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RELIABILITY MEASURES OF A DETERIORATING SYSTEM WITH SEVEN SUBSYSTEMS

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Abstract

This study examines certain dependability properties of a repairable system that can be in three states: normal, deteriorated, and faulty. Deterioration may happen quickly or slowly. It is expected that both repair times and defects are exponential. Utilizing linear 1st order differential equations, we derive precise formulas for the mean system failure time, busy repair duration, profit function, and steady-state availability. Several scenarios are graphically examined to determine how system characteristics affect the MTSF as well as Profit functions.

Keywords: Reliability, Availability, Profit, Deterioration, Busy period analysis, MTSF.

I. Introduction:

In design and operations research, examining the dependability of restored systems is a crucial area of study. Achieving the necessary level of usability and dependability is a crucial prerequisite for system reliability, which is a very important statistic that depends on the system.

Bhardwaj and Chander (2) explained the reliability and cost-effectiveness evaluation of joint repair and waiting time allocation for a two-out-of-three redundant system. Reliability modeling for two of the three systems needing conditional server arrival is presented by Chander and Bhardwaj (4). On two of the three cold stand systems with a likelihood of repair and control, Bhardwaj and Malik (3) carried out MTSF. An evaluation of the availability and reliability traits of two of the three backup systems in a fully repaired state is presented by Yusuf and Hussaini (11). Adhikary et al. (1) investigated coal-fired power stations' cost-effective preventative maintenance scheduling. Abdurrahim & Mujahid (6) examined the best maintenance warranty. Mahmoud and Moshref (5) examine how hardware, human error, and preventative maintenance affect repairable products with rising rates of periodic failure in two cold standby systems. Nourelfath et al (7) suggested an integrated model for planning production as well as preventive maintenance in multiple systems. government systems. Ibrahim Yusuf, Nafiu Hussaini, and Bashir M. Yakasai (11) study some reliability measures in a deteriorating system.

Uemura et al. (9) examined the availability of an intrusion-resistant distributed server system having protective maintenance, while Zuo and Wu (10) assessed the linear & nonlinear preventive maintenance model. Yusuf (12) conducted a comparative evaluation among 2 redundant systems that require support units to operate, while Yusuf et al. (8) addressed the MTSF evaluation of a cold storage system having a two-piece support unit and a service station repaired. I. U.A. Ali & Ibrahim Yusuf (13) discussed several reliability properties of a repairable linear sequential 2 out of 4 system to calculate how the availability, as well as the profit function, could be enhanced by replacing the failed device with an inactive device. Anuradha and S.C. Malik (14) examined the use of the suggested toxic waste inclinator and examined a specific instance of the k out of n: g system for k=2 & n=3 having various policies of repair.

II. Description of the system

We study a linear sequential three-out-of-four system with three modes: modes of operation, modes of deterioration, and modes of failure. Either a slow or fast degradation mode is possible. It has been supposed that the system transfers from operation to slow or quick deterioration with the rate δ_1 and δ_2 respectively. Additionally, it is believed that no two successive units ever fail at the same time.

Monir minimum maintenance is invoked by having a rate λ_1 to regain the system at the initial stage before slow deterioration and major minimum maintenance would be done with a rate λ_2 to regain the system to its initial stage before fast deterioration.

III. States of the system:

State S_0: The system is working; units I, II, and III are operating, but unit IV is on standby.

State S_1: The system is functioning despite undergoing small, minimal maintenance and slow degradation.

State S_2: The system is operating but is deteriorating quickly and is only getting very minimal maintenance.

State S_3: The system is operational, Units II, III, and IV are operating, while Unit I has failed and is being repaired.

State S_4: The system is operational, Units II, III, and IV are in use, while Unit I has failed and is being repaired.

State S_5: Unit I has failed and is being repaired; Units II, III, and IV are operating; and the system is both operational and in a state of rapid degradation.

State S_6: The system is in a fast degradation stage and operational; Unit I has failed and is being repaired; and Units II, III, and IV are operating.

State S_7: The system failed, Units I and II failed and are being repaired, while Units III and IV are idle.

