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PRODUCTION INVENTORY MODEL FOR DECAYING ITEMS WITH PRICE DEPENDENT DEMAND

Neha Singh, Ph.D. Research Scholar, Monad University, Hapur
R.B. Singh, Professor, Monad University, Hapur

ABSTRACT
A production inventory model for decaying items with life-time and price dependent demand for finite time horizon is developed. In this model production rate is taken as linear combination of on-hand inventory and demand both. Shortages are allowed and backlogged. Cost minimization technique is used to get the approximate expressions for total cost and other parameters. Special cases of model are also discussed.

Keywords : Life-time, Price Dependent demand, Linear combination.

INTRODUCTION
Most of the researchers have assumed that as soon as the items arrive in stock, they begin to deteriorate at once, but for many items this is not true. In practice when most of the items arrive in stock they are fresh and new and they begin to decay after a fixed time interval called life-period of items. **Gupta, R and Vrat, P. (1986)** developed an inventory control model under stock dependent consummation rate. **Mandal and Phaujdar (1989)** presented an inventory model for deteriorating items and stock dependent consumption rate. An ordering policy for decaying inventory was developed by **Aggarwal and Jaggi (1989)**. **Su et al (1996)** developed an inventory model under inflation for stock dependent consumption rate and exponential decay. A deterministic production inventory model for deteriorating items and exponential declining damand was represented by **Lin et al (1999). Naresh Kumar and A. K. Sharma (2000)** formulated deterministic production inventory model for deteriorating items with an exponential declining demand.

An approach based on inventory model developed by **Yang, P. and Wee, H. (2003)** and explained a multi-lot size inventory-based system with constant demand and production rates. For different demand rates for different stage of inventory **Goyal, S.K. and Giri, B.C. (2003), Forghani, K., Mirzazadeh, A., & Rafiee, M. (2013), Singh, S.R. and Jain, R. (2009), Manna, S.K. and Chiang, C. (2010) and Singh, S.R. and Diksha (2009)** studied the Economic Production Quantity model under the consideration of partial backlogged. **Aggarwal, V. and Bahari-Hashani, H. (1991)** studied a deteriorating inventory model with exponentially decreasing demand, where author have assumed a finite planning horizon in the model**. Manna, S. K., Lee, C. C., and Chiang, C. (2009)** developed an Economic Production Quantity model with time-varying demand and partially backlogged model. **Sana, S., Goyal, S.K. and Chaudhuri, K.S. (2004)** explored the inventory production-based inventory model by considering shortages. **Singh, S.R. and Saxena, N. (2012)**, investigated and formulated an optimal returned policy-based model by considering reverse logistics with backorders where **Kumar, N., Singh, S.R. and Kumari, R. (2012)** extended these models with new criteria of limited storage facility under inflation. In all inventory models, author seen that the product almost all items either software and hardware have fixed shelf life. Because of the expiration, new technology, time-consuming, more efforts, non-auto start, etc. substantial decay of inventory system cannot be neglected, it was another major feature of the real world. In this field, **Ghare, P.M. and Schrader, G.F. (1963)** developed an Economic Production Quantity model with constant demand and exponentially decaying items. The stock decreases due to demand which is a function of the on-hand inventory and deterioration which is constant. It is seen that items have a lifetime which cessation when advantages become zero of the on-hand inventories. It is noticed that products have a lifetime which cessation when the benefit becomes zero. Considering the concept of permissible delay in payment, **Singh, S.R. and Singh,**

T.J. (2008b) introduced a perishable inventory model with a parabolic rate of demand along with partial backlogging. For the product of low cycle **Dem, H. and Singh, S.R. (2013)** presented a quality consideration deteriorating inventory model. **Sharma, S. and Singh, S.R. (2013)** considered a multivariate demand model for decaying items having shortages. **Sharma, S, Singh, S.R. and Ram, M. (2015)** presented a demand dependent production inventory model with price-sensitive demand and shortages.
U. Mishra, L.E. Cárdenas-Barrón, S.Tiwari, A.A. Shaikh, G Treviño-Garza (2017), introduced an inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. **A. Mashud, M. Khan, M. Uddin, M. Islam (2018)** presented a non-instantaneous inventory model having different deterioration rates with stock and price dependent demand under partially backlogged shortages.
P. Alavian, Y. Eun, S.M. Meerkov, L. Zhang (2020) developed a smart production system: automating decision-making in manufacturing environment. **D. Das, G.C. Samanta, A. Barman, P.K. De, K.K. Mohanta (2022)** introduced a recovery mathematical model for the impact of supply chain interruptions during the lockdown in COVID-19 using two warehouse perishable inventory policies In the last few decades researcher pay attention to the deterioration-based models, they considered the different deterioration rates for different environment conditions. Now author discuss the non – instantaneous deteriorating items in our study so many researchers contribute to this field, but author have discussed some of them.

