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ON FUZZY NEUTROSOPHIC $I-$ BAIRE SPACES

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Abstract:

In this paper, the concept of fuzzy neutrosophic $I-$ Baire spaces is introduced, characterizations and properties of these spaces are stuided. It is shown that (X, τ_N) is Baire if and only if (X, τ_N, I) is fuzzy neutrosophic $I-$ Baire for any ideal space I on X.

Keywords: Fuzzy neutrosophic $*-$dense set, Fuzzy neutrosophic nowhere $*-$dense set, Fuzzy neutrosophic $*-$ residual set, Fuzzy neutrosophic $I-$Baire spaces, Fuzzy neutrosophic $*-$ one and two category.
AMS subject classification: 54A40, 03E72

Introduction:

Fuzzy sets are mathematical framework that extends classical set theory to handle uncertainty and vagueness. Introduced by Lofti A. Zadeh in 1965 [12], fuzzy set provide a way to represent and manipulate imprecise or ambiguous information. In classical set theory, an element can either belong or not belong to a set, with no degrees of membership. However, in many real world situations, the membership of an element to a set is not always clear-cut. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [11] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

Fuzzy sets have found applications in various fields, including control systems, artificial intelligence, decision-making, pattern recognition and data analysis. They provide a powerful tool for modelling and reasoning with imprecise and uncertain information, allowing for more flexible and nuanced representations of real world.

The aim of this paper is to introduce and study fuzzy neutrosophic $I-$ Baire spaces. Some characterizations and properties of fuzzy neutrosophic I- Baire spaces are investigated. If τ_N is a fuzzy neutrosophic topology on X and I is an ideal topology on X, then (X, τ_N, I) is called an fuzzy neutrosophic ideal topological space or simply an fuzzy neutrosophic ideal space.

Preliminaries:

Throughout the present paper, (X, τ_N, I) denote the fuzzy neutrosophic ideal topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic ideal interior and ideal closure of A_N is denoted by $*-(A_N)^+$, $*-(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzy neutrosophic $*-$open set $(fnOS)$ if $A_N \leq *-fn\,(((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic $*-$ open set is called fuzzy neutrosophic $*-$closed set $(fnCS)$.

Definition 2.1 [2]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [2]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Definition 2.3 [2]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$

$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$

Definition 2.4 [2]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [2]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1\ for\ all\ x\ \in X$. It is denoted by 0_N.

Definition 2.6 [2]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x\ \in X$. It is denoted by 1_N.

Definition 2.7 [2]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1]:

A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

$(i)\ \ 0_N, 1_N \in \tau$

$(ii)\ \ A_1 \cap A_2 \in \tau\ \ for\ any\ A_1, A_2 \in \tau$

$(iii)\ \ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\}\ \in \tau$

Satisfying the following axioms.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1]:

Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$int(A) = \cup\{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$

$cl(A) = \cap\{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$

Definition 2.11 [4]:

A map $I : I^X \to I$ is called a fuzzy ideal on X if it satisfies:

(1) $I(\overline{0}) = 1$,

(2) $\lambda \leq \mu \Rightarrow I(\lambda) \geq I(\mu)$ for all $\lambda, \mu \in I^X$,

(3) $I(\lambda \vee \mu) \geq I(\lambda) \wedge I(\mu)$ for all $\lambda, \mu \in I^X$.

If I_1 and I_2 are fuzzy ideals on X, we have I_1 is finer than I_2 (I_2 is coarser than I_1), denoted by $I_1 \leq I_2$ iff $I_1(\lambda) \leq I_2(\lambda)\ \forall \lambda \in I^X$. The triple (X, τ, I) is called a fuzzy ideal topological space.

On Fuzzy Neutrosophic $*-$ Dense Sets and Nowhere $*-$ Dense Sets

Motivated by the classical concept of $*-$ denseness and nowhere $*-$ denseness in [13], and the fuzzy neutrosophic $*-$ dense set and fuzzy neutrosophic nowhere $*-$ dense set we shall now defined.

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Definition 3.1:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological ideal space (X, τ_N, I) is called a fuzzy neutrosophic $* -$ dense if there exist no fuzzy neutrosophic $* -$ closed set B_N in (X, τ_N, I) such that $A_N < B_N < 1$. That is, $fn \; * -(A_N)^- = 1_N$.

