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LUCAS ANTIMAGIC LABELING OF SOME CATERPILLAR GRAPHS

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ABSTRACT

A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \to \{1,2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).

In this paper the Lucas Antimagic Labeling of some Caterpillar graphs are found.

KEYWORDS:
Caterpillar graph, Comb graph, Double comb graph, Hurdle graph, Twig graph, Lucas Antimagic graph.

1.INTRODUCTION

In this paper, graph $G(V,E)$ is considered as finite, simple and undirected with p vertices and q edges. A graph labeling is a fundamental concept in graph theory, where integers are assigned to vertices or edges. Its enormous applications in astronomy, theory of coding and other fields has propelled it to the frontend of research. After referring, the seminal work of Gallian, as showcased in his comprehensive survey [1], we have embarked on this research endeavor. Furthermore, the innovative concept of Antimagic labeling, introduced by N.Hartsfield and G.Ringel in the year 1990, has opened up new avenues of exploration. Inspired by these groundbreaking contributions, we introduced Lucas Antimagic labeling and further Lucas Antimagic labeling has been investigated on various caterpillar graphs (A graph G is known to be a caterpillar if G is a tree such that the elimination of the vertices with degree 1 ends in a path. The resulting path is called the spine of the caterpillar) namely comb, double comb, hurdle, twig, $P_n \odot mK_1$, $S(n_1, n_2, \ldots n_m)$

2.DEFINITIONS

Definition 2.1:[2] Lucas number is defined by the linear recurrence relation
$L_1 = 2, \quad L_2 = 1 \; and \; L_n = L_{n-1} + L_{n-2}, n > 2$
The first few Lucas numbers are 2,1,3,4,7,11,18,29,47,…
Definition 2.2:[2] A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \to \{1,2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).
Definition 2.3:[3] The comb graph is represented by $P_n \odot K_1, n \geq 2$. The P_n is a path graph with (n-1) edges and n vertices. The graph is constructed by connecting each vertex in the path with a pendant edge.
Definition 2.4:[4] A double comb graph is acquired from a path P_n by linking two pendant vertices at each vertex of P_n indicated by $P_n \odot 2K_1, n \geq 2$.
Definition 2.5:[5] A graph acquired from a path P_n by linking a pendant edge to every internal vertices of the path is called Hurdle graph with n-2 hurdles and is signified by $Hd_n, n \geq 3$.
Definition 2.6:[6] A Twig graph is acquired from P_n by including exactly two pendant edges to each internal vertices of the path.
Definition 2.7: $P_n \odot mK_1, n \geq 1, m \geq 3$ is acquired from a path P_n by attaching m pendant vertices at each vertex of P_n.
Definition 2.8:[7] Let $l_1, l_2, \ldots l_m$ be the m vertices of the path P_m. From each vertex

$l_i, i = 1,2, \dots m$ there are $n_i, i = 1,2 \dots m$ pendant vertices say $l_1^i, l_2^i, \dots l_{n_i}^i$.The resultant graph is a Caterpillar.

3.MAIN RESULTS

Theorem 3.1:

The Comb graph $P_n \odot K_1, n \geq 2$ is Lucas antimagic graph.

Proof :

Let G be a Comb graph $P_n \odot K_1$.

Let $V(G) = \{u_i, v_i : i \in [1, n]\}$

$E(G) = \{u_i u_{i+1} : i \in [1, n-1],\ u_i v_i : i \in [1, n]\}$

Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(u_i u_{i+1}) = L_{n+i}, i \in [1, n-1]$

$f(u_i v_i) = L_i \ , i \in [1, n]$

The induced mapping $f^*: V(G) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_{n+1}$

$f^*(u_i) = L_{n+i} + L_{n-1+i} + L_i, \ \ i \in [2, n-1]$

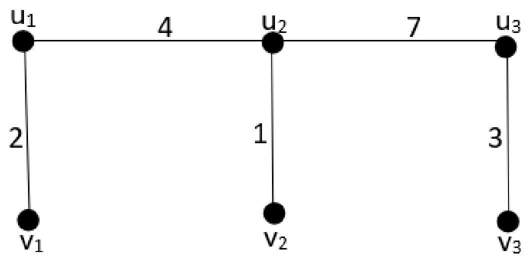
$f^*(u_n) = L_n + L_{2n-1}$

$f^*(v_i) = L_i, \ i \in [1, n]$

We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.1.1: The Comb graph $P_3 \odot K_1$ and its Lucas antimagic labeling.



