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VAGUE REGULAR GENERALIZED - INT AND VAGUE REGULAR GENERALIZED - NBD IN TOPOLOGICAL SPACES

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Abstract:
The purpose of this study is to investigate a novel class of vague regular α generalized interior in vague topological spaces and some of their characterizations are obtained. Further, we introduced and studied vague regular α generalized neighborhood in vague topological spaces. Also several properties were discussed.

1. Introduction

To address the ambiguity, several generalizations of vague sets have been achieved. In 1970 [5], Levine proposed the idea of generalized closed sets and covered topics such as connectedness, set attributes, closed maps, open maps, and separation axioms. In topological spaces H. Maki, K. Balachandran, and R. Devi [6] introduce the notion of generalized closed sets. In recent years, many researchers like, Arockiarani, Maria presenti [7], Amarendra babu, Ahmed allam and Rama rav[8] have worked on vague topological spaces. Bharathi. S, Poongodi.D introduced a new class of generalized closed sets in vague topological spaces in 2022. This paper, explores the notion of VR G– interior (VR g -Int) and VR G– neighbourhood(VR G - Nbd) in vague topological spaces. The basic properties and some characterizations are obtained. Throughout this paper, we have considered Xv, Yv as vague topological spaces. Let Av Xv, the closure and the interior is denoted by Cl(Av) and Int(Av) respectively. Here a vague regular alpha generalized interior and vague regular alpha generalized closure are defined as follows {E: E is a VR GOS and E Av} and{F: F is a VR GCS and Av F}.

2. Preliminaries

Definition 2.1: [4] Consider the universe Xv. A vague set is defined by Av where a true membership function (tAv(x)) and a false membership function (fAv(x)) are used to represent tAv(x) and fAv(x) respectively. The "evidence for x" is used to derive the lower constraint on the grade of membership of x, which is denoted by tAv(x). The "evidence against x" is used to determine the lower bound on the negation of x, which is fAv(x). As a result, a subinterval of [0,1] defines the grade of membership of x in Av. This states that if the actual grade of membership is µv(x), then tAv(x) µv(x) fAv(x) and tAv(x) fAv(x) 1.

Definition 2.2: [8]
i) A VRCS (vague regular closed set) if Av VCl(VInt(Av)).
ii) A V CS (vague α closed set) if VCl(VInt(VCl(A)) Av.
iii) A VGCS (vague generalized closed set) if VCl(Av) Uv when Av Uv.
iv) A VGSCS (vague generalized semi closed set) if VsCl(Av) Uv when Av Uv.
v) A VGPCS (vague generalized pre closed set) if VpCl(Av) Uv when Av Uv.

Definition 2.3: [7, 9]
i) A VG CS if V Cl(Av) Uv when Av Uv.
ii) A VRGCS if VCl(Av) Uv when Av Uv.

Definition 2.4: [3]
A VR GCS(vague regular generalized closed set) if V Cl(Av) Uv when Av Uv and Uv is VROS in Xv. The compliment of VR GCS is VR GOS (vague regular generalized open set)

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Definition 2.5: [2]
A map f: (A, τv)   (B, σv) is called a VR G continuous map if   f-1(F) is a Vr GCS in (A,τv) for every VCS (vague closed set) F of (B, σv).

3. Vague regular generalized neighbourhood of a vague point (VR G - Nbd)

Definition 3.1: Let Nv be a subset of vague topological space Xv and a vague point xv   Xv. A subset Nv is said to be a VR G - Nbd of a vague point xv iff there exists .
Definition 3.2: Let Nv be a subset of a vague topological space Xv, is called a VR G - Nbd of Av Xv iff there exists .
Remark 3.3: The VR G - Nbd of vague point xv Xv need not be a VR G open in Xv.
Theorem 3.4: Every Nbd Nv of vague point xv   Xv is a VR G - Nbd of Xv.
Proof: Consider the vague topological space Xv of Nbd Nv and a vague point xv   Xv. Then an open set Gv so as xv   Gv   Nv. Since every open set is VRαGOS Gv so as xv   Gv   Nv. Therefore, Nv is VR G - Nbd of xv.
Remark 3.5: VR G - Nbd Nv of xv be a Nbd of xv in Xv is need not be in general.

