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EMBRACING UNCERTAINTY: A REVIEW OF QUANTIFICATION TECHNIQUES IN FINITE ELEMENT ANALYSIS

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Abstract
This paper comprehensively explores the role and importance of uncertainty quantification in finite element analysis (FEA). This review paper delves into the various techniques used to quantify uncertainty, highlighting the necessity of these methods in ensuring accurate and reliable results in FEA. The paper discusses the use of surrogate modeling as an effective tool for uncertainty quantification, providing an in-depth analysis of different surrogate types, their functions, and validation procedures. It further explores the application of these techniques in various fields, with a particular focus on their impact on prediction, sensitivity analysis, and optimization. The limitations of deterministic FEA models are also discussed, emphasizing the need for embracing uncertainty in FEA computations. The paper concludes with a discussion of recent advances in this field and provides practical recommendations for further research. This review serves as a useful resource for researchers and students seeking to understand and implement uncertainty quantification techniques in FEA.
Keywords: Uncertainty Quantification Finite, Element Analysis (FEA), Surrogate modeling, Prediction, Sensitivity Analysis, Optimization

Introduction
Finite Element Analysis (FEA) is an effective computational application that allows researchers and engineers to simulate the behaviour of physical systems under various conditions. It is a numerical technique that is used to solve complex engineering and mathematical physics problems. This method involves dividing a complex problem into smaller, simpler parts, known as finite elements. Each of these components has a set of equations that explain its behaviour, and they are connected at locations known as nodes. The FEM provides an approximate solution to a wide range of engineering problems [1] [2]. The beauty of FEM lies in its flexibility. The elements can be assembled in numerous ways, enabling them to represent highly complex shapes. This makes FEM an invaluable tool for solving problems with intricate geometries. FEA is widely utilized in a variety of applications, including structural analysis, heat transfer, fluid flow, mass transport, and electromagnetic potential. It is especially beneficial for problems with complex geometries, loadings, and material properties that cannot be solved analytically.
In structural analysis, for example, FEA can be used to predict how a structure will respond to external forces such as gravity, wind, or seismic loads. It can also be used to determine the distribution of stresses and displacements within the structure [2]. In heat transfer analysis, FEA can be used to simulate the distribution of temperature within a body due to heat sources, conduction, convection,

and radiation. It can also be used to predict how these temperatures will change over time. In fluid flow analysis, FEA can be used to simulate the flow of liquids or gases through pipes or around objects. It can also be used to predict pressure drops, velocity profiles, and turbulence effects [2].
Despite its many advantages, FEA also has some limitations. For example, it requires a significant number of computational resources, especially for large or complex problems [3]. Nevertheless, with advancements in computer technology and software development, FEA has become an indispensable tool in engineering design and analysis. It continues to evolve with new methods and techniques being developed to improve its accuracy and efficiency.