Theorem 3.2:

The double Comb graph $P_n \odot 2K_1, n \geq 2$ is Lucas antimagic graph.

Proof :

Let G be a double Comb graph $P_n \odot 2K_1$.

Let $V(G) = \{u_i, v_i, w_i : i \in [1, n]\}$

$E(G) = \{u_i u_{i+1} : i \in [1, n-1],\ u_i v_i, u_i w_i : i \in [1, n]\}$

Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(u_i u_{i+1}) = L_{n+i}, i \in [1, n-1]$

$f(u_i v_i) = L_i \ , i \in [1, n],$

$f(u_i w_i) = L_{2n-1+i} \ , i \in [1, n]$

The induced mapping $f^*: V(G) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_{n+1} + L_{2n}$

$f^*(u_i) = L_{n+i} + L_{n-1+i} + L_{2n-1+i} + L_i, \ \ i \in [2, n-1]$

$f^*(u_n) = L_n + L_{2n-1} + L_{3n-1}$

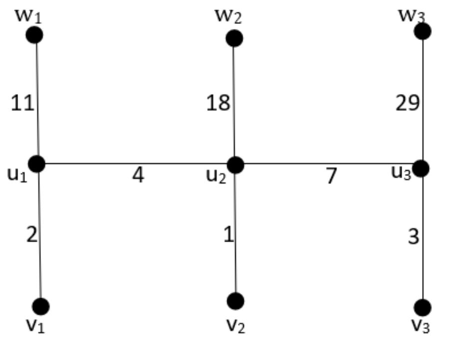
$f^*(v_i) = L_i, i \in [1, n]$

$f^*(w_i) = L_{2n-1+i}, i \in [1, n]$

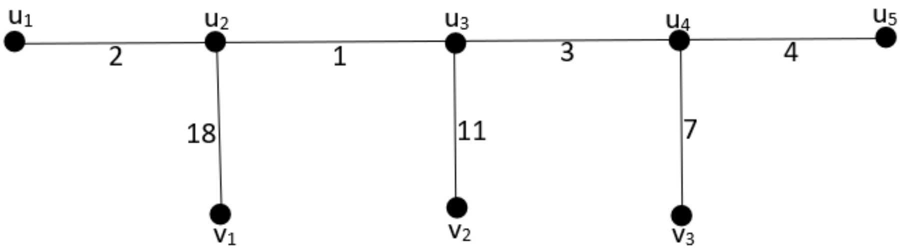
We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.2.1: The double Comb graph $P_3 \odot 2K_1$ and its Lucas antimagic labeling.



Theorem 3.3:
The Hurdle graph $Hd_n, n \geq 3$ is Lucas antimagic graph.
Proof :
Let G be a Hurdle graph Hd_n .
Let $V(G) = \{u_i: i \in [1, n], v_i: i \in [1, n-2]\}$
$E(G) = \{u_i u_{i+1} : i \in [1, n-1]\ ,\ v_i u_{i+1}\ i \in [1, n-2]\}$
Define a function $f: E(G) \rightarrow \{L_1, L_2, \dots L_q\}$ by
$f(u_i u_{i+1}) = L_i\ , i \in [1, n-1]$
$f(v_i u_{i+1}) = L_{2n-2-i}\ , i \in [1, n-2]$
The induced mapping $f^* :\ V(G) \rightarrow \{1,2, \dots \sum L_q\}$ is given by
$f^*(u_1) = L_1$
$f^*(u_i) = L_i + L_{i-1} + L_{2n-1-i}, i \in [2, n-1]$
$f^*(u_n) = L_{n-1}$
$f^*(v_i) = L_{2n-2-i}\ ,\ \ i \in [1, n-2]$
We observe that the vertices are all distinct.
Hence G is Lucas antimagic graph.
Example 3.3.1: The Hurdle graph Hd_5 and its Lucas antimagic labeling.



