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QUOTIENT-4 CORDIAL LABELING OF GENERALIZED PETERSEN GRAPH

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Abstract
Let $G(V,E)$ be a simple graph of order p and size q. Let $\varphi: V(G) \to Z_5 - \{0\}$ be a function. For each edge set $E(G)$ define the labeling $\varphi^*: E(G) \to Z_4$ by $\varphi^*(uv) = \lceil(\frac{\varphi(u)}{\varphi(v)})\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1, 1 \leq i,j \leq 4$, $i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1, 0 \leq k,l \leq 3, k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y. Here some cases of generalized Petersen graphs are quotient-4 cordial labeling.
Keywords:
Generalized Petersen graphs, quotient-4 cordial labeling and quotient-4 cordial graph.

1. INTRODUCTION

Here the graphs considered are finite, simple, undirected and non-trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [4] for more information. The cordial labeling concept was first introduced by Cahit [2]. H- and H2 –cordial labeling was introduced by Freeda S and ChellathuraiR.S [3]. Mean Cordial Labeling was introduced by Albert William, Indira Rajasingh, and S Roy [1]. Quotient-4 cordial labeling was introduced by P.Sumathi and S.Kavitha [5]. A graph G is said to be quotient-4 cordial graph if it receives quotient-4 cordial labeling .Let $v_\varphi(i)$ denotes the number of vertices labeled with i and $e_\varphi(k)$ denotes the number of edges labeled with k, $1 \leq i \leq 4, 0 \leq k \leq 3$.

2. DEFINITIONS

Definition: 2.1[6] Let $G\ (V,E)$ be a simple graph of order p and size q. Let $\varphi: V(G) \to Z_5 - \{0\}$ be a function. For each edge set $E\ (G)$ define the labeling $\varphi^*: E(G) \to Z_4$ by $\varphi^*(uv) = \lceil(\frac{\varphi(u)}{\varphi(v)})\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1$, $1 \leq i,j \leq 4$, $i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1, 0 \leq k,l \leq 3, k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y.

Definition: 2.2[5] A *generalized Petersen graph* $P\ (n,m)$, $n \geq 3, 1 \leq m < \frac{n}{2}$ is a 3-regular graph with $2n$ vertices $x_1, x_2 \dots x_n, y_1, y_2 \dots y_n$ and edges $\{x_i, y_i\}, \{x_i, y_{i+1}\}, \{y_i, y_{i+m}\}$ for all $i \in \{1, 2 \dots n\}$ where the subscripts are reduced modulo n.

3. MAIN RESULTS

Theorem: 3.1 A generalized Petersen graph $P\ (n, 1)$ is quotient-4 cordial if $n\ \geq 3$.
Proof: Let G be a generalized Petersen graph $P(n\ ,1)$.
$V\ (G) = \{x_t, y_t\ :\ 1 \leq t\ \leq n\}$.
$E(G) = \left\{(x_t y_t), (x_t x_{t+m}): 1\ \leq\ t\ \leq\ n, 1 \leq m < \frac{n}{2}\right\} \cup \{y_t y_{t+1}: 1 \leq t \leq n-1\} \cup \{\ y_1 y_n\}$. Where the subscripts are taken modulo n.
Here $|V(G)| = 2n\ , |E(G)| = 3n\ .$
Define $\varphi\ : V(G) \to \{1, 2, 3, 4\}$.

The values of x_t *and* y_t are labeled as follows:

Case 1:

$n \equiv 0, 7$ (modulo 8)

For $1 \le t \le n$.

$\varphi\ (x_t) = 1 \quad if\ t \equiv 2, 4$ (modulo 8).
$\varphi\ (x_t) = 2 \quad if\ t \equiv 6, 7$ (modulo 8).
$\varphi\ (x_t) = 3 \quad if\ t \equiv 0, 1, 3, 5$ (modulo 8).
$\varphi\ (y_t) = 1 \quad if\ t \equiv 1, 3$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 6, 7$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 0, 2, 4, 5$ (modulo 8).

Case 2:

$n \equiv 1$ (modulo 8).

For $1 \le t \le n-2$, the labeling of x_t and y_t values are same as case 1.

$\varphi\ (x_n) = 3, \varphi\ (x_{n-1}) = 2.$
$\varphi\ (y_n) = 4, \varphi\ (y_{n-1}) = 1.$

Case 3:

$n \equiv 2$ (modulo 8).

For $1 \le t \ \le n-3$, the labeling of x_t values are same as case 1.

