NEW APPROACH FOR DESIGNING A CONTROLLER BY USING MODEL ORDER REDUCTION TECHNIQUES

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Abstract –
Physical systems that are modeled typically have higher order systems—generally larger than two. When the system order is high, the controller design for such a physical system becomes laborious. Therefore, using reduced order models to approximate these models is desirable. This work uses the Modified Cauer Form Method and Eigen Spectrum Analysis to obtain a reduced order model. In order to achieve the required requirements, the controller—which is coupled in cascade with the original system—is made for the lower order model. The suggested approach guarantees system stability in the case of the lower order model. A numerical example is provided to illustrate the suggested strategy.

Keywords –
Eigen Spectrum, Order Reduction, Cauer Form, Stability, and Transfer Function, PID Controller.

I. INTRODUCTION

The quality of a reduced order model is judged by the way it is utilized, and the degree of its success in representing the desired characteristics of the system. One of the main objectives of order reduction is to design a controller of low order which can effectively con effectively control the original system high order system so that the overall system is of low order and is easy to understand. It is thus important that the model order reduction methods should reduce the high order controller to a low order controller without incurring too much error. Model reduction is based on open loop considerations while closed loop stability performance is of main concern in controller design. Pade approximation [1, 2] is the method of model order reduction of the higher order system. This gives the simplification of a model after converting it into a reduced order model. A different approach can be used to simplify a model which results are stable model. In this approach the numerator coefficients can be obtained by modified cauer form [3, 5] and denominator coefficients can be obtained by eigen spectrum analysis [6]. Several methods [7, 8] have been developed for designing a PID controller. In this paper a simple algebraic scheme is proposed to design a PID controller for Linear Time Invariant Continuous System. The closed loop transfer function of the reduced order models with PID controller are compared with the reference model transfer function in frequency domain.

II. II. STATEMENT OF THE PROBLEM

2.1. PID Controller Transfer Function PID controller can be mathematically represented as [12],

$$u(t) = k_1 \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d + T_d \frac{d_e(\tau)}{dt} \right] \qquad (1)$$

Where u(t) and e(t) denotes the control and error signals of the system is the proportion gain, and represents the integral and derivative time constants respectively. The corresponding PID controller transfer function is given as

$$G_c(s) = k_1 \left[1 + \frac{1}{T_i s} + T_d s \right] \qquad (2)$$

Equation (2) can be rewritten as

$G_c(s) = k_1 + \frac{k_2}{s} + k_3$ (3)

k1, k2 and k3 are represents the proportional, integral and derivative gain values of the controller

2.2. Higher Order Transfer Function

Let higher order system or process whose performance is unsatisfactory may be described by the transfer function

$$G_c(s) = \frac{N(s)}{D(s)} = \frac{A_{21}+A_{22}s+A_{23}s^2+\ldots\ldots A_{2n}s^{n-1}}{A_{11}+A_{12}s+A_{13}s^2+\ldots\ldots A_{1n+1}s^n} \quad (4)$$

And a reference model having the desired performance is given.

2.3. Lower Order Transfer Function To find a rth lower order model for the above continuous system, where r < n in the following form, such that the lower order model retains the characteristics of the original system and approximates its response as closely as possible for the same type of inputs

$$R(s) = \frac{a_{21}+a_{22}s+a_{23}s^2+\ldots\ldots a_{2n}s^{n-1}}{a_{11}+as+a_{13}s^2+\ldots\ldots a_{1n+1}s^n} \quad (5)$$

Where, a_{2j} and a1i are scalar constants. Objective is to derive a controller such that the performance of the augmented process matches with that of the reference model. To reduce the computational complexities and difficulties of implementation, the higher order of the system is reduced into lower second order system. And PID controller is also derived for reduced order system.

III. REDUCTION METHOD

The reduction procedure for getting the r th-order reduced models consists of the following steps:
The reduction procedure for getting the k^{th}-order reduced models consists of the following two steps:

Step-1: Determination of the denominator polynomial for the kth-order reduced model using Eigen Spectrum Analysis of original system by the following procedure:

Step 1: First, as illustrated in Fig. 1, adjust the Eigen spectrum zone (ESZ) of the HOS.