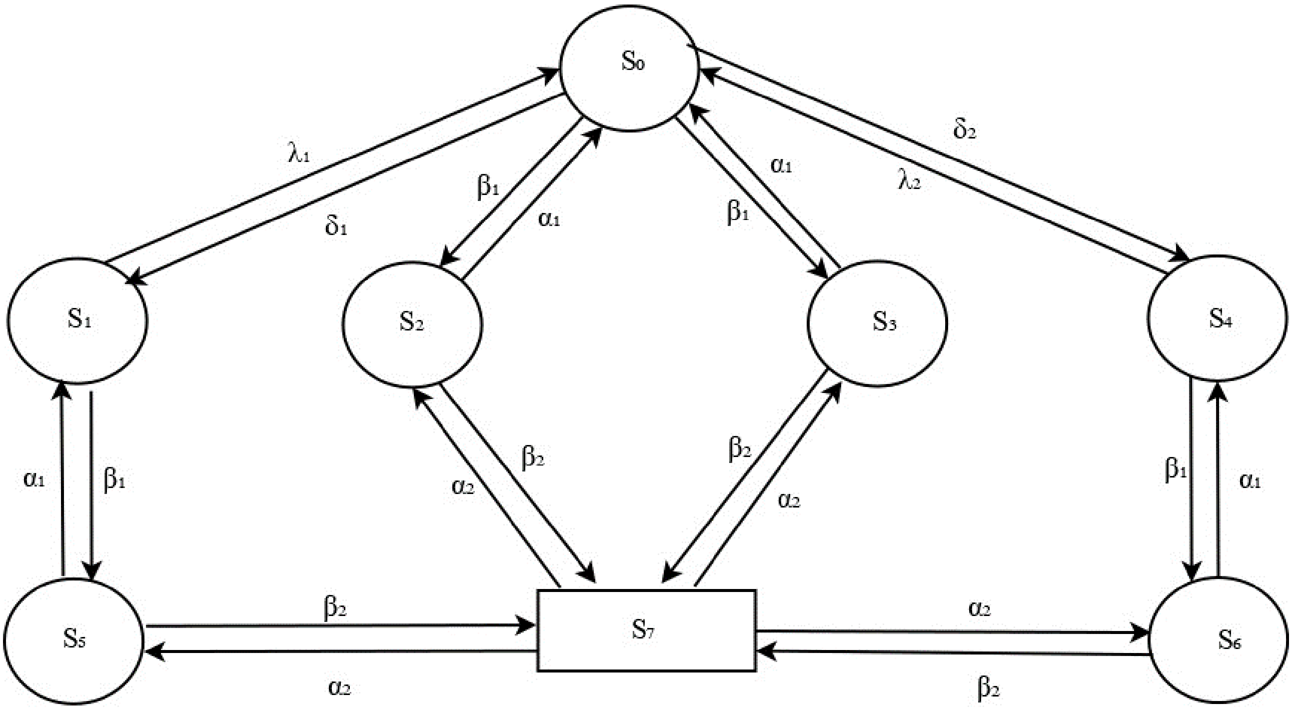

Figure 1: Transition diagram of the system

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IV. Model formulation

If P(t) is the probability row vector at the 't' time, therefore the following are the "problem's initial conditions:

$$P(0)=\left[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0)\right]=[1,0,0,0,0,0,0,0] \quad (1)$$

We derive the differential equations of a system that follows:

$$P_0^1(t)=-(\delta_1+\delta_2+2\beta_1)P_0(t)+\mu_1P_1(t)+\alpha_1P_3(t)+\alpha_1P_4(t)+\mu_2P_2(t) \quad (2)$$

$$P_1^1(t)=-(\mu_1+\beta_1)P_1(t)+\delta_1P_0(t)+\alpha_1P_5(t) \quad (3)$$

$$P_2^1(t)=-(\mu_2+\beta_1)P_2(t)+\delta_2P_0(t)+\alpha_1P_6(t) \quad (4)$$

$$P_3^1(t)=-(\alpha_1+\beta_2)P_3(t)+\beta_1P_0(t)+\alpha_2P_7(t) \quad (5)$$

$$P_4^1(t)=-(\alpha_1+\beta_2)P_4(t)+\beta_1P_0(t)+\alpha_2P_7(t) \quad (6)$$

$$P_5^1(t)=-(\alpha_1+\beta_2)P_5(t)+\beta_1P_1(t)+\alpha_2P_7(t) \quad (7)$$

$$P_6^1(t)=-(\alpha_1+\beta_2)P_6(t)+\alpha_2P_7(t)+\beta_1P_2(t) \quad (8)$$

$$P_7^1(t)=-(4\alpha_2)P_7(t)+\beta_2P_6(t)+\beta_2P_5(t)+\beta_2P_3(t)+\beta_2P_4(t) \quad (9)$$

The above differential equations are transformed into the matrix as $\dot{P}=QP$

Where

$$Q=\begin{bmatrix} -(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0 & 0 \\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 & 0 \\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 \\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0 & \alpha_1 \\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & \alpha_2 \\ 0 & 0 & \beta_2 & \beta_2 & 0 & \beta_2 & \beta_2 & -4\alpha_2 \end{bmatrix} \quad (10)$$

Since evaluating the transient solution would be too difficult, we will limit our computation of the MTSF. The anticipated time to attain a state of absorbing is computed by a transpose matrix of the Q, which we take and remove rows along with the columns for the state of absorbing to create a new matrix known as A. This yields the MTSF.