Theorem 3.4:
The Twig graph $Tg_n, n \geq 3$ is Lucas antimagic graph.
Proof :
Let G be a Twig graph Tg_n.
Let $V(G) = \{u_i: i \in [1, n], v_i, w_i: i \in [1, n-2]\}$
$E(G) = \{u_i u_{i+1}\ i \in [1, n-1]\ ,\ v_i u_{i+1},\ w_i u_{i+1}: i \in [1, n-2]\}$
Define a function $f: E(G) \rightarrow \{L_1, L_2, \dots L_q\}$ by
$f(u_i u_{i+1}) = L_i\ , i \in [1, n-1]\ ,$
$f(v_i u_{i+1}) = L_{2n-2-i}\ , i \in [1, n-2]$
$f(w_i u_{i+1}) = L_{2n-3+i}\ , i \in [1, n-2]$
The induced mapping $f^* :\ V(G) \rightarrow \{1,2, \dots \sum L_q\}$ is given by
$f^*(u_1) = L_1$
$f^*(u_i) = L_i + L_{i-1} + L_{2n-1-i} + L_{2n-4+i},\ i \in [2, n-1]$
$f^*(u_n) = L_{n-1}$

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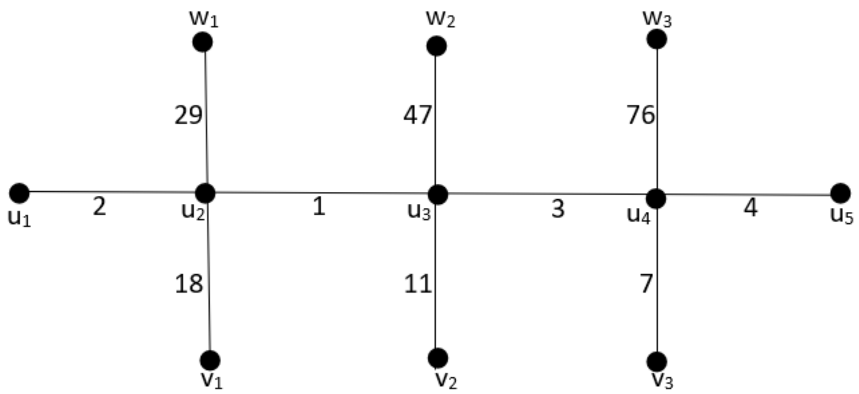
$f^*(v_i) = L_{2n-2-i}$, $i \in [1, n-2]$
$f^*(w_i) = L_{2n-3+i}$, $i \in [1, n-2]$
We observe that the vertices are all distinct.
Hence G is Lucas antimagic graph.
Example 3.4.1: The Twig graph Tg_5 and its Lucas antimagic labeling.



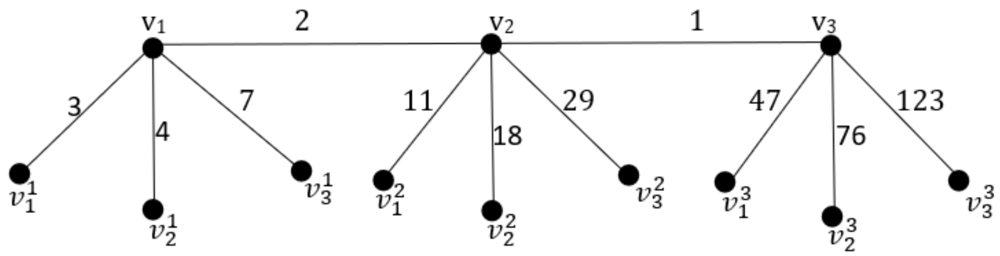
Theorem 3.5:
$P_n \odot mK_1$, $n \geq 1, m \geq 3$ is Lucas antimagic graph.
Proof :
Let G be $P_n \odot mK_1$.
Let $V(G) = \{v_i , v_j^i : i \in [1, n], j \in [1, m]\}$
$E(G) = \{v_i\, v_j^i : i \in [1, n], j \in [1, m]\ ,\ v_i v_{i+1} : i \in [1, n-1]\}$
Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by
$f(v_i v_{i+1}) = L_i$, $i \in [1, n-1]$
$f\left(v_i\, v_j^i\right) = L_{n-1+j+m(i-1)}$, $i \in [1, n], j \in [1, m]$
The induced mapping $f^* :\ V(G) \to \{1,2, \dots \sum L_q\}$ is given by