Theorem 3.6: If Nv is VR G open, then Nv is a VR G – Nbd of each of its points.
Proof: Let us assume that Nv is VRαGOS and xv Nv. We assert that Nv is a VR G - Nbd of xv. Because Nv is a VR GOS so as xv   Nv   Nv. Hence Nv is a VR G – Nbd of each of its points. Since the arbitrary point xv is in Nv.

Theorem 3.7: Consider the vague topological space Xv. If Fv is a vague regular   generalized closed subset, xv FvC. Then a VR G -Nbd Nv of xv such as Nv   Fv .

Proof: Consider a vague regular   generalized closed subset Fv of Xv, xv   FvC. Therefore, FvC is VR GOS of Xv. By previous theorem we have FvC   VR G -Nbd of each of its points. Then a VR G -Nbd Nv of xv such as Nv   FC. i.e, Nv   Fv .
Definition 3.8: Consider a vague point xv in Xv. The VR G -Nbd system at xv is the collection of all VR G -Nbd, and it is represented by the symbol VR G –Nv(x).
Theorem 3.9: Consider the vague topological space Xv. Let VR G Nv (x) be the collection of all VR G -Nbd of xv and for each xv   Xv The results are as follows. (i) ∀ xv   Xv, VRαG –Nv (x) .
(ii) Nv   VR G –Nv (x) ⇒ xv   Nv.
(iii) Nv   VR G –Nv (x), Mv   Nv ⇒ Mv   VR G –Nv (x).
(iv) Nv   VR G –Nv   (x), Mv   VR G –Nv (x) ⇒ Nv   Mv   VR G –Nv (x).
(v) Nv   VR G –Nv (x) ⇒ there exists Mv   VR G –Nv (x) such that Mv   Nv and Mv   VR G –Nv (y) to each yv   Mv.
Proof: (i) As Xv is a VR G open set, every xv Xv is a VR G -Nbd of Xv. Hence, for each xv Xv, there exists at least one VR G -Nbd (specifically, - Xv). Hence, for every xv Xv, VR G Nv(x) .
(ii) Nv is a VR G -Nbd of xv, if Nv   VR G –Nv (x). Hence by the definition VR G – Nbd, xv Nv.
(iii) Let Nv   VR G –Nv (x) and Mv   Nv. A VR G open set Gv follows, such as xv   G¬v   Nv. As Nv   Mv, xv   Gv   Mv. Mv is hence VR G -Nbd of xv. Thus Mv   VR G –Nv (x).
(iv) Consider Nv   VR G –Nv(x) and let Mv   Nv(x). According to the specification of VR G -Nbd, VR G open sets Gv1 and Gv2 such as xv   Gv1   Nv and xv   Gv2   Mv. So xv   Gv1   Gv2   Nv   Mv (*). Because Gv1   Gv2 is a VR G open set, follows from (*) that Nv   Mv is a VR G - Nbd of xv. Hence Nv   Mv   VR G –Nv (x)
(v) If Nv   VR G - Nv (x), then a VR G open set Mv such as xv   Mv   Nv exists. Mv is VR G -Nbd of each of its points since it is a VR G open set. So, Mv   VR G –Nv (y) by yv   Mv.