ASSUMPTIONS AND NOTATIONS:
A Production inventory model for decaying items with lifetime and price dependent demand for finite time horizon is developed under following assumptions and notations:

(i) A single item is considered over the prescribed period T units of time, which is subject to a linear deterioration rate.
(ii) D (p) is the demand rate when the selling price is p.
(iii) I (t) is the inventory level at any time t, $t \geq 0$.
(iv) Production rate is the linear combination of on hand inventory and demand rate. i.e. $P(t,p) = I(t) + bD(p)$, $0 \leq b < 1$, $t \geq 0$
(v) θ (t) is the linear deterioration rate s.t. $\theta(t) = \alpha t$, $0 < \alpha << 1$
(vi) μ is the life time of items.
(vii) C_1 is the inventory carrying cost per unit time.
(viii) C_2 is the shortage cost per unit time.
(ix) C_3 is the setup cost for each new cycle.
(x) C_d is the cost of deteriorated items.
(xi) $T(= t_1 + t_2 + t_3 + t_4)$ is the cycle time.
(xii) Q_1 is the maximum inventory level.
(xiii) Q_2 is the unfilled order backlog.
(xiv) q be the inventory level at time μ , $0 \leq \mu \leq t_1$
(xv) No replacement or repair of deteriorated items is made during a given cycle.
(xvi) Shortages are allowed and backlogged.
(xvii) K is the total average cost of the system.
(xviii) Deterioration of the items is considered only after the life-time of items.

MATHEMATICAL MODEL AND ANALYSIS FOR THE SYSTEM:
Initially the inventory level is zero. The production starts at time t = 0 with the concept of life-time and after life-time μ when inventory level become q deterioration can take place and after

t_1 units of time, it reaches to maximum inventory level Q_1. After this production stopped and at time, $t = t_2$ the inventory level becomes zero. At this time shortage starts developing at time $t = t_3$, it reaches to maximum shortage level Q_2. At this fresh production starts to clear the backlog by the time $t = t_4$. Our aim is to find the optimum values of t_1, t_2, t_3, t_4, Q_1 and Q_2 that minimize the total average cost (K) over the time horizon (0, T).

The inventory level I (t) at time t ($0 \le t \le T$) satisfies the differential equations:

$$I'(t) = P(t,p) - D(p), \qquad 0 \le t \le \mu \qquad \dots (1)$$

$$I'(t) = -\theta(t)I(t) + P(t,p) - D(p), \qquad \mu \le t \le t_1 \qquad \dots (2)$$

$$I'(t) = -\theta(t)I(t) - D(p), \qquad 0 \le t \le t_2 \qquad \dots (3)$$

$$I'(t) = -D(p), \qquad 0 \le t \le t_3 \qquad \dots (4)$$

$$I'(t) = P(t,p) - D(p), \qquad 0 \le t \le t_4 \qquad \dots (5)$$

The boundary conditions are

$$I(t) = 0 \text{ at } t = 0,\ t_1 + t_2 \text{ and } T \qquad \dots (6)$$

$$I(\mu) = q,\ I(t_1) = Q_1,\ I(t_1 + t_2 + t_3) = Q_2 \qquad \dots (7)$$

By equation (1), $\frac{dI(t)}{dt} = P(t,p) - D(p) \qquad 0 \le t \le \mu$

$$\Rightarrow \quad \frac{dI(t)}{dt} - I(t) = (b-1).D(p), \text{ since } P(t,p) = I(t) + bD(p)$$

Solution of (1) is given by

$$I(t).e^{-t} = \int (b-1)D(p)e^{-t}dt + A$$

where A is some constant.