$\varphi\ (x_n) = \varphi\ (x_{n-1}) = 3, \varphi\ (x_{n-2}) = 2.$

For $1 \le t \ \le n$, the labeling of y_t values are same as case 1.

Case 4:

$n \equiv 3$ (modulo 8)

For $1 \le t \le n$, the labeling of x_t values are same as case 1.

For $1 \le \mathrm{t} \ \le \mathrm{n}-4$, the labeling of y_t values are same as case 1.

$\varphi\ (y_n) = \varphi\ (y_{n-2}) = 4, \varphi\ (y_{n-1}) = 1, \varphi\ (y_{n-3}) = 2.$

Case 5:

$n \equiv 4$ (modulo 8)

For $1 \le t \ \le n-5$, the labeling of x_t and $y_t$values are same as case 1.

$\varphi\ (x_n) = 1, \varphi\ (x_{n-1}) = \varphi\ (x_{n-2}) = 3, \varphi\ (x_{n-3}) = 4, \varphi\ (x_{n-4}) = 2.$
$\varphi\ (y_n) = \varphi\ (y_{n-2}) = 4, \varphi\ (y_{n-1}) = 1, \varphi\ (y_{n-3}) = 3, \varphi\ (y_{n-4}) = 2.$

Case 6:

$n \equiv 5$ (modulo 8)

For $1 \le \mathrm{t} \ \le \mathrm{n}-3$, the labeling of x_t and y_t values are same as case 1.

$\varphi\ (x_n) = 3, \varphi\ (x_{n-1}) = 1, \varphi\ (x_{n-2}) = 4.$
$\varphi\ (y_n) = \varphi\ (y_{n-1}) = 2, \varphi\ (y_{n-2}) = 4.$

Case 7:

$\mathrm{n} \ \equiv 6$ (modulo 8)

For $1 \le \mathrm{t} \ \le \mathrm{n}-6$, the labeling of x_t and y_t values are same as case 1.

$\varphi\ (x_n) = \varphi\ (x_{n-1}) = 3, \varphi\ (x_{n-2}) = 1, \varphi\ (x_{n-3}) = 4, \varphi\ (x_{n-4}) = \varphi\ (x_{n-5}) = 2.$
$\varphi\ (y_n) = 3, \varphi\ (y_{n-1}) = \varphi\ (y_{n-3}) = 1, \varphi\ (y_{n-2}) = \varphi\ (y_{n-4}) = 4, \varphi\ (y_{n-5}) = 2.$

The following table shows that n concurrence is realized with modulo 8.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0,2,4,6$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$
$n \equiv 1,7$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}-1$	$\frac{n+1}{2}-1$
$n \equiv 3$	$\frac{n+1}{2}$	$\frac{n+1}{2}-1$	$\frac{n+1}{2}$	$\frac{n+1}{2}-1$
$n \equiv 5$	$\frac{n+1}{2}$	$\frac{n+1}{2}-1$	$\frac{n+1}{2}-1$	$\frac{n+1}{2}$

Table 1: Vertex labeling of $P(n,1)$ graph

The following table shows that n concurrence is realized with modulo 8.

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 0,4$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$
$n \equiv 1,5$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}-1$
$n \equiv 2$	$\frac{3n+2}{4}$	$\frac{3n+2}{4}$	$\frac{3n+2}{4}-1$	$\frac{3n+2}{4}-1$
$n \equiv 3$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$
$n \equiv 6$	$\frac{3n+2}{4}$	$\frac{3n+2}{4}$	$\frac{3n+2}{4}$	$\frac{3n+2}{4}-1$
$n \equiv 7$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}+1$

Table 2: Edge labeling of $P(n,1)$ graph

The above tables 1 and 2 show that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the generalized Petersen graph $P(n,1)$ is quotient-4 cordial labeling.

Theorem: 3.2 If m is even and $n \equiv 0$ (modulo 8), $n \geq 3$ then the generalized Petersen graph $P(n,m)$ is quotient-4 cordial.

Proof: Let G be a generalized Petersen graph $P(n,m)$.

$V(G) = \{x_t, y_t : 1 \leq t \leq n\}$.

$E(G) = \left\{(x_t y_t), (x_t x_{t+m}) : 1 \leq t \leq n, 1 \leq m < \frac{n}{2}\right\} \cup \{y_t y_{t+1} : 1 \leq t \leq n-1\} \cup \{y_1 y_n\}$. Where the subscripts are taken modulo n.

Here $|V(G)| = 2n$, $|E(G)| = 3n$.

Define $\varphi : V(G) \to \{1,2,3,4\}$.