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$$E\left[T_{P(0)\to P(absorbing)}\right]=P(0)\left(-A^{-1}\right)\begin{pmatrix}1\\1\\1\\1\\1\\1\\1\end{pmatrix} \qquad (11)$$

Where

$$A=\begin{bmatrix}-(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0\\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0\\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1\\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0\\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0\\ 0 & \beta_{\partial 1} & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0\\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2)\end{bmatrix} \qquad (12)$$

$$MTTF=\frac{N_1}{D_1} \qquad (13)$$

$$N_1=\beta_1^2\beta_2^3+2\beta_1\beta_2^3\lambda_1+2\beta_1\beta_2^3\lambda_2+7\alpha_1^3\lambda_1\lambda_2+3\beta_2^3\lambda_1\lambda_2+3\alpha_1\beta_1^2\beta_2^2+7\alpha_1\beta_1\beta_2^2\lambda_1+5\alpha_1^2\beta_1\beta_2\lambda_1+7\alpha_1\beta_1\beta_2^2\lambda_2+5\alpha_1^2\beta_1\beta_2\lambda_2$$
$$+13\alpha_1\beta_2^2\lambda_1\lambda_2+17\alpha_1^2\beta_2\lambda_1\lambda_2$$

$$D_1=\beta_1\beta_2\begin{pmatrix}2\beta_1^2\beta_2^2+\beta_1\beta_2^2\delta_1+\beta_1\beta_2^2\delta_2+2\beta_1\beta_2^2\lambda_1+2\beta_1\beta_2^2\lambda_2+\alpha_1^2\delta_1\lambda_2+\alpha_1^2\delta_2\lambda_1+\beta_2^2\delta_1\lambda_2+\beta_2^2\delta_2\lambda_1+2\alpha_1^2\lambda_1\lambda_2\\ +2\beta_2^2\lambda_1\lambda_2+\alpha_1\beta_1\beta_2\delta_1+\alpha_1\beta_1\beta_2\delta_2+2\alpha_1\beta_1\beta_2\lambda_2+2\alpha_1\beta_2\delta_2\lambda_1+4\alpha_1\beta_2\lambda_1\lambda_2\end{pmatrix}$$

V. Availability Analysis:

The initial conditions for this problem are the same as for the reliability case:

$$P(0)=[1,0,0,0,0,0,0], \qquad (14)$$

The differential equations can be expressed as:

$$\begin{bmatrix}\dot{P}_0\\ \dot{P}_1\\ \dot{P}_2\\ \dot{P}_3\\ \dot{P}_4\\ \dot{P}_5\\ \dot{P}_6\\ \dot{P}_7\end{bmatrix}=\begin{bmatrix}-(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0 & 0\\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 & 0\\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0\\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0 & \alpha_1\\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & \alpha_2\\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2\\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & \alpha_2\\ 0 & 0 & \beta_2 & \beta_2 & 0 & \beta_2 & \beta_2 & -4\alpha_2\end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\end{bmatrix} \qquad (15)$$

The process below could be utilized to observe the steady state availability. The state probabilities derivatives approach 0 in a steady state. We can use that to compute steady-state probabilities.

$$A(\infty)=P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty) \qquad (16)$$

$$QP(\infty)=0 \qquad (17)$$

Or in the form of a matrix form

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$$\begin{bmatrix} -(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0 & 0 \\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 & 0 \\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 \\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0 & \alpha_1 \\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & \alpha_2 \\ 0 & 0 & \beta_2 & \beta_2 & 0 & \beta_2 & \beta_2 & -4\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

To get this " $P_0(\infty), P_1(\infty), P_2(\infty), P_3(\infty), P_4(\infty), P_5(\infty), P_6(\infty), P_7(\infty)$ we solve the equation (17) and the following normalizing conditions

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)=1 \quad (19)$$

To get (17) yield, we replace each redundant row in the equation with the equation (19).

$$\begin{bmatrix} -(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0 & 0 \\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 & 0 \\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 \\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0 & \alpha_1 \\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & \alpha_2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The availability of steady state $A(\infty)$ has been provided by

$$A(\infty)=P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)=\frac{N_2}{D_2} \quad (20)$$

Where

$$\begin{aligned} N_2 &= \alpha_1^4\left(\delta_2\lambda_1+(\delta_1+\lambda_1)\lambda_2\right)+3\alpha_2\beta_1\beta_2^2\left(2\beta_1^2+\delta_2\lambda_1+(\delta_1+2\lambda_1)\lambda_2+\beta_1\left(\delta_1+\delta_2+2(\lambda_1+\lambda_2)\right)\right) \\ &\quad +\alpha_1^3\left((3\alpha_2+2\beta_2)(\delta_2\lambda_1+(\delta_1+\lambda_1)\lambda_2)+\beta_1\left(\delta_2\lambda_1+\delta_1\lambda_2+2\lambda_1\lambda_2+\beta_2(\delta_1+\delta_2+\lambda_1+\lambda_2)\right)\right) \\ &\quad +\alpha_1^2\begin{pmatrix} \beta_2(6\alpha_2+\beta_2)(\delta_2\lambda_1+(\delta_1+\lambda_1)\lambda_2)+\beta_1^2\beta_2\left(\beta_2+\delta_1+\delta_2+2(\lambda_1+\lambda_2)\right) \\ +\beta_1\begin{pmatrix} \beta_2^2(\delta_1+\delta_2+\lambda_1+\lambda_2)+3\alpha_2\left(\delta_2\lambda_1+(\delta_1+2\lambda_1)\lambda_2\right) \\ +2\beta_2\left(\delta_2\lambda_1+(\delta_1+2\lambda_1)\lambda_2\right)+\alpha_2\beta_2\left(3\delta_1+3\delta_2+4(\lambda_1+\lambda_2)\right) \end{pmatrix} \end{pmatrix} \\ &\quad +\alpha_1\beta_2\begin{pmatrix} \beta_1\beta_2\left(2\beta_1^2+\delta_2\lambda_1+(\delta_1+2\lambda_1)\lambda_2+\beta_1\left(\delta_1+\delta_2+2(\lambda_1+\lambda_2)\right)\right) \\ +\alpha_2\begin{pmatrix} 3\beta_2\left(\delta_2\lambda_1+(\delta_1+\lambda_1)\lambda_2\right)+\beta_1\begin{pmatrix} 3\beta_2\delta_1+3\beta_2\delta_2+6\delta_2\lambda_1+6(\delta_1+2\lambda_1)\lambda_2 \\ +4\beta_2(\lambda_1+\lambda_2) \end{pmatrix} \\ +\beta_1^2\left(5\beta_2+3\left(\delta_1+\delta_2+2(\lambda_1+\lambda_2)\right)\right) \end{pmatrix} \end{pmatrix} \end{aligned}$$