$$f^*(v_1) = L_1 + \sum_{j=1}^{m} L_{n-1+j}$$

$f^*(v_i) = L_{i-1} + L_i + \sum_{j=1}^{m} L_{n-1+j+m(i-1)}$, $i \in [2, n-1]$

$$f^*(v_n) = L_{n-1} + \sum_{j=1}^{m} L_{j+(n-1)(m+1)}$$

$f^*\left(v_j^i\right) = L_{n-1+j+m(i-1)}$, $i \in [1, n]$, $j \in [1, m]$
We observe that the vertices are all distinct.
Hence G is Lucas antimagic graph.
Example 3.5.1: The graph $P_3 \odot 3K_1$ and its Lucas antimagic labeling.



Theorem 3.6:
The Caterpillar $S(n_1, n_2, \dots n_m), m \geq 1$ is Lucas antimagic graph.
Proof :

Let G be the Caterpillar $S(n_1, n_2, \ldots n_m)$.
Let $V(G) = \{v_i, v_j^i : i \in [1,m], j \in [1,n_i]\}$
$E(G) = \{v_i v_j^i : i \in [1,m], j \in [1,n_i]\ ,\ v_i v_{i+1} : j \in [1,m-1]\}$
Define a function $f: E(G) \to \{L_1, L_2, \ldots L_q\}$ by
$f(v_i v_{i+1}) = L_{n_1+n_2+\cdots+n_m+i}, i \in [1,m-1]$
$f(v_i v_j^i) = L_{n_1+n_2+\cdots+n_{i-1}+j}, i \in [1,m], j \in [1,n_i]$
The induced mapping $f^*: V(G) \to \{1,2, \ldots \sum L_q\}$ is given by

$$f^*(v_1) = L_{n_1+n_2+\cdots+n_m+1} + \sum_{j=1}^{n_1} L_j$$

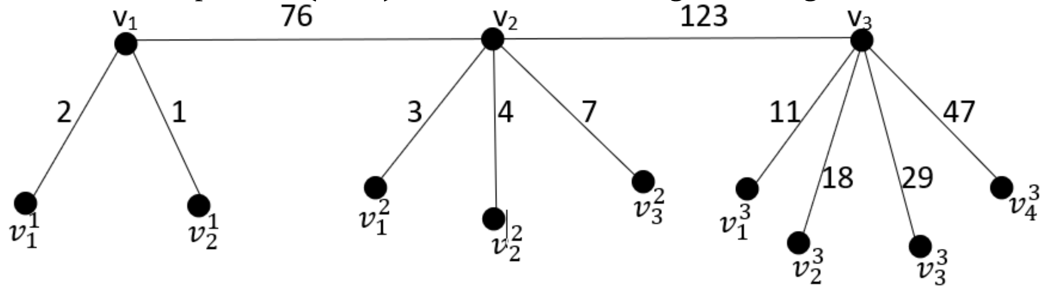
$$f^*(v_i) = L_{n_1+n_2+\cdots+n_m+i-1} + L_{n_1+n_2+\cdots+n_m+i} + \sum_{j=1}^{n_i} L_{n_1+n_2+\cdots+n_{i-1}+j}, i \in [2,m-1]$$

$$f^*(v_m) = L_{n_1+n_2+\cdots+n_m+m-1} + \sum_{j=1}^{n_m} L_{n_1+n_2+\cdots+n_{m-1}+j}$$

$$f^*(v_j^i) = L_{n_1+n_2+\cdots+n_{i-1}+j}, i \in [1,m], j \in [1,n_i]$$

We observe that the vertices are all distinct.
Hence G is Lucas antimagic graph.

Example 3.6.1: The Caterpillar $S(2,3,4)$ and its Lucas antimagic labeling.



4.CONCLUSION:

In this article, It is proved that various caterpillar graphs are Lucas Antimagic. Similar investigations are in process.

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