Definition 3.2:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological ideal space (X, τ_N, I) is called a fuzzy neutrosophic nowhere $* -$ dense if there exist no non-zero fuzzy neutrosophic $* -$ open set B_N in (X, τ_N, I) such that $B_N < \; fn \; * -(A_N)^-$. That is, $fn * - (((A_N)^-)^+) = 0_N$.

Example 3.1:

Let $X = \{a, b, c\}$ and consider the fuzzy neutrosophic sets A_N, B_N are defined as follows.

$A_N = \{\langle a, 0.6, 0.6\rangle, \langle b, 0.6, 0.6\rangle, \langle c, 0.3, 0.4\rangle\}$

$B_N = \{\langle a, 0.6, 0.5\rangle, \langle b, 0.6, 0.5\rangle, \langle c, 0.4, 0.5\rangle\}$

Then $\tau_N = \{0_N, 1_N, A_N, B_N\}$ is a fuzzy neutrosophic ideal topological space on X.

Thus (X, τ_N, I) is a fuzzy neutrosophic ideal topological space, $1 - A_N, 1 - B_N$ are fuzzy neutrosophic nowhere $* -$ dense sets.

Definition 3.3:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic ideal topological space (X, τ_N, I) is called a fuzzy neutrosophic $* -$ semi-open if $A_N \leq \; fn * - \left(((A_N)^+)^- \right)$. The complement of A_N in (X, τ_N, I) is called a fuzzy neutrosophic $* -$ semi-closed set in (X, τ_N, I).

Definition 3.4:

Let (X, τ_N, I) be a fuzzy neutrosophic ideal topological space. A fuzzy neutrosophic set A_N in (X, τ_N, I) is called fuzzy neutrosophic $* -$ one category set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere $* -$ dense sets in (X, τ_N, I). A fuzzy neutrosophic which is not fuzzy neutrosophic $* -$ one category set is called a fuzzy neutrosophic $* -$ two category set in (X, τ_N, I).

Definition 3.5:

Let A_N be a fuzzy neutrosophic $* -$ one category set in (X, τ_N, I). Then $1 - \; A_N$ is called fuzzy neutrosophic $* -$ residual set in (X, τ_N, I).

Proposition 3.1:

If (X, τ_N, I) be a fuzzy neutrosophic ideal topological space. If A_N is a fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I) then

(1) $fn \; * - (A_N)^+ = 0$.

(2) $(1 - A_N)$ is fuzzy neutrosophic $* -$ dense set in (X, τ_N, I).

(3) $fn \; * -(A_N)^-$ is fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I).

(4) $1 - \; fn * -(A_N)^-$ is fuzzy neutrosophic $* -$ dense set in (X, τ_N, I).

(5) A_N is a fuzzy neutrosophic $* -$ semi closed set in (X, τ_N, I).

Proof:

(1) Let A_N be a fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I). Then $fn * - (A_N)^+ = A_N$. Now $fn * - \left(((A_N)^-)^+ \right) = \; fn * - (A_N)^+ = 0_N$. And hence we have, $fn * - (A_N)^+ = 0_N$.

(2) Let A_N be a fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I). Then by proposition 3.1(1) we have $fn * - (A_N)^+ = 0_N$. Now $1 - (fn * - (1 - A_N)^+) = 1 - 0 = 1_N$. Therefore $(1 - A_N)$ is fuzzy neutrosophic $* -$ dense set in (X, τ_N, I).

(3) Let A_N be a fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I). Now $fn * - (A_N)^- \leq A_N$ gives that $fn * - (A_N)^- \leq \; fn * - \left(((A_N)^-)^+ \right) = 0_N$. Hence by the hypothesis, $fn * -(A_N)^-$ is fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I).

(4) By proposition 3.1(3), $fn*-(A_N)^-$ is fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I). By proposition 3.1(2), we have $1-(fn*-(1-A_N)^-)$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

(5) Let A_N be a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I). Then $fn*-(((A_N)^-)^+)=0_N$. And therefore $fn*-(((A_N)^-)^+)\leq A_N$. Hence, A_N a fuzzy neutrosophic $*-$ semi closed set in (X,τ_N,I).