$$I(t).e^{-t} = -(b-1)D(p)e^{-t} + A$$

Using boundary conditions, we get the solution

$$I(t) = (b-1)D(p)(e^{t} - 1), \qquad 0 \le t \le \mu \qquad \dots (8)$$

By equation (2),

$$\frac{dI(t)}{dt} + \theta(t)I(t) = P(t,p) - D(p), \qquad \mu \le t \le t_1$$

$$\Rightarrow \quad \frac{dI(t)}{dt} + (\alpha t - 1)I(t) = (b-1)D(p) \qquad [\text{since } \theta(t) = \alpha t]$$

Solution of this equation is given by

$$I(t).e^{\frac{\alpha t^2}{2} - t} = \int (b-1)D(p).e^{\frac{\alpha t^2}{2} - t}dt + B$$

where B is some constant.

$$\Rightarrow \quad I(t).e^{\frac{\alpha t^2}{2} - t} = (b-1)D(p)\int \left(1 + \frac{\alpha t^2}{2}\right)e^{-t}dt + B$$

[leaving the terms of higher powers of α]

$$\Rightarrow \quad I(t)e^{\frac{\alpha t^2}{2} - t} = (1-b)D(p)e^{-t}\left[1 + \frac{\alpha}{2}(t^2 + 2t + 2)\right] + B$$

Using boundary conditions at $t = \mu$, we get

$$I(t)e^{\frac{\alpha t^2}{2}} = (1-b)D(p)\left[1+\frac{\alpha}{2}(t^2+2t+2)\right] + qe^{\frac{\alpha\mu^2}{2}}e^{t-\mu}$$
$$-(1-b)D(p)e^{t-\mu}\left[1+\frac{\alpha}{2}(\mu^2+2\mu+2)\right]$$

$$\Rightarrow \quad I(t) = (1-b)D(p)\left[1+\frac{\alpha}{2}(t^2+2t+2)\right]\left(1-\frac{\alpha t^2}{2}\right) + qe^{\frac{\alpha(\mu^2-t^2)}{2}}e^{t-\mu}$$
$$-(1-b)D(p)e^{t-\mu}\left[1+\frac{\alpha}{2}(\mu^2+2\mu+2)\right]\left(1-\frac{\alpha t^2}{2}\right)$$

[leaving the terms of higher powers of α]

$$\Rightarrow \quad I(t) = (1-b)D(p)[1+\alpha(t+1)] + qe^{t-\mu}\left[1+\frac{\alpha}{2}(\mu^2-t^2)\right]$$
$$-(1-b)D(p)e^{t-\mu}\left[1+\alpha(\mu+1)+\frac{\alpha}{2}(\mu^2-t^2)\right], \ \mu \le t \le t_1 \quad \ldots (9)$$

By equation (3),

$$\frac{dI(t)}{dt} = -\theta(t)I(t) - D(p), \qquad 0 \le t \le t_2$$

whose solution is given by

$$I(t).e^{\frac{\alpha t^2}{2}} = -\int D(p)e^{\frac{\alpha t^2}{2}}dt + C$$, where C is some constant.

$$\Rightarrow \quad I(t).e^{\frac{\alpha t^2}{2}} = -D(p)\left(t+\frac{\alpha t^3}{6}\right) + C$$

By using boundary conditions, we get the solution.

$$I(t).e^{\frac{\alpha t^2}{2}} = -D(p)\left(t+\frac{\alpha t^3}{6}\right) + D(p)\left(t_2+\frac{\alpha t_2{}^3}{6}\right)$$

$$\Rightarrow I(t) = D(p)\left[(t_2-t)+\frac{\alpha}{6}\left(t_2^3-t^3\right)\right]\left(1-\frac{\alpha t^2}{2}\right)$$

$$\Rightarrow \quad I(t) = D(p)\left[(t_2-t)+\alpha\left\{\frac{t_2{}^3}{6}-\frac{t^2t_2}{2}+\frac{t^3}{3}\right\}\right], \quad 0 \le t \le t_2 \qquad \ldots(10)$$

By equation (4)

$$\frac{dI(t)}{dt} = -D(p), \qquad 0 \le t \le t_3$$

whose solution is given by

$$I(t) = -D(p)t, \qquad 0 \le t \le t_3 \qquad \ldots(11)$$

Also by equation (5), we have

$$\frac{dI(t)}{dt} = P(t,p) - D(p), \qquad 0 \le t \le t_4$$

$$\Rightarrow \quad \frac{dI(t)}{dt} - I(t) = (b-1)D(p)$$

and solution is given by

$$I(t)e^{-t} = -(b-1)D(p)e^{-t} + E$$

where E is some constant. Using boundary conditions we get the solution

$$I(t)e^{-t} = -(b-1)D(p)e^{-t} + (b-1)D(p)e^{-t_4}$$

$$\Rightarrow \quad I(t) = (b-1)D(p)(e^{t-t_4} - 1) \qquad 0 \le t \le t_4 \qquad \ldots(12)$$