The ESZ is formed by the two lines that cut through the nearest (Reλ1) and farthest (Reλp) real poles when cut by two lines that pass through the farthest imaginary pole pairs (±Im(max)). This is the case if poles -λi (i = 1,n) are positioned at – (Reλi ± Imλi) (i = 1,p) within the ESZ.

Step 2: Measurement of the HOS stiffness and pole centroid:

The mean of the real parts of the poles is known as the pole centroid, and it may be represented as

$$\lambda_m = \frac{\sum_{i-1}^{p} R_e\lambda_i}{p} \quad (6)$$

System stiffness is defined as the ratio of the nearest to the farthest pole of a system in terms of real parts only and is put as

$$\lambda_s = \frac{R_e\lambda_1}{R_e\lambda_p} \quad (7)$$

Step 3: Determination of eigen spectral points of LOS:

If λ'_m and $\lambda'_s$are pole centroid and system stiffness of LOS such that $\lambda'_m = \lambda_m$ and $\lambda'_s = \lambda_s$ then following situation arise:

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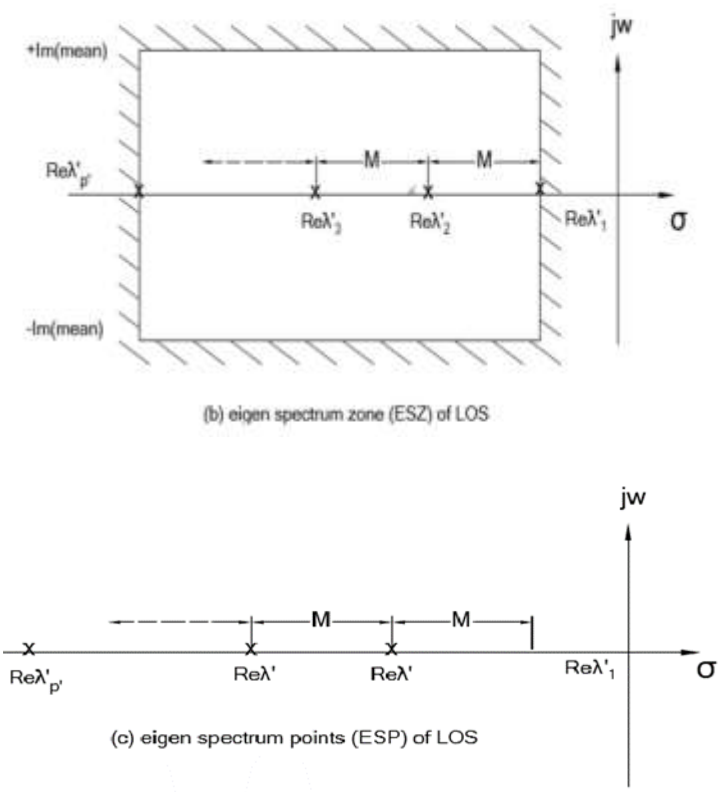
(a) eigen spectrum zone (ESZ) of HOS

$$\lambda'_s = \frac{R_e\lambda'_1}{R_e\lambda'_{p'}} = \lambda_s \qquad (8)$$

$$\lambda'_m = \frac{R_e\lambda'_1 + R_e\lambda'_2 + R_e \ldots\ldots + R_e\lambda'_{p'}}{p'} = \lambda_m \qquad (9)$$



(b) eigen spectrum zone (ESZ) of LOS

(c) eigen spectrum points (ESP) of LOS

Fig.1.Eigen Spectrum zones and points of system

Where λ'_i(i=1,r) are the poles of LOS located at – (Reλ'_i ± Imλ'_i) i=1,p'. Now if,

$$\frac{R_e\lambda'_{p'} - R_e\lambda'_1}{p'-1} = M\,, \qquad (9)$$

i.e. $R_e\lambda'_1$+M=$R_e\lambda'_2$, $R_e\lambda'_2$+M=$R_e\lambda'_3$ and so on till $R_e\lambda'_{p'-1} + M = R_e\lambda'_{p'}$ then Eq. (9) can be put as

$$\lambda_m = \frac{R_e\lambda'_1 + R_e\lambda'_{p'} + - - \mp(R_e\lambda'_1 + (p'-2)M)}{p'}$$

