- **27 *64.3225+135 = 1871.7075**
- **$[9K_1+ 3K_3 + K_5] = 64.3225$….[V];**

- **$[9K_4 + 27K_2 + K_6] = 135$**
- **2*9+27*4 +9= 135**
- **$K_4 = 2$; $K_2 = 4$; $K_6 = 9$**
- **From[IV] - [V];We have**
- **$16K_1 + 4K_3 + K_5 = 129.312$**

(-)

$9K_1 + 3K_3 + K_5 = 64.3225$

$7K_1 + K_3 = 64.9895$

- **7*(9)+1.9895=64.9895**
- **$K_1 = 9$;$K_3 = 1.9895$;**
- **$K_5 = 64.9895-3*1.9895 -9*9$**
- **$K_5 = -21.979$**

Substituting , K_0, K_1, K_2 K_3, K_4, K_5, K_6 ,in standard equation, we have **$Y^* = K_0 + K_1 (\sum\alpha^2{}_1\ \alpha22) + K2 (\prod\alpha21) + K3 (\sum\alpha21\ \alpha22)2 + K4 (\sum\alpha21\ \alpha22)(\prod\alpha21) + K5 (\sum\alpha21\ \alpha22)3 + K6 (\prod\alpha21)2$**

$K_0=0.7655$, $K_1= 9$. $K_2= 4$; K3= 1.9895; $K_4= 2$; $K_5= -21.989$; $K_6 = 9$

$TC^* = 0.7655 + 9(\sum\alpha^2{}_1\ \alpha^2{}_2) + 4(\prod\alpha^2{}_1) + 1.9895 (\sum\alpha^2{}_1\ \alpha^2{}_2)^2 + 2(\sum\alpha^2{}_1\ \alpha^2{}_2)(\prod\alpha^2{}_1) - 21.989(\sum\alpha^2{}_1\ \alpha^2{}_2)^3 + 9(\prod\alpha^2{}_1)^2$ ....[VI];

- **[VI] Above Is Generalised Estimation of iron based super alloy By An Expansion Into Direction Cosines A_1,A_2,A_3 With Respect To The Crystal Axes**
- **FOR <100> Directions, TC* = 0.7655**
- **FOR <110> Directions, TC*\ = 0.7655+9/4 +1.9895/16-21.989/64 = 2.786**
- **FOR <111> Directions, TC*=0.7655+9/3 +4/27 +1.9895/9 +2/81-21.989/27 +9/729 =3.333**

IV Conclusion.

Magnetostriction, Fracture Toughness and Texture Factor of Iron, Nickel and cobalt based super alloys and Generlised Equation can be utilized to obtain its value at any crystallographic direction with the provision of directional cosines α_1, α2, α3 along that particular crystallographic direction. It was found that Magnetostriction, Fracture Toughness and Texture Factor is least for <111> direction and reasonable for <110> direction and least for <100> directions respectively for iron, cobalt, nickel based superalloys.

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