Now by using boundary conditions (6) and (7). We get

$I(t) = Q_1$, when $t = t_1$, $\mu \le t \le t_1$

Also $I(t) = Q_1$, when $t = 0$, $0 \le t \le t_2$

By (9) when $t = t_1$

$$Q_1 = (1-b)D(p)[1+\alpha(t_1+1)] + qe^{t_1-\mu}\left[1+\frac{\alpha}{2}(\mu^2 - t_1^{\,2})\right]$$
$$-(1-b)D(p)e^{t_1-\mu}\left[1+\alpha(\mu+1)+\frac{\alpha}{2}(\mu^2 - t_1^{\,2})\right]$$

and by equation (10), when t=0

$$Q_1 = D(p)\left(t_2 + \frac{\alpha t_2^{\,3}}{6}\right)$$
$$= (1-b)D(p)[1+\alpha(t_1+1)] + qe^{t_1-\mu}\left[1+\frac{\alpha}{2}(\mu^2 - t_1^{\,2})\right]$$
$$-(1-b)D(p)e^{t_1-\mu}\left[1+\alpha(\mu+1)+\frac{\alpha}{2}(\mu^2 - t_1^{\,2})\right] \qquad \ldots (13)$$

From this equation we observe that variables t_1 and t_2 are not independent variables. Therefore we can write

$$t_2 = f_1(t_1) \qquad \ldots (14)$$

When $t = t_3$, $I(t) = Q_2$, $0 \le t \le t_3$

And when $t = 0$, $I(t) = Q_2$, $0 \le t \le t_4$

By equation (11), $Q_2 = -D(p)t_3$

and also by equation (12), $Q_2 = (b-1)D(p)(e^{-t_4} - 1)$

$$\Rightarrow \quad Q_2 = -D(p)t_3 = (b-1)D(p)(e^{-t_4} - 1) \qquad \ldots (15)$$

From this equation we observe that variables t_3 and t_4 are dependent. Therefore, we can write

$$t_3 = (1-b)(e^{-t_4} - 1) = f_2(t_4) \qquad \ldots (16)$$

The deterioration cost for the period (0, T) is given by

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$$C_d\left[\int_{\mu}^{t_1}\theta(t)I(t)dt+\int_{0}^{t_2}\theta(t)I(t)dt\right]$$

$$=C_d\left[\int_{\mu}^{t_1}\alpha t\left\{(1-b)D(p)\big(1+\alpha(t+1)\big)+qe^{t-\mu}\left(1+\frac{\alpha}{2}(\mu^2-t^2)\right)\right.\right.$$

$$\left.-(1-b)D(p)e^{t-\mu}\left(1+\alpha(\mu+1)+\frac{\alpha}{2}(\mu^2-t^2)\right)\right\}dt$$

$$\left.+\int_{0}^{t_2}\alpha t\left\{D(p)\left[(t_2-t)+\alpha\left(\frac{t_2^3}{6}-\frac{t^2t_2}{2}+\frac{t^3}{3}\right)\right]\right\}dt\right]$$

$$=C_d\left[(1-b)D(p)\frac{\alpha}{2}(t_1^2-\mu^2)+q\alpha\int_{\mu}^{t_1}te^{t-\mu}dt\right.$$

$$\left.-(1-b)D(p)\alpha\int_{\mu}^{t_1}te^{t-\mu}dt+\alpha D(p)\frac{t_2^3}{6}\right]$$

[leaving the terms of higher powers of α].

$$=C_d\left[\alpha\left\{e^{t_1-\mu}(t_1-1)-(\mu-1)\right\}\left\{q-(1-b)D(p)\right\}\right.$$

$$\left.+\alpha D(p)\left\{(1-b)\left(\frac{t_1^2-\mu^2}{2}\right)+\frac{t_2^3}{6}\right\}\right] \quad \ldots (17)$$