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$$D_2=\alpha_1^4\left(\delta_2\lambda_1+\left(\delta_1+\lambda_1\right)\lambda_2\right)+\beta_1\beta_2^2\left(3\alpha_2+\beta_2\right)\left(2\beta_1^2+\delta_2\lambda_1+\left(\delta_1+2\lambda_1\right)\lambda_2+\beta_1\left(\delta_1+\delta_2+2\left(\lambda_1+\lambda_2\right)\right)\right)$$
$$+\alpha_1^3\left(\left(3\alpha_2+2\beta_2\right)\left(\delta_2\lambda_1+\left(\delta_1+\lambda_1\right)\lambda_2\right)+\beta_1\left(\delta_2\lambda_1+\delta_1\lambda_2+2\lambda_1\lambda_2+\beta_2\left(\delta_1+\delta_2+\lambda_1+\lambda_2\right)\right)\right)$$
$$+\alpha_1^2\left(\begin{array}{l}\beta_2\left(6\alpha_2+\beta_2\right)\left(\delta_2\lambda_1+\left(\delta_1+\lambda_1\right)\lambda_2\right)+\beta_1^2\beta_2\left(\beta_2+\delta_1+\delta_2+2\left(\lambda_1+\lambda_2\right)\right)\\+\beta_1\left(\begin{array}{l}\beta_2^2\left(\delta_1+\delta_2+\lambda_1+\lambda_2\right)+3\alpha_2\left(\delta_2\lambda_1+\left(\delta_1+2\lambda_1\right)\lambda_2\right)+3\beta_2\left(\delta_2\lambda_1+\left(\delta_1+2\lambda_1\right)\lambda_2\right)\\+\alpha_2\beta_2\left(3\delta_1+3\delta_2+4\left(\lambda_1+\lambda_2\right)\right)\end{array}\right)\end{array}\right)$$
$$+\alpha_1\beta_2\left(\begin{array}{l}\beta_1\beta_2\left(2\beta_1^2+3\left(\delta_2\lambda_1+\left(\delta_1+2\lambda_1\right)\lambda_2\right)+2\beta_1\left(\delta_1+\delta_2+2\left(\lambda_1+\lambda_2\right)\right)\right)\\+\alpha_2\left(\begin{array}{l}3\beta_2\left(\delta_2\lambda_1+\left(\delta_1+\lambda_1\right)\lambda_2\right)+\beta_1\left(2\beta_2\delta_1+3\beta_2\delta_2+6\delta_2\lambda_1+6\left(\delta_1+2\lambda_1\right)\lambda_2+4\beta_2\left(\lambda_1+\lambda_2\right)\right)\\+\beta_1^2\left(5\beta_2+3\left(\delta_1+\delta_2+2\left(\lambda_1+\lambda_2\right)\right)\right)\end{array}\right)\end{array}\right)$$

VI. Busy period analysis:

Similar to the reliability case, the starting conditions for the given problem are as follows:

$$P(0)=\left[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0)\right]=\left[1,0,0,0,0,0,0,0\right]$$

The form of differential equation can be stated as follows:

$$\begin{bmatrix}\dot{P}_0\\\dot{P}_1\\\dot{P}_2\\\dot{P}_3\\\dot{P}_4\\\dot{P}_5\\\dot{P}_6\\\dot{P}_7\end{bmatrix}=\begin{bmatrix}-(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0 & 0\\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 & 0\\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0\\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0 & \alpha_1\\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & \alpha_2\\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2\\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & \alpha_2\\ 0 & 0 & \beta_2 & \beta_2 & 0 & \beta_2 & \beta_2 & -4\alpha_2\end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\end{bmatrix}$$

The following process can be used to find a steady state busy period. The state probabilities derivatives approach 0 in a steady state. This enables us to determine the probability of a steady state using

$$B(\infty)=P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)=\frac{N_3}{D_2} \qquad (21)$$

$$QP(\infty)=0 \qquad (22)$$

Or, in the matrix form

$$\begin{bmatrix}-(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0 & 0\\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 & 0\\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0\\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0 & \alpha_1\\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & \alpha_2\\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2\\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & \alpha_2\\ 0 & 0 & \beta_2 & \beta_2 & 0 & \beta_2 & \beta_2 & -4\alpha_2\end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