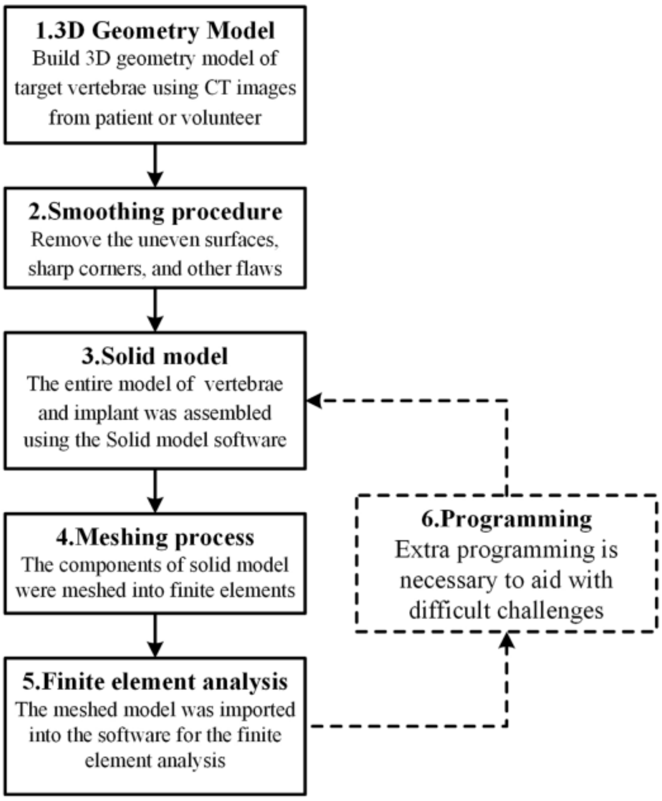

Figure 1**:** A flowchart of the modeling method [3]

As computing technology has advanced, numerical methods like FEA have overcome computational accuracy and stability challenges. This has led to significant developments and widespread applications. In terms of software, there are several commercial finite element software packages available for research and design purposes. These include ANSYS, ABAQUS, FLAC, LS-DYNA, ADINA, and Marc among others [4]. The fundamental idea behind the FEM is that an effective region can be analytically model or estimated by replacing it with a set of discrete elements. [4]. As a result, Finite Element Analysis (FEA) has become an indispensable numerical tool for solving complicated engineering and mathematical physics issues.

Literature Review

Uncertainty quantification (UQ) in finite element analysis (FEA) is a rapidly evolving field that aims to quantify the impact of uncertainties in model inputs on the outputs. This is crucial in many engineering and scientific applications where precise predictions are necessary despite the presence of uncertainties.

Uncertainty in Finite Element Analysis (FEA) is a numerical method for solving problems in engineering and mathematical physics. However, uncertainties can arise in FEA from various sources such as material properties, boundary conditions, and loading conditions. These uncertainties can significantly affect the reliability and accuracy of FEA predictions.

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Quantification Techniques have been developed for quantifying uncertainty in FEA. These include probabilistic methods, interval methods, and fuzzy methods. Probabilistic methods treat uncertainties as random variables and use statistical methods to quantify uncertainty. Interval methods represent uncertainties as intervals and use interval arithmetic for uncertainty quantification. Fuzzy methods represent uncertainties as fuzzy numbers and use fuzzy arithmetic for uncertainty quantification.

Probabilistic Methods are the most widely used techniques for UQ in FEA. They include Monte Carlo simulation, First and Second Order Reliability Methods (FORM/SORM), and stochastic finite element methods. These methods provide a probabilistic description of uncertainty, which is particularly useful when the statistical properties of the uncertainties are known.

Interval and Fuzzy Methods are useful when the statistical properties of the uncertainties are not known. Interval methods provide a range of possible values for the outputs, while fuzzy methods provide a degree of possibility for each output value.

Challenges and Future Directions Despite the advancements in UQ techniques in FEA, several challenges remain. These include the high computational cost of UQ techniques, the difficulty in handling multiple types of uncertainties, and the lack of robust UQ techniques for complex FEA models. Future research in this field could focus on developing efficient UQ techniques, integrating different types of uncertainties, and applying UQ techniques to complex FEA models. Embracing uncertainty through quantification techniques in FEA is crucial for making reliable predictions. While significant progress has been made in this field, further research is needed to overcome the existing challenges and enhance the applicability of UQ techniques in FEA.

Surrogate Modelling

Surrogate modeling is a technique used in engineering when a result of interest cannot be readily observed or computed, and an approximate numerical model of the outcome is utilized instead. To evaluate design objectives and constraints functions as an outcome of the design variables, most engineering design challenges necessitate experiments and simulations. [5]. However, for many real-world difficulties, a single model can take several minutes, hours, or even weeks to complete. Because they call for dozens or even millions of simulation evaluations, and common activities it becomes difficult to perform design optimization, design space exploration, sensitivity evaluation, and "what-if" analysis. Building approximation models, often referred to as surrogate models, or meta-models, that closely imitate the behavior of the simulation model while being computationally less expensive to analyze is one technique to lessen this burden.

