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SUPRA RESIDUAL SETS AND SUPRA BAIRE SPACE FUNCTIONS ON FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract

In this paper several characterization of fuzzy neutrosophic supra residual sets are established and the criteria of the fuzzy neutrosophic supra residual of fuzzy neutrosophic supra sets on fuzzy neutrosophic supra topological space.

Keywords: Fuzzy neutrosophic supra residual set, Fuzzy supra Semi F_σ –set, Fuzzy supra Semi G_δ –set, Fuzzy neutrosophic supra nowhere dense set, Fuzzy neutrosophic supra first and second category, Fuzzy neutrosophic supra Baire spaces.

1.INTRODUCTION

The notion of a fuzzy set introduced by L,A. Zadeh(14)in 1965, both pure and applied Mathematicians. Since then, the notion of fuzziness has been applied for the study in all branches of Mathematics. The concept of fuzzy topological spaces , introduced by Chang in 1968(5) has been extensively studied and applied in many field , including artificial intelligence, decision making, and images processing .

Supra Baire spaces, introduced by Haworth and McCoy in 1977 [6], are an important concept in topology. In 1983, Mashhour et al. [7] extended the notion of supra topological spaces. Abd El-Monsef and Ramadan in 1987 [1] investigated the concept of fuzzy supra topological spaces, which has since been further studied by researchers such as Ahmed, Chandra Chetia, and others [2].

The concept of supra Baire spaces in fuzzy setting was introduced and studied by E. Poongothai and G. Thangaraj in (8). Also introduced and studied fuzzy supra semi Baire spaces by (9).

Building upon previous research, Arockiarani l and Martina Jency [4] further studied fuzzy neutrosophic sets and fuzzy neutrosophic topological spaces.

In this paper, we explain the concept of supra residual sets on fuzzy neutrosophic topological space. We investigate several fundamental properties and characterization of these sets.

PRELIMINARIES

Definition 2.1 [3]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ $x \in X$ where $T, I, F: X \to [0,1]$ and $0 \le \{T_A(x) + I_A(x) + F_A(x)\} \le 3$.

Definition 2.2 [3]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$

Definition 2.3 [3]:

Let X be a non-empty set, and $A = T_A(x), I_A(x), F_A(x)$, $B = T_B(x), I_B(x), F_B(x)$ be two fuzzy neutrosophic sets. Then

$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$

$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$

Definition 2.4 [3]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [3]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T(x) = 0, I(x) = 0, F_A(x) = 1$ for all $x \in X$ It is denoted by 0_N .

Definition 2.6 [3]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T(x) = 1, I(x) = 1, F_A(x) = 0$ for all $x \in X$ It is denoted by 0_N.

Definition 2.7 [3]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$

where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$

Definition 2.8 [12]:

Let $(X_N{}^*, T_N{}^*)$ be a fuzzy neutrosophic supra topological space. Then $(X_N{}^*, T_N{}^*)$ is called fuzzy neutrosophic Baire Space if $fn\left(int^*(\vee_{i=1}^{\infty}(X_{N_i}{}^*))\right) = 0$ where $(X_{N_i}{}^*)$'s are fuzzy neutrosophic supra nowhere dense set in $(X_N{}^*, T_N{}^*)$.

Definition 2.9 [4]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Theorem 2.10 [12]:

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. Then the following are equivalent:

(1) (X_N, T^*) is a fuzzy neutrosophic supra Baire Space

(2) $fn\ int^*(A_N) = 0$ for every fuzzy neutrosophic supra first category set (A_N) in (X_N, T^*)

(3) $fn\ cl^*(B) = 1$ for every fuzzy neutrosophic supra residual set (B_N) in (X_N, T^*).

3. FUZZY NEUTROSOPHIC SUPRA RESIDUAL SET

Proposition 3.1

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. A fuzzy set A_N in (X_N, T^*) is called a fuzzy neutrosophic supra residual set if $A_N = \wedge_{i=1}^{\infty} A_{N_i}$, where the fuzzy sets (A_{N_i}),s are such that $fn\ cl^*\ [fn\ int^*\ (A_{N_i})] = 1_N$ in (X_N, T^*)

Proposition 3.2

If A_N is a fuzzy neutrosophic supra G_δ-set in a fuzzy neutrosophic supra topological space (X_N, T^*) such that $fn\ cl^*(A_N) = 1$, then A_N is a fuzzy neutrosophic supra residual set in (X_N, T^*).