To attain $P_1(\infty),P_2(\infty),P_3(\infty),P_4(\infty),P_5(\infty),P_6(\infty),P_7(\infty)$ we solve the equation (22) and the following normalizing condition:

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$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)=1 \tag{23}$$

We substitute the equation (23) in any one of the redundant rows in equation (22) to yield

$$\begin{bmatrix} -(\delta_1+2\beta_1+\delta_2) & \mu_1 & \mu_2 & \alpha_1 & \alpha_1 & 0 & 0 & 0 \\ \delta_1 & -(\mu_1+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 & 0 \\ \delta_2 & 0 & -(\mu_2+\beta_1) & 0 & 0 & 0 & \alpha_1 & 0 \\ \beta_1 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & 0 & \alpha_1 \\ \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & 0 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & \alpha_2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady-state busy period $B(\infty)$ is expressed as $B(\infty)=\dfrac{N_3}{D_2}$

Where

$$\begin{aligned} N_3 = {} & \alpha_1^4(\delta_2\lambda_1+\delta_1\lambda_2)+\alpha_1^3\big((3\alpha_2+2\beta_2)(\delta_2\lambda_1+\delta_1\lambda_2)+\beta_1(\beta_2(\delta_1+\delta_2)+\delta_2\lambda_1+\delta_1\lambda_2+2\lambda_1\lambda_2)\big) \\ & +\beta_1\beta_2^2(3\alpha_2+\beta_2)\big(2\beta_1^2+\delta_2\lambda_1+(\delta_1+2\lambda_1)\lambda_2+\beta_1(\delta_1+\delta_2+2(\lambda_1+\lambda_2))\big) \\ & +\alpha_1\beta_2\begin{pmatrix} \beta_1\beta_2\big(2\beta_1^2+3(\delta_2\lambda_1+(\delta_1+2\lambda_1)\lambda_2)+2\beta_1(\delta_1+\delta_2+2(\lambda_1+\lambda_2))\big) \\ +\alpha_2\begin{pmatrix} 3\beta_2(\delta_2\lambda_1+\delta_1\lambda_2)+\beta_1(3\beta_2\delta_1+3\beta_2\delta_2+6\delta_2\lambda_1+6(\delta_1+2\lambda_1)\lambda_2+2\beta_2(\lambda_1+\lambda_2)) \\ +\beta_1^2(4\beta_2+3(\delta_1+\delta_2+2(\lambda_1+\lambda_2))) \end{pmatrix} \end{pmatrix} \\ & +\alpha_1^2\begin{pmatrix} \beta_2\big(\beta_2(\delta_2\lambda_1+\delta_1\lambda_2)+\beta_1(\beta_2(\delta_1+\delta_2)+3\delta_2\lambda_1+3(\delta_1+2\lambda_1)\lambda_2)+\beta_1^2(\delta_1+\delta_2+2(\lambda_1+\lambda_2))\big) \\ +\alpha_2\big(6\beta_2(\delta_2\lambda_1+\delta_1\lambda_2)+\beta_1(3\delta_2\lambda_1+3(\delta_1+2\lambda_1)\lambda_2+\beta_2(3\delta_1+2\delta_2+2(\lambda_1+\lambda_2)))\big) \end{pmatrix} \end{aligned}$$

VII. Profit analysis:

In steady-state, the anticipated overall profit /time acquired by the system has been provided by:

Profit = total revenue generated – accumulated cost acquired because of the maintenance at deterioration and repair due to failure.

$$PF(\infty)=C_0A(\infty)-C_1B(\infty) \tag{24}$$

PF	Profit incurred by the system
C_1	Cost per unit during the period that the system is being maintained and repaired
C_0	Revenue per unit uptime of the system

VIII. Results and discussions:

The mean time to system failure along with the profit function results for each of the created models were obtained numerically and presented in this section. To maintain consistency throughout the simulations, the following set of parameter values have been fixed for the model analysis:

$\alpha_1=0.02, \alpha_2=0.04, \beta_1=0.05, \beta_2=0.06, \lambda_1=0.02, \lambda_2=0.03, \delta_1=0.2, \delta_2=0.3, C_1=1000, C_2=500$

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S.No.	α_1	MTSF	Profit	α_2	Profit
1.	0	12.4832	166.6667	0	-187.9369
2.	0.01	18.1195	228.3563	0.01	40.1389
3.	0.02	23.7832	279.9909	0.02	158.5366
4.	0.03	29.4688	323.6173	0.03	231.0348
5.	0.04	35.1724	360.7595	0.04	279.9909
6.	0.05	40.8908	392.5895	0.05	315.2702
7.	0.06	46.6216	420.0284	0.06	341.9015
8.	0.07	52.3629	443.8105	0.07	362.7175
9.	0.08	58.1131	464.5277	0.08	379.4354
10.	0.09	63.8710	482.6610	0.09	393.1571
11.	0.1	69.6355	498.6040	0.1	404.6217