Surrogate modeling is a powerful tool for uncertainty quantification. It provides a way to approximate complex systems, analyze different surrogate types, understand their functions, and validate their accuracy. This makes surrogate modeling an indispensable tool in various engineering and scientific fields. Surrogate models are engineering methods that are used when an important outcome is difficult to measure or compute, so a hypothetical mathematical model of the outcome is used instead. They are particularly useful in the field of Finite Element Analysis (FEA) computations, where they can significantly reduce the computational burden associated with data-intensive tasks.

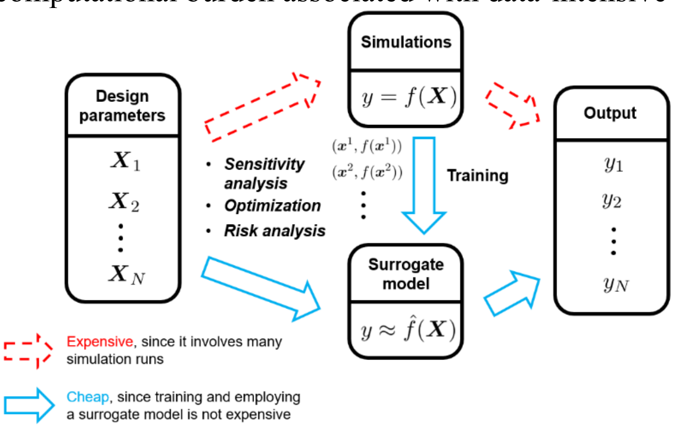

Figure 2: Procedure of surrogate modeling

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2.2 Validation Procedures of Surrogate Models

Validating surrogate models involves assessing their performance and selecting the most suitable one from a pool of candidates, often by tuning their hyperparameters. Predict the response at an unobserved point. This response is the output of the surrogate model. It could also be the outcome of a simulation model, numerical calculation, or another method. Calculate the true analysis at the unknown point. The actual reaction is usually the outcome of a computerized model or an actual test. Contrast and evaluate the expected and observed responses. The comparison of the two responses will reveal how similar they are [6].

In terms of comparison, surrogate models like PRS and kriging often require fewer data points to construct an accurate model compared to methods like artificial neural networks [6][7]. However, methods like artificial neural networks may provide more accurate predictions when large amounts of training data are available. The choice of surrogate model often depends on the specific requirements of the FEA computation task at hand. Factors such as the complexity of the system being modeled, the amount of available data, and computational resources can all influence which type of surrogate model is most appropriate.

Surrogate models play a crucial role in FEA computations by providing an efficient way to approximate complex systems. The choice of surrogate model depends on various factors and each type has its strengths and weaknesses. As research in this field continues, we can expect to see further improvements in surrogate modeling techniques that will enhance their accuracy and efficiency.

Surrogate models are mathematical constructs that are used to approximate the behavior of complex systems, and they play a crucial role in Finite Element Analysis (FEA) computations. They're useful for things like anticipating, analysis of sensitivity, unresolved evaluation, and surrogate-assisted optimization.