The values of x_t and y_t are labeled as follows:

When $n \equiv 0$ (modulo 8)
For $1 \leq t \leq n$.

Case 1:

$m \equiv 0$ (modulo 8).

Sub Case 1.1:

When $n = md$ where d is even multiples.
For $1 \leq t \leq md$.
$\varphi\ (x_t) = 1 \quad if\ t \equiv 4, 7$ (modulo 8).
$\varphi\ (x_t) = 2 \quad if\ t \equiv 1, 2$ (modulo 8).
$\varphi\ (x_t) = 3 \quad if\ t \equiv 5, 6$ (modulo 8).
$\varphi\ (x_t) = 4 \quad if\ t \equiv 0, 3$ (modulo 8).
For $mk + 1 \leq t \leq ml$ where $k \equiv 0$ (modulo 2) and $l \equiv 1$ (modulo 2).
For $0 \leq k \leq \left\lfloor \frac{n}{m} \right\rfloor - 1$ and $0 \leq l \leq \left\lfloor \frac{n}{m} \right\rfloor$.
$\varphi\ (y_t) = 1 \quad if\ t \equiv 0, 5$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 1, 2$ (modulo 8).
$\varphi\ (y_t) = 3 \quad if\ t \equiv 3, 4$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 6, 7$ (modulo 8).
For $mk + 1 \leq t \leq ml$ where $k \equiv 1$ (modulo 2) and $l \equiv 0$ (modulo 2).
For $1 \leq k \leq \left\lfloor \frac{n}{m} \right\rfloor$ and $2 \leq l \leq \left\lfloor \frac{n}{m} \right\rfloor$.
$\varphi\ (y_t) = 1 \quad if\ t \equiv 3, 6$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 1, 2$ (modulo 8).
$\varphi\ (y_t) = 3 \quad if\ t \equiv 0, 7$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 4, 5$ (modulo 8).

Sub Case 1.2:

When $n = md + r$ where d is even multiples and r is multiples of 8 and less than m.
For $1 \leq t \leq md$, the labeling of x_t values are same as sub case 1.1.
For $md + 1 \leq t \leq md + r$.
$\varphi\ (x_t) = 1 \quad if\ t \equiv 3, 6$ (modulo 8).
$\varphi\ (x_t) = 2 \quad if\ t \equiv 0, 1$ (modulo 8).
$\varphi\ (x_t) = 3 \quad if\ t \equiv 4, 5$ (modulo 8).
$\varphi\ (x_t) = 4 \quad if\ t \equiv 2, 7$ (modulo 8).
For $1 \leq t \leq md$, the labeling of y_t values are same as sub case 1.1.
For $md + 1 \leq t \leq md + r$.
$\varphi\ (y_t) = 1 \quad if\ t \equiv 4, 7$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 1, 2$ (modulo 8).
$\varphi\ (y_t) = 3 \quad if\ t \equiv 5, 6$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 0, 3$ (modulo 8).

Sub Case 1.3:

When $n = md$ where d is odd multiples.
For $1 \leq t \leq md - m$, the labeling of x_t values are same as sub case 1.1.
For $md - m + 1 \leq t \leq md$.
$\varphi\ (x_t) = 1 \quad if\ t \equiv 3, 6$ (modulo 8).
$\varphi\ (x_t) = 2 \quad if\ t \equiv 0, 1$ (modulo 8).
$\varphi\ (x_t) = 3 \quad if\ t \equiv 4, 5$ (modulo 8).
$\varphi\ (x_t) = 4 \quad if\ t \equiv 2, 7$ (modulo 8).
For $1 \leq t \leq md - m$, the labeling of y_t values are same as sub case 1.1.

For $md - m + 1 \leq t \leq md$.
$\varphi\ (y_t) = 1 \quad if\ t \equiv 4, 7$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 1, 2$ (modulo 8).
$\varphi\ (y_t) = 3 \quad if\ t \equiv 5, 6$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 0, 3$ (modulo 8).

Case 2:

$m \equiv 2$ (modulo 8).
$\varphi\ (x_t) = 1 \quad if\ t \equiv 2, 3$ (modulo 8).
$\varphi\ (x_t) = 2 \quad if\ t \equiv 6, 7$ (modulo 8).
$\varphi\ (x_t) = 3 \quad if\ t \equiv 5$ (modulo 8).
$\varphi\ (x_t) = 4 \quad if\ t \equiv 0, 1,4$ (modulo 8).
$\varphi\ (y_t) = 1 \quad if\ t \equiv 1, 4$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 6, 7$ (modulo 8).
$\varphi\ (y_t) = 3 \quad if\ t \equiv 2, 3, 5$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 0$ (modulo 8).