S.No.	β_1	MTSF	Profit	β_2	Profit
1.	0	128.8660	666.6667	0	538.4615
2.	0.01	62.7451	437.5940	0.01	483.0939
3.	0.02	40.9454	359.2699	0.02	434.1912
4.	0.03	30.1724	319.7445	0.03	390.1135
5.	0.04	23.7832	295.9184	0.04	350.0767
6.	0.05	19.5707	279.9909	0.05	313.5181
7.	0.06	16.5931	268.5944	0.06	279.9909
8.	0.07	14.3817	260.0371	0.07	249.1273
9.	0.08	12.6773	253.3760	0.08	220.6191
10.	0.09	11.3253	248.0440	0.09	194.2049
11.	0.1	128.8660	243.6797	0.1	169.6613

S.No.	δ_1	MTSF	Profit	δ_2	MTSF	Profit
1.	0	27.1673	282.1623	0	28.2853	282.8871
2.	0.01	25.3629	281.0003	0.01	26.6065	281.8001
3.	0.02	23.7832	279.9909	0.02	25.1157	280.8419
4.	0.03	22.3887	279.1058	0.03	23.7832	279.9909
5.	0.04	21.1487	278.3235	0.04	22.5849	279.2300
6.	0.05	20.0389	277.6270	0.05	21.5016	278.5457
7.	0.06	19.0397	277.0029	0.06	20.5174	277.9269
8.	0.07	18.1354	276.4406	0.07	19.6194	277.3647
9.	0.08	17.3132	275.9312	0.08	18.7967	276.8516
10.	0.09	16.5622	275.4676	0.09	18.0402	276.3815
11.	0.1	15.8737	275.0439	0.1	17.3423	275.9492

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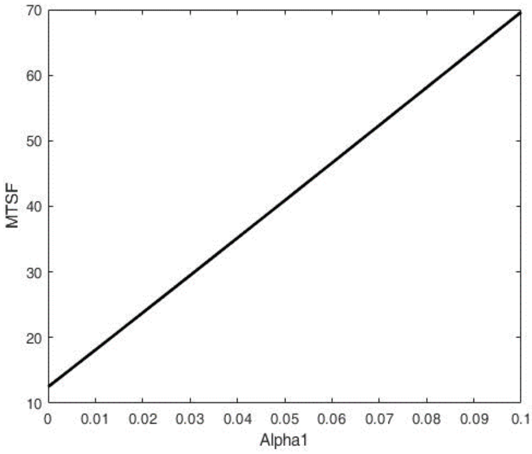