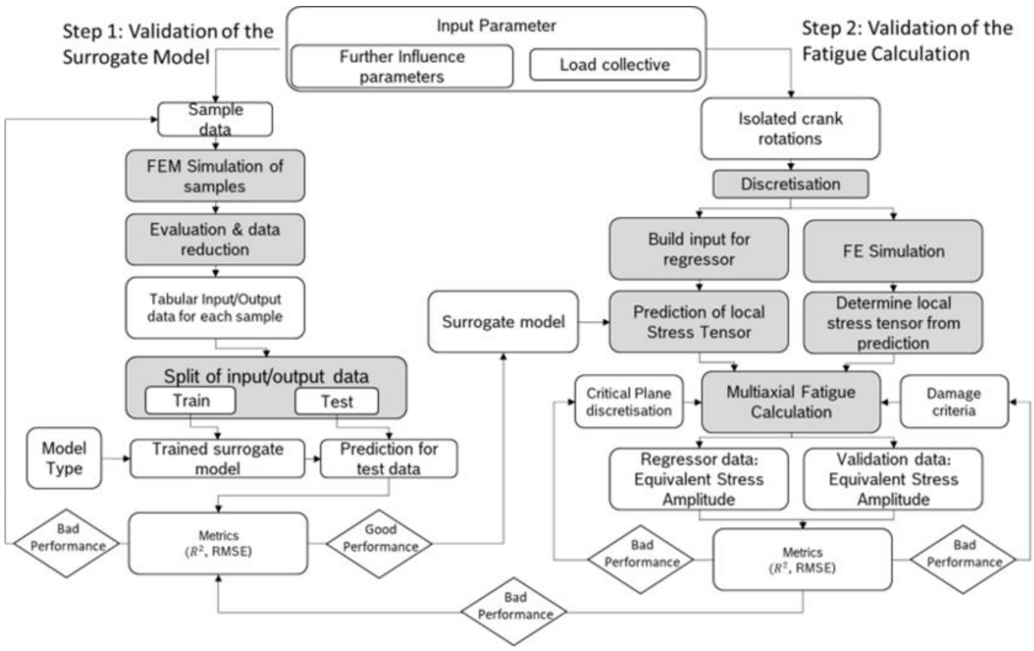

Figure 3: Validation process of the surrogate model and the usage for the fatigue calculation [8]

2.3 Uncertainty Quantification

Uncertainty quantification (UQ) in Finite Element Analysis (FEA) is a crucial aspect of engineering and technology with significant implications for the interpretation of results and decision-making processes. The process of mathematical and physical simulation involves decomposing and refining complex systems to reveal underlying principles. Often these complex mathematical models like partial differential equations (PDE) lack closed-form solutions. Therefore, numerical simulation methods like finite element or finite difference schemes are employed to obtain results. Simulation is especially important because it allows parameters in models to be changed to better understand the

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cause and effect of complex phenomena that would be too expensive or dangerous to investigate using traditional experimental methods. These processes, however, introduce a great deal of uncertainty [8]. The primary causes of experiment uncertainty are monitoring errors and random disturbance of specific experimental settings. Due to the complexity of reality, the incompleteness of knowledge, and cognitive limitations, mathematical models frequently overlook certain contributing aspects and can only represent real behavior to a certain degree of accuracy. This creates contradictions between the real system and the mathematical model, introducing uncertainty.
UQ aims to determine the impact of uncertainty and variability on the response of the model. To accurately interpret results, it is necessary to quantify these errors, which includes recognizing the key causes of uncertainty, evaluating how uncertainty expands in complicated systems, finding stable optimal solutions across a wide variety of inputs, and making better judgments with a known degree of confidence. This can help to cut developmental time, model costs, and unexpected failures. It is crucial to evaluate the uncertainties related to model predictions since it leads to higher confidence.in the predictions and a more accurate estimation of risks associated with particular design choices. This improves decision-making support for robust or reliability-based design [8][9][10].

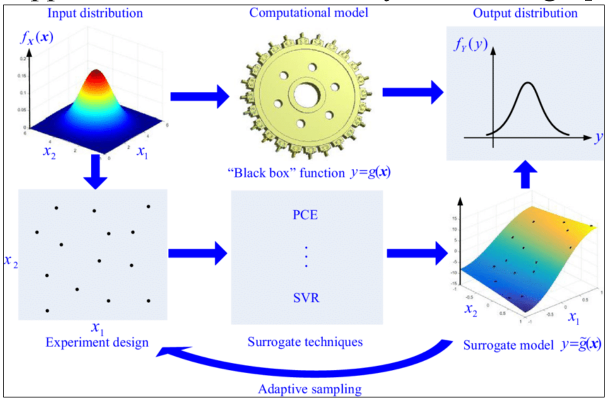

Figure 4: Process of uncertainty quantification (UQ) in finite element analysis

Uncertainty quantification in FEA is an essential aspect that needs to be considered carefully It not only helps in understanding the system behavior under uncertain conditions but also aids in making informed decisions based on a known level of confidence.