Inventory carrying cost over the period (0, T) is given by

$$C_1\left[\int_{0}^{\mu}I(t)dt+\int_{\mu}^{t_1}I(t)dt+\int_{0}^{t_2}I(t)dt\right]$$

$$=C_1\left[\int_{0}^{\mu}(b-1)D(p)(e^t-1)dt+\int_{\mu}^{t_1}\left\{(1-b)D(p)\big(1+\alpha(t+1)\big)\right.\right.$$

$$\left.+qe^{t-\mu}\left(1+\frac{\alpha}{2}(\mu^2-t^2)\right)-(1-b)D(p)e^{t-\mu}\left(1+\alpha(\mu+1)+\frac{\alpha}{2}(\mu^2-t^2)\right)\right\}dt$$

$$\left.+\int_{0}^{t_2}\left\{D(p)\left[(t_2-t)+\alpha\left(\frac{t_2^3}{6}-\frac{t^2t_2}{2}+\frac{t^3}{3}\right)\right]\right\}\right.$$

$$=C_1\left[(b-1)D(p)(e^{\mu}-\mu-1)+(1-b)D(p)\left\{(1+\alpha)(t_1-\mu)+\frac{\alpha}{2}\left(t_1^2-\mu^2\right)\right\}\right.$$

$$+\left\{q\left(1+\frac{\alpha\mu^2}{2}\right)-(1-b)D(p)\left(1+\alpha(\mu+1)+\frac{\alpha}{2}\mu^2\right)\right\}\left\{e^{t_1-\mu}-1\right\}$$

$$-\frac{\alpha}{2}\{q-(1-b)D(p)\}\left\{e^{t_1-\mu}(t_1{}^2-2t_1+2)-(\mu^2-2\mu+2)\right\}$$

$$+D(p)\left(\frac{t_2{}^2}{2}+\frac{\alpha t_2{}^4}{12}\right)\Bigg]\Bigg] \quad \ldots (18)$$

Shortage cost is given by

$$C_2\left[-\int_0^{t_3} I(t)dt+\int_0^{t_4} I(t)dt\right]$$

$$= C_2\left[D(p)\frac{t_3{}^2}{2}+(b-1)D(p)(1-t_4-e^{-t_4})\right] \quad \ldots (19)$$

Hence the total average cost of the inventory system is, K = (Set up cost + Deterioration cost + Inventory carrying cost + Shortage cost)/T.

$$=\frac{C_3}{T}+\frac{C_d}{T}\Big[\alpha\left\{e^{t_1-\mu}(t_1-1)-(\mu-1)\right\}\{q-(1-b)D(p)\}+\alpha D(p)$$

$$\left\{(1-b)\frac{t_1{}^2-\mu^2}{2}+\frac{t_2{}^3}{6}\right\}\Bigg]+\frac{C_1}{T}\Big[(b-1)D(p)(e^{\mu}-\mu-1)+(1-b)D(p)$$

$$\left\{(1+\alpha)(t_1-\mu)+\frac{\alpha}{2}(t_1{}^2-\mu^2)\right\}+\left\{q\left(1+\frac{\alpha\mu^2}{2}\right)\right.$$

$$\left.-(1-b)D(p)\left[1+\alpha(\mu+1)+\frac{\alpha\mu^2}{2}\right]\right\}\left(e^{t_1-\mu}-1\right)$$

$$-\frac{\alpha}{2}\{q-(1-b)D(p)\}\Big\{e^{t_1-\mu}(t_1^2-2t_1+2)$$

$$-(\mu^2-2\mu+2)\Big\}+D(p)\left\{\frac{t_2^2}{2}+\frac{\alpha t_2^4}{12}\right\}\Bigg]+\frac{C_2D(p)}{T}\Bigg[\frac{t_3^2}{2}$$

$$+(b-1)(1-t_4-e^{-t_4})\Bigg] \quad \ldots (20)$$

APPROXIMATE SOLUTION PROCEDURE:

Equation (20) contains four variables t_1, t_2, t_3 and t_4. However these variables are not independent and are related by (14) and (16). Also we have K > 0.