Fig.2: α_1 effect on MTSF

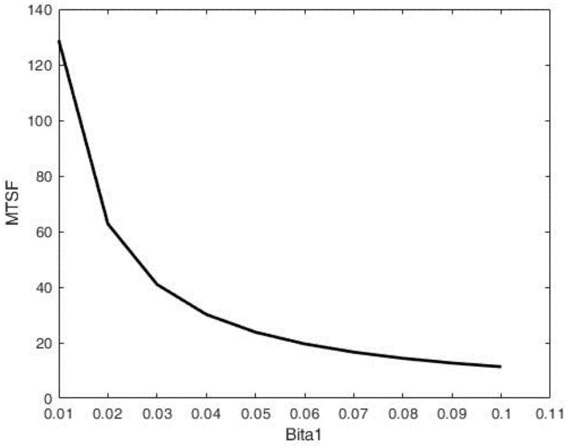

Fig.3: β_1 effect on MTSF

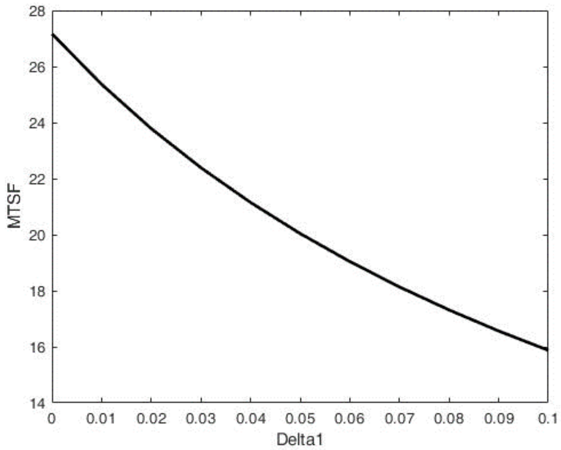

Fig.4: δ_1 effect on MTSF

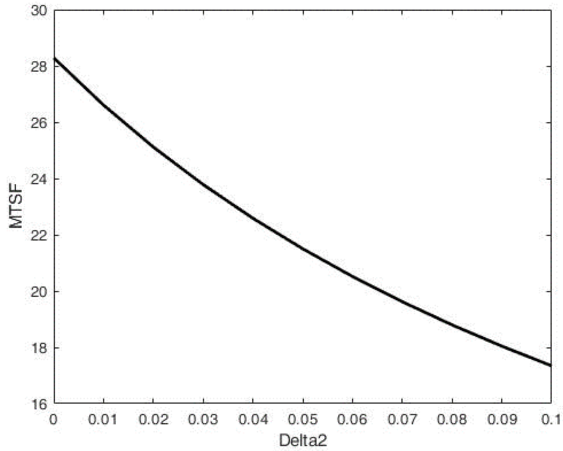

Fig.5: δ_2 effect on MTSF

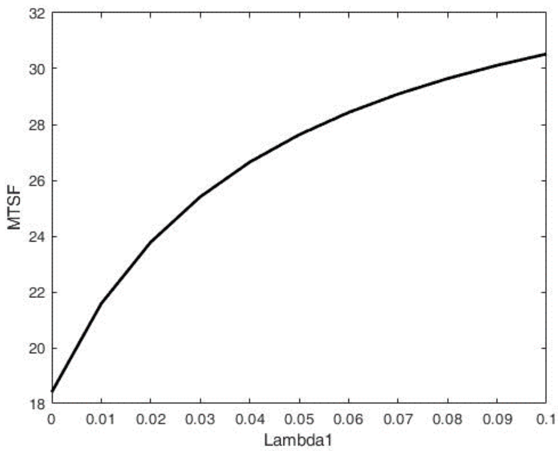

Fig.6 λ_1 effect on MTSF

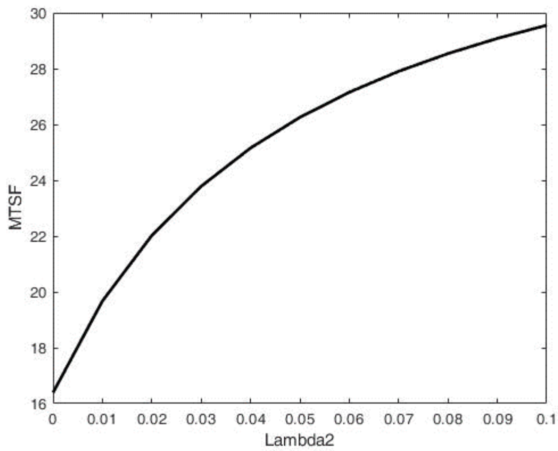

Fig.7: λ_2 effect on MTSF

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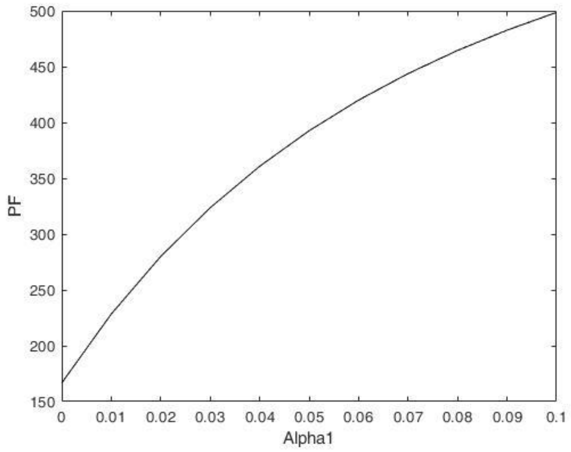