2.4 Uncertainty Quantification Techniques

Uncertainty quantification (UQ) is a scientific field that focuses on quantifying and estimating uncertainties in both mathematical and real-world applications. When some aspects of the system are not precisely known, it aims at estimating the probability of certain outcomes [11].
There are various causes of unpredictability, including observation error, random disruption of experimental settings, the complexity of reality, information inadequacy, and cognitive limitations [12]. Uncertainty may enter mathematical frameworks and measuring experiments in a variety of circumstances. Consider parametric unpredictability, parametric uncertainty, fundamental uncertainty, and algorithms uncertainty when categorizing causes of uncertainty. The source of parameter uncertainty is the model factors that are supplied to the computational systems model but whose precise values are unknown to experimentalists and cannot be managed in physical tests. Metric uncertainty is generated by the variability of the model's input variables [12]. Structural uncertainty, also known as model inadequacy, model bias, or model disparities, arises from a lack of knowledge of the underlying physics of the situation. Algorithmic uncertainty, also known as mathematical uncertainty or discrete uncertainty, is caused by numerical inaccuracies and numerical approximations made during computer model implementation.
Several techniques are used to quantify uncertainty. These consist of the design of the experiment, the surrogate model, Bayesian inference, model calibration, sensitivity evaluation, uncertainties propagation, and model uncertainty analysis. Sensitivity analysis is used to assess the effects of various independent variable values on a specific dependent variable under a specific set of underlying

assumptions. Uncertainty propagation involves the process of determining how uncertainty in the inputs of a model propagates through the model to affect the outputs [13]. Model calibration is the process of refining a model by comparing its outputs with observed data. A statistical inference technique known as Bayesian inference uses Bayes' theorem to adjust a hypothesis' probability as new data or evidence becomes available. Any information-gathering activity that incorporates variation, whether or not the experimenter has complete control over it, is considered to be designed experimentally [13A surrogate model is a means of approximating the behavior of a sophisticated mathematical model while being considerably simpler and easier to use. Model uncertainty analysis is the process of detecting and quantifying uncertainties in the structure and parameters of a model.

Theoretical frameworks for handling uncertainty that are frequently employed include interval analysis, fuzzy set theory, probability theory, evidence theory, info-gap decision theory, and hybrid approaches [14]. These theories provide a framework for dealing with uncertainty and making informed decisions. Uncertainty quantification is a critical aspect of many scientific and engineering fields. It involves understanding the sources of uncertainty, categorizing them, and using various techniques to quantify them. The ultimate goal is to make more informed decisions and predictions about the system or process being studied.

2.5 Uncertainty Quantification Methods

Several UQ methods are used to quantify uncertainty. These consist of experimental design, surrogate model, Bayesian inference, model calibration, sensitivity analysis, uncertainty propagation, and model uncertainty analysis.

2.5.1 Sensitivity Analysis

Sensitivity analysis is used to assess, under a specific set of assumptions, the effects of varying values of an independent variable on a given dependent variable. This method is crucial in FEA as it helps engineers understand which parameters have the most significant impact on the system’s behavior [14].

2.5.2 Uncertainty Propagation

Uncertainty propagation involves determining how uncertainty in the inputs of a model propagates through the model to affect the outputs. This method is essential in FEA as it allows engineers to estimate the range of possible outcomes given the uncertainties in the input parameters.

2.5.3 Model Calibration

Model calibration is the process of refining a model by comparing its outputs with observed data. This method is vital in FEA as it helps improve the model's accuracy by adjusting its parameters to match observed data.

2.5.4 Bayesian Inference

A statistical inference technique known as Bayesian inference uses Bayes' theorem to adjust a hypothesis' probability as new data or evidence becomes available. [15].

2.5.5 Experimental Design

Whether or not the experimenter has complete control over the exercise, designing an information-gathering exercise with variation is known as experimental design. This method is crucial in FEA as it helps engineers design experiments that can provide the most information about the system.