For minimum K, we must have

$$\frac{\partial K}{\partial t_1}=0 \text{ and } \frac{\partial K}{\partial t_4}=0 \quad \ldots (21)$$

Provided these values of t_j satisfy the conditions

$$\frac{\partial^2 K}{\partial t_1^2}>0,\ \frac{\partial^2 K}{\partial t_4^2}>0 \text{ and } \frac{\partial^2 K}{\partial t_1^2}\frac{\partial^2 K}{\partial t_4^2}-\left(\frac{\partial^2 K}{\partial t_1\partial t_4}\right)^2>0$$

Now differentiating (20) with respect to t_1 and t_4, we get

$$
\begin{aligned}
&\{1+f_1'(t_1)\}\Big[C_3+C_d\Big\{\alpha\Big(e^{t_1-\mu}(t_1-1)-(\mu-1)\Big)\Big(q-(1-b)D(p)\Big)\\
&+\alpha D(p)\left((1-b)\frac{t_1^2-\mu^2}{2}+\frac{f_1^3(t_1)}{6}\right)\Big\}+C_1\Big[(b-1)D(p)(e^{\mu}-\mu-1)\\
&+(1-b)D(p)\left((1+\alpha)(t_1-\mu)+\frac{\alpha}{2}(t_1^2-\mu^2)\right)\\
&+\left\{q\left(1+\frac{\alpha\mu^2}{2}\right)-(1-b)D(p)\left(1+\alpha(\mu+1)+\frac{\alpha\mu^2}{2}\right)\right\}\left(e^{t_1-\mu}-1\right)\\
&-\frac{\alpha}{2}\{q-(1-b)D(p)\}\Big\{e^{t_1-\mu}(t_1^2-2t_1+2)-(\mu^2-2\mu+2)\Big\}\\
&+D(p)\left\{\frac{f_1^2(t_1)}{2}+\frac{\alpha f_1^4(t_1)}{12}\right\}\Big]+C_2\Big[D(p)\frac{f_2^2(t_4)}{2}\\
&+(b-1)D(p)(1-t_4-e^{-t_4})\Big]\Big]-C_dT\Big[\alpha t_1e^{t_1-\mu}\{q-(1-b)D(p)\}\\
&+\alpha D(p)\left\{(1-b)t_1+\frac{f_1^2(t_1)f_1'(t_1)}{2}\right\}\Big]-C_1T\Big[(1-b)D(p)\{(1+\alpha)+\alpha t_1\}\\
&+\left\{q\left(1+\frac{\alpha\mu^2}{2}\right)-(1-b)D(p)\left(1+\alpha(\mu+1)+\frac{\alpha\mu^2}{2}\right)\right\}e^{t_1-\mu}\\
&-\frac{\alpha}{2}\Big(q-(1-b)D(p)\Big)t_1^2e^{t_1-\mu}+D(p)\left\{f_1(t_1)f_1'(t_1)+\frac{\alpha f_1^3(t_1)f_1'(t_1)}{3}\right\}\Big]=0
\end{aligned}
\qquad \ldots (22)
$$

and

$$
\begin{aligned}
&\{1+f_2'(t_4)\}\Big[C_3+C_d\Big\{\alpha\Big(e^{t_1-\mu}(t_1-1)-(\mu-1)\Big)\Big(q-(1-b)D(p)\Big)\\
&+\alpha D(p)\left((1-b)\frac{t_1^2-\mu^2}{2}+\frac{f_1^3(t_1)}{6}\right)\Big\}+C_1\Big[(b-1)D(p)(e^{\mu}-\mu-1)\\
&+(1-b)D(p)\left((1+\alpha)(t_1-\mu)+\frac{\alpha}{2}(t_1^2-\mu^2)\right)\\
&+\left\{q\left(1+\frac{\alpha\mu^2}{2}\right)-(1-b)D(p)\left(1+\alpha(\mu+1)+\frac{\alpha\mu^2}{2}\right)\right\}\left(e^{t_1-\mu}-1\right)\\
&-\frac{\alpha}{2}\{q-(1-b)D(p)\}\Big\{e^{t_1-\mu}(t_1^2-2t_1+2)-(\mu^2-2\mu+2)\Big\}\\
&+D(p)\left\{\frac{f_1^2(t_1)}{2}+\frac{\alpha f_1^4(t_1)}{12}\right\}\Big]+C_2\Big[D(p)\frac{f_2^2(t_4)}{2}
\end{aligned}
$$

$$+(b-1)D(p)(1-t_4-e^{-t_4})]\Big]-C_2T\Big\{D(p)f_2(t_4)f_2{}'(t_4)$$
$$+(b-1)D(p)(e^{-t_4}-1)\Big\}=0 \qquad \ldots (23)$$

Where $f_1(t_1)$ and $f_2(t_4)$ are given by equations (14) and (16). From these two simultaneous non-linear equations, the optimum values of t_1 and t_4 can be found out. The optimum values of t_2, t_3, Q_1, Q_2 and minimum average cost K can be obtained from (13), (15) and (20).