Fig.8: α_1 effect on PF

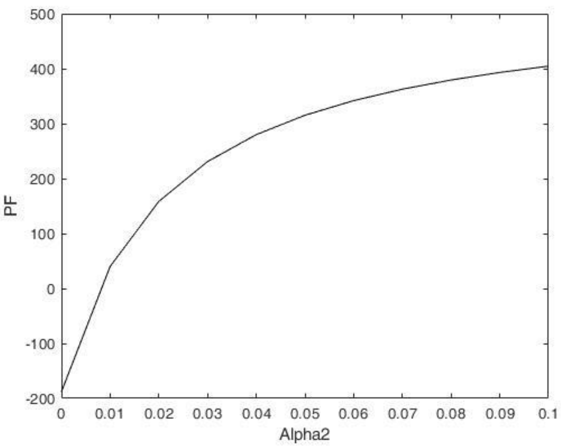

Fig.9: α_2 effect on PF

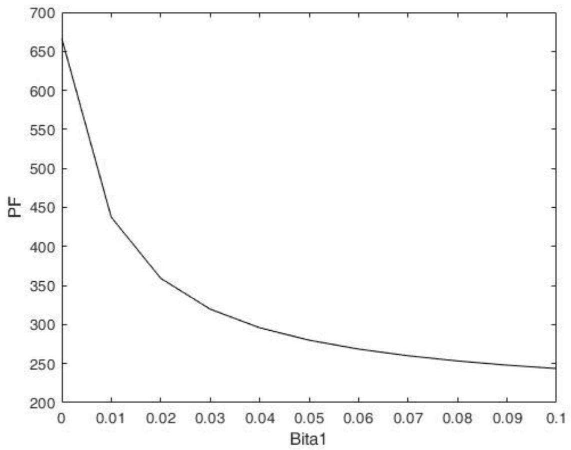

Fig.10: β_1 effect on PF

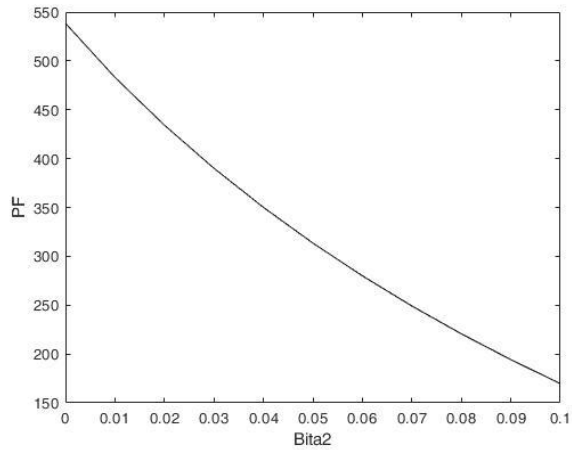

Fig.11: β_2 effect on PF

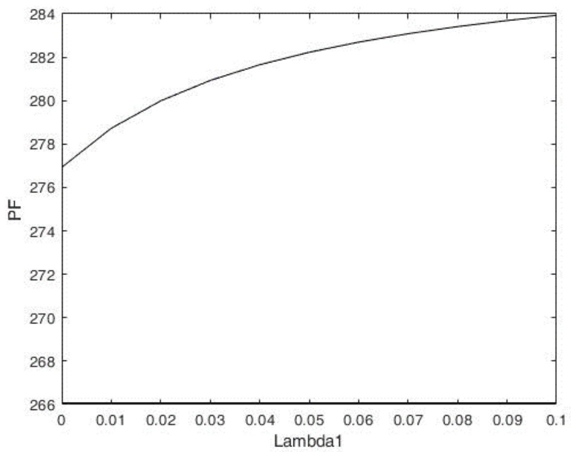

Fig.12: λ_1 effect on PF

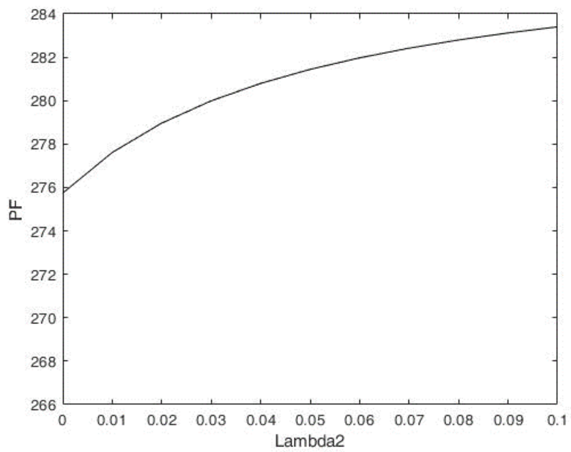

Fig.13: λ_2 effect on PF

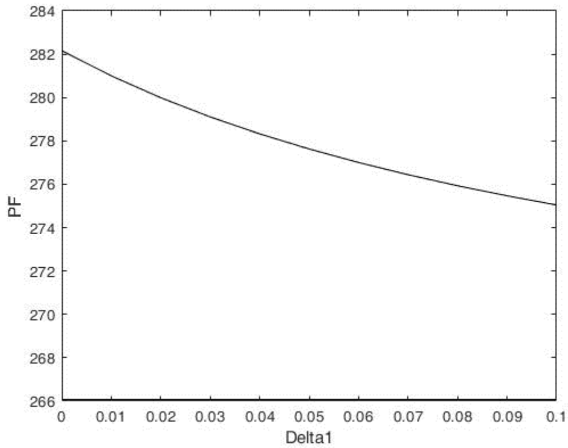

Fig.14: δ_1 effect on PF

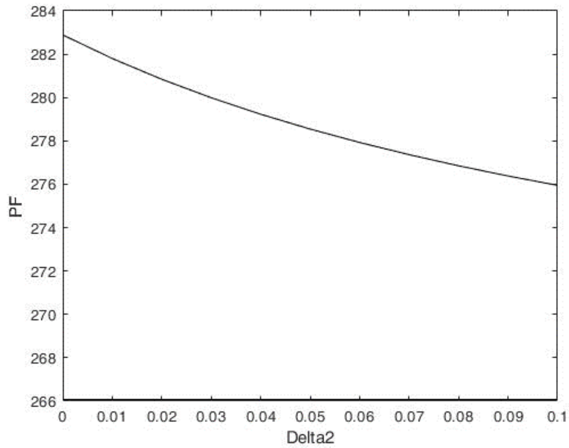

Fig.15: δ_2 effect on PF

Based on the above tables and graphs, I observed the following results:

	$\alpha_1 \uparrow$	MTSF Increases
	$\beta_1 \uparrow$	MTSF Decreases
	$\delta_1 \uparrow$	MTSF Decreases
	$\delta_2 \uparrow$	MTSF Decreases
	$\lambda_1 \uparrow$	MTSF Increases
	$\lambda_2 \uparrow$	MTSF Increases

	$\alpha_1 \uparrow$	Profit increases
	$\alpha_2 \uparrow$	Profit increases
	$\beta_1 \uparrow$	Profit decreases
	$\beta_2 \uparrow$	Profit decreases
	$\delta_1 \uparrow$	Profit increases
	$\delta_2 \uparrow$	Profit increases
	$\lambda_1 \uparrow$	Profit decreases
	$\lambda_2 \uparrow$	Profit decreases

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