Proposition 3.2:
If A_N is a fuzzy neutrosophic $*-$ dense, fuzzy neutrosophic $*-$ open set in (X,τ_N,I) such that $B_N\leq 1-A_N$, then B_N is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

Proof:
Let A_N be a fuzzy neutrosophic $*-$ open set in (X,τ_N,I) such that $fn*-(A_N)^-=1_N$. Now $B_N\leq 1-A_N$ implies that $fn*-(B_N)^-\leq fn*-(1-A_N)^-=(1-A_N)$ $[1-A_N$ is a fuzzy neutrosophic $*-$ closed set in $(X,\tau_N,I)]$ Then we have $fn*-(((B_N)^-)^+)\leq fn*-(1-A_N)^+=1-(fn*-(A_N)^-)=1-1=0_N$. And hence $fn*-(((B_N)^-)^+)=0_N$. Therefore B_N is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

Proposition 3.3:
If A_N is a fuzzy neutrosophic nowhere $*-$dense set and fuzzy neutrosophic $*-$ open set in (X,τ_N,I), then $1-A_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

Proof:
Let A_N be a fuzzy neutrosophic $*-$ open set in (X,τ_N,I) such that $fn*-(A_N)^-=1$. Now $fn*-(((1-A_N)^-)^+)=1-\left(fn*-(((1-A_N)^+)^-)\right)=1-fn*-(A_N)^-=1-1=0_N$. Hence $1-A_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

Proposition 3.4:
Let A_N be a fuzzy neutrosophic $*-$ dense set in (X,τ_N,I). If B_N is any fuzzy neutrosophic set in (X,τ_N,I), then B_N is a fuzzy neutrosophic nowhere $*-$dense set in (X,τ_N,I), if and only if $A_N\wedge B_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

Proof:
Let B_N be a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

Now, $fn*-(((A_N\wedge B_N)^-)^+)=fn*-\left(fn*-(A_N)^-\wedge fn*-(B_N)^-\right)^+$
$=(fn*-(1\wedge fn*-(B_N)^-))^+$
$=fn*-(((B_N)^-)^+)=0_N$.

Therefore $A_N\wedge B_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).
Conversely, let $A_N\wedge B_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I). Then $fn*-(((A_N\wedge B_N)^-)^+)=0_N$ implies that $fn*-((fn*-(A_N)^-\wedge fn*-(B_N)^-))^+$.
Hence $fn*-((1\wedge fn*-(B_N)^-))^+=0_N$ and therefore $fn*-(((B_N)^-)^+)=0_N$ which means that B_N is a fuzzy neutrosophic nowhere $*-$ dense set in (X,τ_N,I).

Proposition 3.5:
If A_N is a fuzzy neutrosophic $*-$ one category set in (X,τ_N,I) then $1-A_N=\wedge_{i=1}^{\infty}B_{N_i}$, where $fn*-(B_{N_i})^-=1$.

Proof:
Let A_N be a fuzzy neutrosophic $*-$ one category set in (X,τ_N,I). Then $A_N=\vee_{i=1}^{\infty}A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere $*-$ dense sets in (X,τ_N,I). Now $(1-A_N)=1-\vee_{i=1}^{\infty}A_{N_i}=\wedge_{i=1}^{\infty}1-A_{N_i}$. Now $A_{N_i}$is a fuzzy neutrosophic nowhere $*-$ dense sets

in (X, τ_N, I). Then, by proposition 3.1 (4), we have $1 - A_{N_i}$ is a fuzzy neutrosophic $* -$ dense set in (X, τ_N, I). Let us put $B_{N_i} = 1 - A_{N_i}$. Then $1 - A_N = \wedge_{i=1}^{\infty} B_{N_i}$ where $fn * - (B_{N_i})^- = 1$.

Proposition 3.6:

Every fuzzy neutrosophic nowhere $* -$ dense sets is a fuzzy neutrosophic $* -$ closed set.

Proof:

Let A_N be any fuzzy neutrosophic nowhere $* -$ dense set in a fuzzy neutrosophic ideal topological space (X, τ_N, I). Therefore, we have $fn * - (((A_N)^-)^+) = 0_N$ and it means that there does not exist any fuzzy neutrosophic $* -$ open set in between A_N and $fn * -(A_N)^-$. Also, let us suppose that $A_N \leq B_N$, where B_N is fuzzy neutrosophic $* -$ open set and obviously $fn * - (A_N)^- \leq B_N$. Therefore B_N is a fuzzy neutrosophic $* -$ closed set.

Fuzzy Neutrosophic $I -$ Baire Space

Definition 4.1:

A fuzzy neutrosophic ideal topological space (X, τ_N, I) is called fuzzy neutrosophic $I -$ Baire space if $fn * - (\vee_{i=1}^{\infty}(A_{N_i}))^+ = 0_N$, where $A_{N_i}'s$ are fuzzy neutrosophic nowhere $* -$ dense sets in (X, τ_N, I).