4. Vague regular α generalized interior (VRαG- Int(Av))

Definition 4.1: Consider a subset Av of the vague topological space Xv. The VR G - Int (VR G -

interior) point xv of Av is characterized by the vague union of all vague regular generalized open subsets of Av. So VR G - Int (Av) { Gv: Gv is VR GOS, Gv Av}
Theorem 4.2: Consider a subset Av of Xv. Then VR G - Int(Av) { Gv: Gv is VRαGOS, Gv Av}
Proof: Let Xv be a vague topological space and let a subset Av Xv.
xv VR G ¬ - Int(Av) a VR G interior point xv of Av
Av is VR G - Nbd of xv.
a VR GOS Gv such as xv Gv Av.
xv { Gv: G¬v is VR GOS, Gv Av}. Thus VR GOS - Int(Av) { Gv: Gv is VR GOS,Gv Av}.
Theorem 4.3: Consider Av & Bv be a vague subsets of a vague topological space Xv. Then
(i) VR G ¬ -Int(Xv) Xv & VR G - Int ( ) .
(ii) VR G - Int (Av) Av.
(iii) If Bv is any VR GOS Av, then Bv VR G - Int (Av).
(iv) If Av Bv, then VR G - Int (Av) VR G - Int (Bv).
(v) VR G - Int (VR G - Int (Av) VR G - Int (Av).

Proof: (i) As the VR G open sets are Xv and , by the above Theorem 4.2 VR G - Int (Xv) { Gv: Gv is VR GOS, Gv Xv} Xv {set of all VR GOS} Xv and VR G - Int ( ) .
ii) We take xv VR G − Int (Av). ⇒ xv is an interior point of Av. ⇒ Av is a Nbd of xv.
⇒ xv Av. Thus, xv VR G − Int (Av). Hence VR G − Int (Av) Av.

(iii) Let us consider a VR GOS Bv such that Bv Av and xv Bv. Since Bv is a VR GOS contained in Av. xv is VR G − Int of Av. i.e. xv VR G − Int(Av). Thus Bv VR G − Int(Av).
(iv) VR G − Int(Av) is the largest VR GOS containing Av. Since Bv Av, VR G − Int(Bv) is the union of all VR GOS containing Bv. Hence VR G - Int (Av) VR G - Int (Bv).
(v) Since VR G − Int(Av) is VR GOS. Clearly VR G - Int (VR G - Int (Av)) VR G - Int (Av).

Theorem 4.3: If Av is VR GOS, then VR G - Int (Av) Av.

Proof: Consider Av is a VR GOS of Xv. We Know That VR G - Int (Av) Av. Further VR GOS contained in Av. By theorem we have Av VR G - Int (Av). Thus VR G - Int (Av) Av.
Theorem 4.4: VR G - Int (Av) VR G - Int (Bv) VR G - Int(Av Bv).
Proof: WKT Av Av Bv & Bv Av Bv. By above theorem we have VR G - Int (Av) VR G - Int (Av Bv), VR G - Int (Bv) VR G - Int (Av Bv). Hence VR G - Int (Av) VR G - Int (Bv) VR G - Int (Av Bv).
Theorem 4.5: VR G - Int (Av Bv) VRαG - Int (Av) VRαG - Int (Bv).
Proof: WKT Av Bv Av & Av Bv Bv. We have VR G - Int (Av Bv) VR G - Int (Av) and VR G - Int (Av Bv) VRG - Int (Bv). This implies that VR G - Int (Av Bv) VR G - Int (Av) VR G - Int (Bv). (*)
Let xv VR G - Int (Av) VR G - Int (Bv). Then xv VR G - Int (Av) and xv VR G - Int (Bv). Hence each of sets Av and Bv having the VR G - Int point xv. So Av & Bv is VR G - Nbds of xv, Av Bv is also VR G - Nbds of xv. Thus xv VR G − Int (Av Bv). Therefore, VR G - Int (Av) VR G - Int (Bv) VR G - Int (Av Bv) (**). From (*) and (**), VR G - Int (Av Bv) = VR G - Int (Av) VR G - Int (Bv).

Conclusion:
This paper is proposed a new class of interior, closure and neighbourhood like VR G - Int and VR G - Nbd in vague topological spaces. Several properties and characterizations are discussed. Further these concepts can be expanded to the future work like connectedness and compactness.

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