Or $\lambda_m p' = R_e\lambda'_1 + R_e\lambda'_{p'} + (M + 2M + \cdots \ldots + (p'-2)M)$

Or N= $R_e\lambda'_1$(p'-1) + $R_e\lambda'_{p\prime}$+ QM (10)

Where N=λ'_m and QM = M+2M+ …………………….+ (p'-2) M.

By putting$R_e\lambda'_1 = \lambda_s R_e\lambda'_{p\prime}$. Equation (9) and (10) as under:

$$R_e\lambda'_{p\prime} - \lambda_s R_e\lambda'_{p'} = M(p' - 1) \qquad (11)$$

$$R_e\lambda'_{p'}(p' - 1) + R_e\lambda'_{p\prime}\text{+QM} \qquad (12)$$

Eqs. (11) and (12) can be put as

$$\lambda'_{p'}(1 - \lambda_s) + M(1 - p') = 0$$

$$R_e\lambda'_{p'}[\lambda_s(p' - 1) + 1] + MQ = N \qquad or$$

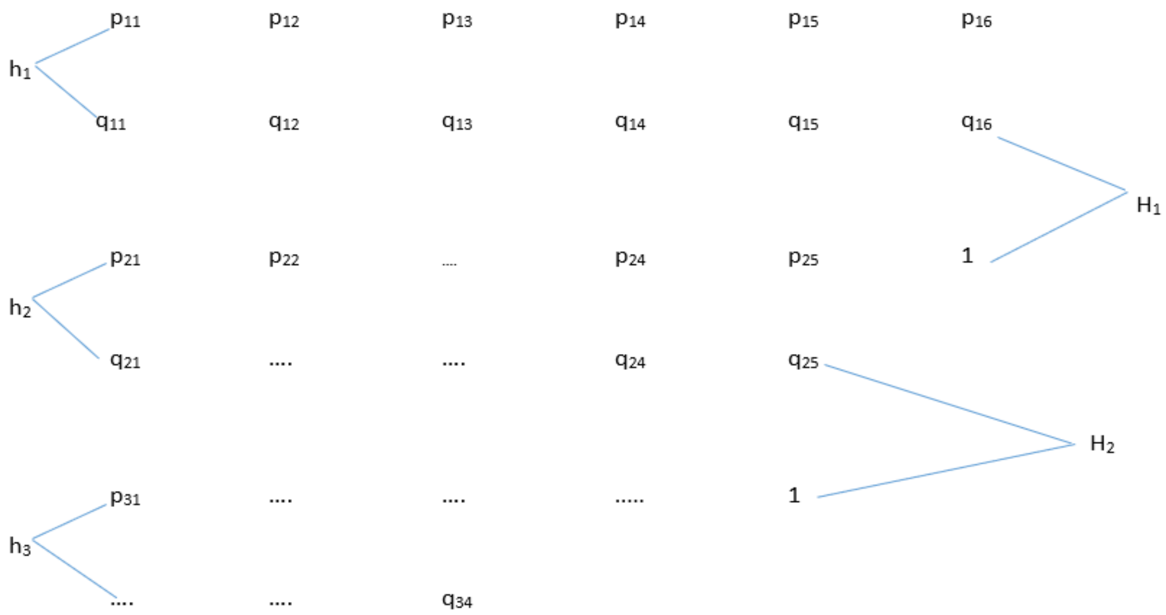
$$\begin{bmatrix} \lambda_s(p' - 1) + 1 & Q \\ (1 - \lambda_s) & (1 - p') \end{bmatrix} \begin{bmatrix} R_e\lambda'_{p'} \\ M \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix} \qquad (13)$$

Eq. (12) can be solved for $R_e\lambda'_{p'}$ and M enabling thereby to locate the Eigen spectral points (ESP) as shown in Fig. 1.