Example 4.1:

Let $X = \{a, b, c\}$ and consider the fuzzy neutrosophic sets A_N, B_N are defined as follows.

$A_N = \{\langle a, 0.6, 0.5\rangle, \langle b, 0.6, 0.5\rangle, \langle c, 0.3, 0.4\rangle\}$

$B_N = \{\langle a, 0.6, 0.6\rangle, \langle b, 0.6, 0.4\rangle, \langle c, 0.3, 0.5\rangle\}$

Then $\tau_N = \{0_N, 1_N, A_N, B_N, A_N \vee B_N, A_N \wedge B_N\}$ is a fuzzy neutrosophic ideal topological space on X. Thus (X, τ_N, I) is a fuzzy neutrosophic ideal topological space. $1 - A_N, 1 - B_N, 1 - A_N \vee B_N \vee 1 - A_N \wedge B_N$ are fuzzy neutrosophic nowhere $* -$ dense sets and $[1 - A_N \vee 1 - B_N \vee 1 - A_N \vee B_N \vee 1 - A_N \wedge B_N] = 1 - A_N \wedge B_N$ is a fuzzy neutrosophic $* -$ one category set. This implies that $fn * - (1 - A_N \wedge B_N)^+ = 0$ this implies that $1 - A_N \wedge B_N$ is a fuzzy neutrosophic $I -$ Baire space.

Definition 4.2:

A fuzzy neutrosophic ideal topological space (X, τ_N, I) is called fuzzy neutrosophic $* -$ one category space if $1_X = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere $* -$ dense sets in (X, τ_N, I). A fuzzy neutrosophic ideal topological space which is not of fuzzy neutrosophic $* -$ one category, is said to be of fuzzy neutrosophic $* -$ second category.

Proposition 4.1:

Let (X, τ_N, I) be a fuzzy neutrosophic ideal topological space. Then the following are equivalent.

i) (X, τ_N, I) is a fuzzy neutrosophic $I -$ Baire space.

ii) $fn * - (A_N)^+ = 0_N$, for every fuzzy neutrosophic $* -$ one category set A_N in (X, τ_N, I).

iii) $fn * - (B_N)^+ = 1_N$, for every fuzzy neutrosophic $* -$ residual set B_N in (X, τ_N, I).

Proof:

$(i) \Rightarrow (ii)$

Let A_N be a fuzzy neutrosophic $* -$ one category set in (X, τ_N, I). Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where A_{N_i}' s are fuzzy neutrosophic nowhere $* -$ dense sets in (X, τ_N, I). Now $fn * - (A_N)^+ = fn * - (\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$. Since (X, τ_N, I) is a fuzzy neutrosophic $I -$ Baire space. Therefore $fn * - (A_N)^+ = 0_N$.

$(ii) \Rightarrow (iii)$

Let B_N be a fuzzy neutrosophic $* -$ residual set in (X, τ_N, I). Then $1 - B_N$ is a fuzzy neutrosophic $* -$ one category set in (X, τ_N, I). By hypothesis, $fn * - (1 - B_N)^+ = 0_N$ which implies that $fn * - (1 - (1 - A_N)^-) = 0_N$. Hence $fn * - (A_N)^- = 1_N$.

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$(iii) \Rightarrow (i)$

Let A_N be a fuzzy neutrosophic $*-$one category set in (X, τ_N, I). Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where $A_{N_i}{}'$ s are fuzzy neutrosophic nowhere $*-$dense sets in (X, τ_N, I). Now A_N is a fuzzy neutrosophic $*-$ one category set implies that $1 - A_N$ is a fuzzy neutrosophic $*-$ residual set in (X, τ_N, I). By hypothesis, we have $1 - fn * -(1 - A_N) = 1_N$ which implies that $1 - fn * -(A_N)^+ = 1_N$. Hence $fn * -(A_N)^+ = 0_N$. That is, $fn * -(\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where $A_{N_i}{}'$ s are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Hence (X, τ_N, I) is a fuzzy neutrosophic $I-$Baire space.

Proposition 4.2:

If $fn * -(\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$ where $fn * -(A_{N_i})^+ = 0_N$ and $A_{N_i} \in \tau_N$, then (X, τ_N, I) is a fuzzy neutrosophic $I-$Baire space.