SPECIAL CASES.

CASE I. If b = 0, then the discussed model reduces to production inventory model in which production rate depends on inventory level.

Equation (20) reduces to

$$K=\frac{C_3}{T}+\frac{C_d}{T}\left[\alpha\left\{e^{t_1-\mu}(t_1-1)-(\mu-1)\right\}\{q-D(p)\}+\alpha D(p)\left\{\frac{t_1{}^2-\mu^2}{2}+\frac{t_2{}^3}{6}\right\}\right]$$
$$+\frac{C_1}{T}\Big[D(p)(1+\mu-e^{\mu})+D(p)\ \left\{(1+\alpha)(t_1-\mu)+\frac{\alpha}{2}(t_1{}^2-\mu^2)\right\}$$
$$+\left\{q\left(1+\frac{\alpha\mu^2}{2}\right)-D(p)\left[1+\alpha(\mu+1)+\frac{\alpha\mu^2}{2}\right]\right\}\left(e^{t_1-\mu}-1\right)$$
$$-\frac{\alpha}{2}\{q-D(p)\}\left\{e^{t_1-\mu}(t_1^2-2t_1+2)\ -(\mu^2-2\mu+2)\right\}$$
$$+D(p)\left\{\frac{t_2^2}{2}+\frac{\alpha t_2^4}{12}\right\}\Big]+\frac{C_2D(p)}{T}\left[\frac{t_3^2}{2}-(1-t_4-e^{-t_4})\right]$$

CASE II. If $\alpha=0$, b = 0 then the discussed model reduces to production inventory model without deterioration and in which production rate depends on inventory level.

Equation (20) reduces to

$$K=\frac{C_3}{T}+\frac{C_1}{T}\Big[D(p)(1+\mu-e^{\mu})+D(p)(t_1-\mu)$$
$$+\{q-D(p)\}(e^{t_1-\mu}-1)+D(p)\frac{t_2^2}{2}\Big]+\frac{C_2D(p)}{T}\left[\frac{t_3^2}{2}-(1-t_4-e^{-t_4})\right]$$

CASE III. If $\mu=0$, then the discussed model reduces to the production inventory model without life time.

Then equation (20) becomes

$$K=\frac{C_3}{T}+\frac{C_d}{T}\left[\alpha\left\{e^{t_1}(t_1-1)+1)\right\}\{q-(1-b)D(p)\}+\alpha D(p)\left\{(1-b)\frac{t_1{}^2}{2}+\frac{t_2{}^3}{6}\right\}\right]$$

$$+\frac{C_1}{T}\Big[(1-b)D(p)\left\{(1+\alpha)t_1+\frac{\alpha}{2}{t_1}^2\right\}+\{q-(1-b)D(p)(1+\alpha)\}\left(e^{t_1}-1\right)$$

$$-\frac{\alpha}{2}\{q-(1-b)D(p)\}\left\{e^{t_1}(t_1^2-2t_1+2)-2\right\}+D(p)\left\{\frac{t_2^2}{2}+\frac{\alpha t_2^4}{12}\right\}\Bigg]$$

$$+\frac{C_2 D(p)}{T}\left[\frac{t_3^2}{2}+(b-1)(1-t_4-e^{-t_4})\right]$$

CASE IV. If $\mu=0$, b = 0 and $\alpha=0$ then the discussed model reduces to the production inventory model without life time and deterioration in which production rate depends on inventory level.

Then equation (20) becomes

$$K=\frac{C_3}{T}+\frac{C_1}{T}\left[D(p)t_1+\{q-D(p)\}(e^{t_1}-1)+D(p)\frac{t_2^2}{2}\right]$$

$$+\frac{C_2 D(p)}{T}\left\{\frac{t_3^2}{2}-(1-t_4-e^{-t_4})\right\}$$

CONCLUSION:

In this paper, a production inventory model for deteriorating items with lifetime and price dependent demand is developed for a fixed and finite time horizon. Shortages and excess demand is backlogged. In the present model production rate is taken as linear combination of on-hand inventory and demand both. Special cases of model are also discussed.

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