STEP-2:

By applying the algorithm [10], the first 'r' quotients of the modified Cauer form of a continued fraction, viz. h1, H1, h2, H2 are evaluated.

$$G_c(s) = \frac{N(s)}{D(s)} = \frac{A_{21} + A_{22}s + A_{23}s^2 + \dots \dots \dots \dots \dots \dots \dots . A_{2n}s^{n-1}}{A_{11} + A_{12}s + A_{13}s^2 + \dots \dots \dots \dots \dots \dots \dots . A_{1n+1}s^n}$$



IV. METHOD FOR COMPARISON

In order to check the accuracy of the proposed method the relative integral square error ISE index in between the transient parts of the reduced models and the original system is calculated using Matlab / Simulink.
The integral square error ISE is defined as

$$ISE = \int_0^\infty [y_n(t) - y_k(t)]^2 \, dt$$

V.GENERAL ALGORITHM FOR DESIGNING THE PID CONTROLLER

Step 1. Construction of a reference model M(s) on the basis of specifications whose closed loop system must approximate to that of the original closed loop response. Let it be specified as:

$$M(s) = \frac{a_0 + a_1 s + a_2 s^2 + \ldots\ldots\ldots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \ldots\ldots\ldots + b_n s^n} \quad (16)$$

Step 2. Determine an equivalent open loop specification model $\widetilde{M(s)}$

$$\widetilde{M(s)} \qquad = \frac{M(s)}{1 - M(s)}$$

Step 3. Specified the structure of the controller Let the controller structure is given by

$$R_c(s) = \frac{p_0 + p_1 s + A_{23} s^2 + \ldots\ldots\ldots p_k s^k}{A_{11} + A_{12} s + A_{13} s^2 + \ldots\ldots\ldots A_{1n+1} s^n} \qquad (16)$$

Step 4. For determining the unknown controller parameters, the response of the closed loop system is matched with reference model as

$$R_c(s) R_p(s) = \widetilde{M(s)}$$

$$R_c(s) = \frac{\widetilde{M(s)}}{R_c(s)} = \sum_{i=0}^{\infty} e_i s^i \qquad (17)$$

Where ei are the power series expansion coefficients about s=0. Step 5 Now the unknown control parameters pi and qi are obtained by equating the equation (16) and (17) in Pade sense

$$p_0 = q_0 e_0$$
$$p_1 = q_0 e_1 + q_1 e_0$$
$$p_2 = q_0 e_2 + q_1 e_1 + q_2 e_0$$
$$p_i = q_0 e_i + q_1 e_{i-1} + \cdots \ldots \ldots \ldots \ldots \ldots + q_i e_0$$
$$0 = q_0 e_{i+1} + q_1 e_i + \cdots \ldots \ldots \ldots \ldots \ldots + q_{i+1} e_0$$
$$0 = q_0 e_{i+j} + q_1 e_{i+j-1} + \cdots \ldots \ldots \ldots \ldots \ldots + q_j e_0$$

The controller having the desired structure is obtained by solving above linear equations Step 6. After obtaining the controller parameters, the close loop transfer function can be obtained as

$$R_{c1} = \frac{R_c(s) R_k(s)}{1 + R_c(s) R_k(s} \qquad (18)$$

VI.NUMERICAL EXAMPLE

The proposed method explains by considering numerical example, taken from the literature. The goodness of the proposed method is measured by calculating the rise time (t_r), settling time (t_s) and maximum overshoot (M_p) and compare with the original system.

Example-: Consider a fourth order system from the literature

$$G(s) = \frac{72+54s+12s^2+s^3}{100+180s+97s^2+18s^3+s^4} \qquad (19)$$

Step 1:- Denominator of reduced order model is determine using following Eigen Spectrum Analysis of original system where $\lambda_1 = -1, \lambda_2 = -2, \quad \lambda_3 = -5\,, \ \lambda_4 = -10$

Correcting the HOS's ESZ:

It will be a line connecting the closest and furthest poles because all poles are real.