Proof:

Now $A_{N_i} \in \tau_N$ implies that A_{N_i} is a fuzzy neutrosophic $*-$ open sets in (X, τ_N, I). Since $fn * -(A_{N_i})^+ = 0_N$. By proposition 3.1(1), A_{N_i} is a fuzzy neutrosophic nowhere $*-$dense sets in (X, τ_N, I). Therefore $fn * -(\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where $A_{N_i}{}'s$ is a fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Hence (X, τ_N, I) is a fuzzy neutrosophic $I-$Baire space.

Proposition 4.3:

If $fn * -(\vee_{i=1}^{\infty} A_{N_i})^+$ where $fn * -(A_{N_i})^+ = 0_N$ and A_{N_i}'s are fuzzy neutrosophic $*-$closed sets in fuzzy neutrosophic ideal topological space in (X, τ_N, I) then (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Proof:

Let $A_{N_i}{}'s$ be fuzzy neutrosophic $*-$closed sets in (X, τ_N, I). Since $fn * -(A_{N_i})^+ = 0_N$, by proposition 3.1(1) $A_{N_i}{}'s$ are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Thus $fn * -(\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where $A_{N_i}{}'s$ are fuzzy neutrosophic nowhere $*-$dense sets in (X, τ_N, I). Hence (X, τ_N, I) is a fuzzy neutrosophic $I-$Baire space.

Proposition 4.4:

If $fn * -(\wedge_{i=1}^{\infty} A_{N_i},)^- = 1_N$, where $A_{N_i}{}'s$ are fuzzy neutrosophic $*-$dense and fuzzy neutrosophic $*-$open sets in fuzzy neutrosophic ideal topological space (X, τ_N, I) if and only if (X, τ_N, I) is a fuzzy neutrosophic $I-$Baire space.

Proof:

Let $A_{N_i}{}'s$ be fuzzy neutrosophic $*-$dense sets in (X, τ_N, I). Then $fn * -(\wedge_{i=1}^{\infty} A_{N_i},)^- = 1_N$ which implies that $1 - fn * -(\wedge_{i=1}^{\infty} A_{N_i},)^- = 0_N$. That is $(1 - fn * -\wedge_{i=1}^{\infty} A_{N_i})^+ = 0_N$ implies that $(1 - fn * -\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$. Since $A_{N_i}{}'s$ be fuzzy neutrosophic $*-$ dense, $* fn - (A_{N_i})^- = 1_N$. Hence $fn * -\left(1 - A_{N_i}\right)^+ = 1 - fn * -\left(A_{N_i}\right)^- = 0_N$. Consequently $fn * -(1 - \vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where $fn * -(1 - A_{N_i})^+ = 0_N$ and $A_{N_i}{}'s$ be fuzzy neutrosophic $*-$closed sets in (X, τ_N, I). By proposition 4.3, (X, τ_N, I) is a fuzzy neutrosophic $I-$Baire space.

Conversely, let $A_{N_i}{}'s$ are fuzzy neutrosophic $*-$dense and fuzzy neutrosophic $*-$open sets in (X, τ_N, I). By proposition 3.3, $1 - A_{N_i}{}'s$ are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Then $A_N = fn * -(\vee_{i=1}^{\infty} 1 - A_{N_i})$ is a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I). Now $fn * -(A_N)^+ = fn * -\left(\vee_{i=1}^{\infty}(1 - A_{N_i})\right)^+ = fn * -(1 - \wedge_{i=1}^{\infty} A_{N_i})^+ = (1 - fn * -\left(\vee_{i=1}^{\infty} A_{N_i}\right))^-$.

Since (X, τ_N, I) is a fuzzy neutrosophic $I -$ Baire space, by proposition 4.1, $fn * -(A_N)^+ = 0_N$. Then $(1 - fn * - (\wedge_{i=1}^{\infty} A_{N_i})^{-}) = 0_N$. This implies that $(fn * - (\wedge_{i=1}^{\infty} A_{N_i})^{-}) = 1_N$.

Conclusion:

In this paper, the concept of a new class of sets, spaces are called them fuzzy neutrosophic $* -$ dense, fuzzy neutrosophic nowhere $* -$ dense, fuzzy neutrosophic $* -$ residual set, fuzzy neutrosophic $* -$ one category set, fuzzy neutrosophic $* -$ two category sets, fuzzy neutrosophic $I -$Baire spaces, fuzzy neutrosophic $* -$ one category space, fuzzy neutrosophic $* -$ two category space. Some of its characterizations of fuzzy neutrosophic $I -$ Baire spaces are also studied. This shall be extended in the future research studies.

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