2.5.6 Surrogate Model

A surrogate model is an approximation method that mimics the behavior of a complex mathematical model but is much simpler and easier to use. This method is essential in FEA as it allows engineers to make predictions about the system’s behavior without having to solve the complex mathematical model.

2.5.7 Model Uncertainty Analysis

Model uncertainty analysis involves the process of identifying and quantifying the uncertainties in the model’s structure and parameters. This method is crucial in FEA as it helps engineers understand the limitations of their models and make more informed decisions [15].

UQ is critical in the assessment and verification of computing design models and simulations because it increases trust in the predictive capacity of computational models.. Decision-makers will be able to assess the accuracy of a forecast and take proactive steps to save time, distribute resources, and lower the possibility of the system performing, being too safe, or being too reliable if the total uncertainty in the simulation is quantified [16][17]. UQ methods are essential in ensuring the accuracy and reliability of FEA results. They help engineers understand the sources of uncertainty, quantify them, and make more informed decisions about the system or process being studied.

2.6 Recent Advances in Uncertainty Quantification in Finite Element Analysis (FEA)

Uncertainty quantification (UQ) has been widely used in the last few decades to ensure the robustness of engineering designs. The research on uncertainty in deterministic engineering modeling has been studied since the early 1980s. UQ has been successfully applied in numerous fields and has played a significant role after nearly forty years of development [18].

One of the recent advances in UQ methods is the use of meta-modeling methods suitable for engineering applications. The two most popular meta-modeling methods are the Polynomial Chaos Method and Gaussian Process. An engineering test problem with several uncertainties has been tackled using these techniques. The test problem considered here is a supersonic nozzle under operational uncertainties [19]. For the deterministic solution, the freely available computational fluid dynamics (CFD) solver SU2 is used. To measure uncertainty and sensitivity, the UQ algorithms are developed in MATLAB and then linked with SU2. The mean as well as the standard deviation of the output quantities are given as the results.

Another recent breakthrough is the creation of a complete data-driven computational technique for studying the UQ and likelihood propagation in gathered tensegrity structures [19]. To manage the deformation of the elastic structure, a surrogate optimization model was constructed.

Future Recommendations and Conclusion

The field of UQ in FEA is vast and continuously evolving. There are several areas where future research could be directed. More work could be done on developing and refining meta-modeling methods for UQ. While the Polynomial Chaos Method and Gaussian Process are popular, other methods may be more suitable for certain types of engineering problems. There is a need for more comprehensive testing of UQ methods on a wider range of engineering problems. This would help to validate these methods and ensure their robustness. More research could be done on integrating UQ methods with different types of solvers. While SU2 was used in the study by Dinesh Kumar et al. [20], other solvers could also be explored. There is a need for more research on data-driven approaches to UQ. The study by Springer [20] showed promising results, but more work needs to be done to validate these approaches and explore their potential applications. While significant advances have been made in the field of UQ in FEA, there is still much work to be done. Future research should focus on refining existing methods, validating these methods through comprehensive testing, exploring the integration with different solvers, and investigating data-driven approaches.

Uncertainty quantification (UQ) in Finite Element Analysis (FEA) plays a crucial role in ensuring the robustness of engineering designs. It provides a systematic framework to understand and manage uncertainties in numerical simulations. The importance of UQ in FEA cannot be overstated, as it allows engineers to make informed decisions about the safety and reliability of their designs.

Recent advances in UQ methods, such as the use of meta-modeling methods and data-driven approaches, have shown promising results. These methods have been successfully applied to a range of engineering problems, demonstrating their versatility and robustness. However, there is still a lot of work to be done despite these advancements. Future research should focus on refining existing methods, validating these methods through comprehensive testing, exploring the integration with different solvers, and investigating data-driven approaches. UQ in FEA is a rapidly evolving field with significant potential for future development. The advances made so far are encouraging, but there is still a long way to go. With continued research and development, it is hoped that UQ methods will become even more robust and widely used in the field of engineering.

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