Case 3:

$m \equiv 4$ (modulo 8).
$\varphi\ (x_t) = 1 \quad if\ t \equiv 1, 3, 6$ (modulo 8).
$\varphi\ (x_t) = 2 \quad if\ t \equiv 0, 4$ (modulo 8).
$\varphi\ (x_t) = 3 \quad if\ t \equiv 2$ (modulo 8).
$\varphi\ (x_t) = 4 \quad if\ t \equiv 5, 7$ (modulo 8).
$\varphi\ (y_t) = 1 \quad if\ t \equiv 2$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 4, 5$ (modulo 8).
$\varphi\ (y_t) = 3 \quad if\ t \equiv 3, 6, 7$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 0, 1$ (modulo 8).

Case 4:

$m \equiv 6$ (modulo 8).
$\varphi\ (x_t) = 1 \quad if\ t \equiv 1, 5$ (modulo 8).
$\varphi\ (x_t) = 2 \quad if\ t \equiv 0, 6$ (modulo 8).
$\varphi\ (x_t) = 3 \quad if\ t \equiv 2, 4, 7$ (modulo 8).
$\varphi\ (x_t) = 4 \quad if\ t \equiv 3$ (modulo 8).
$\varphi\ (y_t) = 1 \quad if\ t \equiv 2, 4$ (modulo 8).
$\varphi\ (y_t) = 2 \quad if\ t \equiv 6, 7$ (modulo 8).
$\varphi\ (y_t) = 3 \quad if\ t \equiv 5$ (modulo 8).
$\varphi\ (y_t) = 4 \quad if\ t \equiv 0, 1, 3$ (modulo 8).

The following table shows that n & m concurrence is realized with modulo 8.

Nature of n and m	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0$ $m \equiv 0, 2, 4, 6$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$

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Table 3: Vertex labeling of $P(n, m)$ graph with $n \equiv 0$ (mod 8) and m is even.
The following table shows that n & m concurrence is realized with modulo 8.

Nature of n and m	$e_{\varphi}(0)$	$e_{\varphi}(1)$	$e_{\varphi}(2)$	$e_{\varphi}(3)$
$n \equiv 0$ $m \equiv 0, 2, 4, 6$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$

Table 4: Edge labeling of $P(n, m)$ graph with $n \equiv 0$ (mod 8) and m is even.
The above tables 3 and 4 show that $|v_{\varphi}(i) - v_{\varphi}(j)| \leq 1$ and $|e_{\varphi}(k) - e_{\varphi}(l)| \leq 1$. Hence the generalized Petersen graph $P(n, m)$ graph with $n \equiv 0$ (mod 8) and m is even is quotient-4 cordial labeling.
Illustration: 3.3 Figure 1 gives the quotient-4 cordial labeling for the graph $P(8, 2)$.

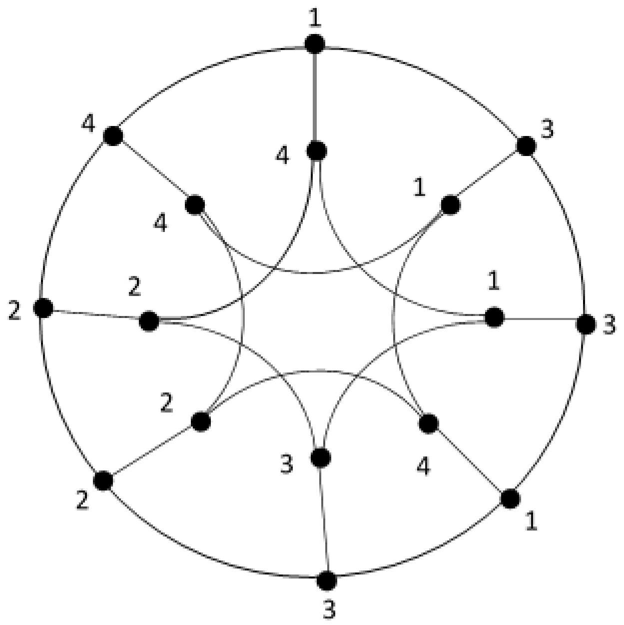

Figure 1.

4. CONCLUSION
In this paper, it is proved that some cases of generalized Petersen graphs which admits quotient-4 cordial. The existence of quotient-4 cordial labeling of different families of graphs will be the future work.

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