Step 2: Measurement of the HOS stiffness and pole centroid:

$$\lambda_m = \frac{\sum_{i=1}^{10} \lambda_i}{10} = 2.5$$

$$\lambda_s = \frac{\lambda_1}{\lambda_4} = 0.25$$

Step 3: Determination of Eigen spectral points of LOS:

Eq. (10) can be formed as under:

$$\begin{bmatrix} 1.1 & 0 \\ 0.90 & -1 \end{bmatrix} \begin{bmatrix} \lambda'_{2'} \\ M \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$\lambda'_{2'} = 4.5454$$
$$\lambda'_{1'} = 0.4546$$

Therefore $\widetilde{D}_2(s) = s^2 + 5s + 2.0663$

Step-4 Determination of numerator using Modified Cauer Form

$h_1 = 1.38$	100	180	97	18	1	
	72	54	12	1		$1 = H_1$
h_2 = -25.24	105	80.33	16.61	1		
	-4.16	-26.33	-33			-33=H_2
	584.24	816.32	1			

Construct the modified Routh array as described in reduction method:

2.0663	5	1	
1.49	1		H_1 =1
	1		

$N_2(s) = 1.49s + 1$

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$R_2(s) = \frac{N2(s)}{D2(s)} = \frac{1.49s+1}{s^2+5s+2.0663}$ (20)

Applying steady state correction to reduced order model.

SSO = $\frac{72}{100}$ = 0.72

SSR = $\frac{1}{2.0663}$ = 0.4839

K2 = SS0/SSR2 = 0.72/0.4839 = 1.4879

So, that the finally second order transfer function are shown below in eq.

$R_2(s) = \frac{2.2169s + 1.4879}{s^2+5s+2.0663}$

PID Controller Design Using Reduced Order Model Consider a reference model

$$M(s) = \frac{4.242s + 25}{s^2 + 7.07s + 25}$$

The equivalent open loop transfer function is

$$\widetilde{M(s)} = \frac{4.242s + 25}{s^2 + 2.828s + 50}$$

The reduced controller transfer function is

$$R_c(s) = \frac{\widetilde{M(s)}}{R_2(s)} = \frac{51.6578 + 133.7652s + 46.210s^2 + 4.242s^3}{4.2077s + 7.7572s^2 + 2.2169s^3}$$

$$R_c(s) = \frac{12.2768 + 9.1574s - 12.3683s^2}{s}$$

It is compared by PID controller transfer function $= k_1 + \frac{k_2}{s} + k_3$

And the value of controller parameters are obtained as

K1= 9.1574, K2= 12.2768, K3= -12.3683

Thus corresponding closed loop transfer function is

$$R_{cL}(s) = \frac{R_c(s) * R_2(s)}{1 + R_c(s) * R_2(s)}$$

$$R_{LC}(s) = \frac{1.8983s^2+40.8416s+18.2666}{s^3+6.8983s^2+42.9082s+18.2666}$$

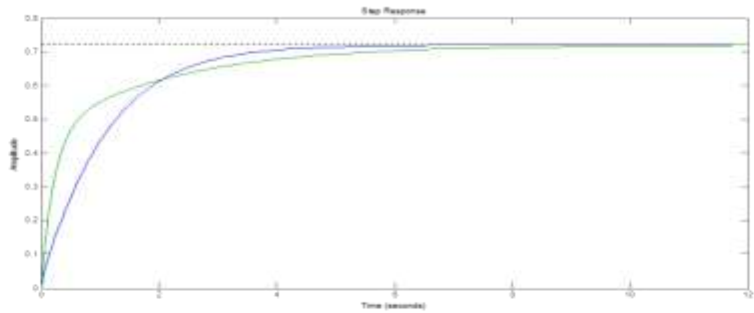

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Fig. 1. Comparison of original plant and reduced order system for step response

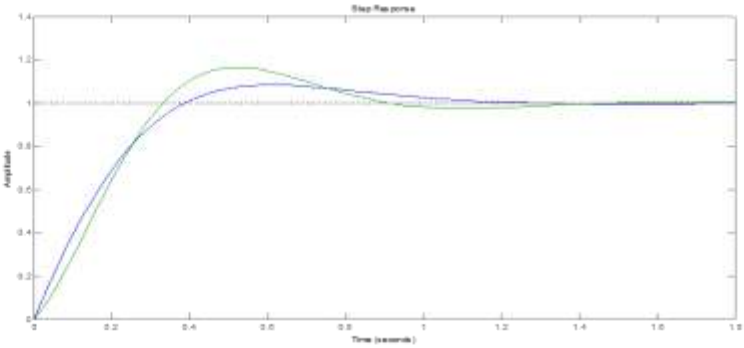

Fig. 2. Comparison of original plant and reduced order system for step response

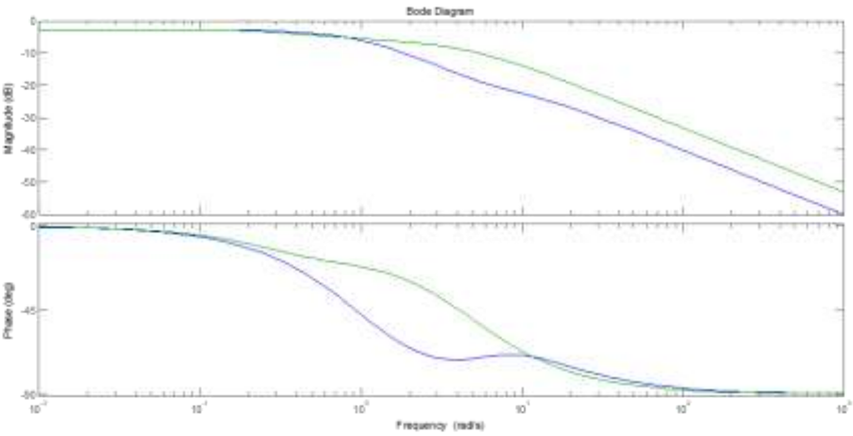

Fig. 3. Comparison of original plant and reduced order system for frequency response

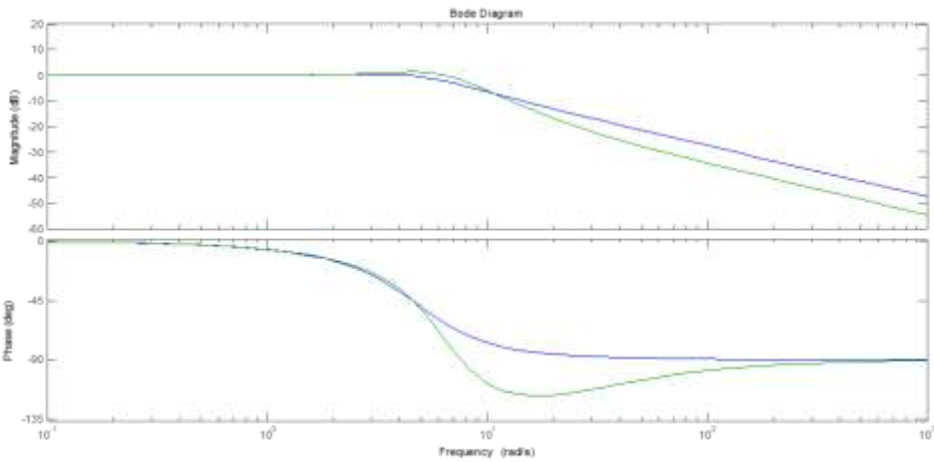

Fig. 4. Comparison of original plant and reduced order system for frequency response

TABLE A

Comparison of different parameters of original plant G(s), reduced order plant $R_2(s)$, and closed loop plant $R_{CL}(s)$

S.No.	Parameter	$G(s)$	$R_2(s)$	$R_{CL}(s)$

1	Rise Time	2.3155	2.7737	0.2455
2	Settling Time	4.0227	6.3493	1.2193
3	Peak	0.7195	0.7194	1.1644
4	Peak Time	7.3222	12.9480	0.5291

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