Proof,

Let A_N be a fuzzy neutrosophic supra G_δ-set in (X_N, T^*). Then $A_N = \wedge_{i=1}^{\infty}(A_{N_i})$, where $A_{N_i} \in T^*$. Now $fn\ cl^*(A_N) = fn\ cl^*[\wedge_{i=1}^{\infty}(A_{N_i})]$ in (X_N, T^*). But $fn\ cl^*[\wedge_{i=1}^{\infty}(A_{N_i})] \le \wedge_{i=1}^{\infty} fn\ cl^*(A_{N_i})$ in $(X_N T^*)$. Then $fn\ cl^*(A_N) \le \wedge_{i=1}^{\infty} fn\ cl^*(A_{N_i})$ in (X_N, T^*). Since A_N is a fuzzy neutrosophic supra dense set in (X_N, T^*), $fn\ cl^*(A_N) = 1$ and hence $1 \le fn\ cl^*[\wedge_{i=1}^{\infty}(A_{N_i})]$. That is, $fn\ cl^*[\wedge_{i=1}^{\infty}(A_{N_i})] = 1$ in (X_N, T^*). This implies that $fn\ cl^*(A_{N_i}) = 1$ in (X_N, T^*) and hence $fn\ cl^*[fn\ int^*(A_{N_i})] = fn\ cl^*(A_{N_i}) = 1$ in (X_N, T^*). Thus $A = \wedge_{i=1}^{\infty}(A_{N_i})$, where $fn\ cl^*[fn\ int^*(A_{N_i})] = 1$, implies that A_N is a fuzzy neutrosophic supra residual set in (X_N, T^*).

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Definition 3.3

Let (X_N, T^*) fuzzy neutrosophic supra topological space. A fuzzy set A_N is called a fuzzy neutrosophic supra semi-open set if $A_N \leq fn\, cl^*[fn\, int^*(A_N)]$. The complement of a fuzzy neutrosophic supra semi-open set is called a fuzzy neutrosophic supra semi-closed set in (X_N, T^*).

Definition 3.4

A fuzzy set A in a fuzzy neutrosophic supra topological space (X_N, T^*), is called a fuzzy neutrosophic supra semi G_δ-set in (X_N, T^*) if $A = \wedge_{i=1}^{\infty}(A_{N_i})$, where (A_{N_i})'s are fuzzy neutrosophic supra semi-closed sets in (X_N, T^*).

Definition 3.5

A fuzzy set A in a fuzzy neutrosophic supra topological space (X_N, T^*) is called a fuzzy neutrosophic supra semi-F_σ-set in (X_N, T^*) if $A = \vee_{i=1}^{\infty}(A_i)$ where (A_i)'s are fuzzy neutrosophic supra semi-closed sets in (X_N, T^*).

Proposition 3.6

If A_N is a fuzzy neutrosophic supra residual set in a fuzzy neutrosophic supra topological space (X_N, T^*) , then A_N is a fuzzy neutrosophic supra semi-.G_δ set.

Proof,

Let A_N be a fuzzy neutrosophic supra residual set in a fuzzy neutrosophic supra topological space (X_N, T^*). Then, $A_N = \wedge_{i=1}^{\infty}(A_{N_i})$, where fuzzy sets (A_{N_i})'s are such that $fn\, cl^*[fn\, int^*(A_{N_i}) = 1_N]$ in (X_N, T^*) . Now $fn\, cl^*[fn\, int^*(A_{N_i})] = 1_N$ in (X_N, T^*), implies that $A_{N_i} \leq fn\, cl^*[fn\, int^*(A_{N_i})]$. Then (A_{N_i})'s are fuzzy neutrosophic supra semi-open sets in $(X_N T^*)$ and thus $\wedge_{i=1}^{\infty}(A_{N_i})$ is a fuzzy neutrosophic supra semi G_δ -set in (X_N, T^*).

Proposition 3.7

If A_N is a fuzzy neutrosophic supra first category set in a fuzzy neutrosophic supra topological space (X_N, T^*), then A_N is a fuzzy neutrosophic supra semi-F_σ set in (X_N, T^*).

Proof:

Let If A_N is a fuzzy neutrosophic supra first category set in(X_N, T^*). Then $1 - A_N$ is a fuzzy neutrosophic supra residual set in (X_N, T^*) and hence, by property (3.6), $1 - A_N$ is a fuzzy neutrosophic supra semi-G_δ set in (X_N, T^*). Therefore A_N is a fuzzy neutrosophic supra semi-F_σ set in (X_N, T^*).

Proposition 3.8

If $fn\, cl^*\ (B_N)$ is a fuzzy neutrosophic supra first category set in a fuzzy neutrosophic supra Baire Space (X_N, T^*), then A is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proof:

If $fn\, cl^*\ (B_N)$ is a fuzzy neutrosophic supra first category set in (X_N, T^*). Since (X_N, T^*) is a fuzzy neutrosophic supra Baire Space, by theorem (2.10) $fn\, int^*[fncl^*(A)] = 0$ in (X_N, T^*). Therefore A is an fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proposition 3.9

If $fn\, int^*(B_N)$ is A fuzzy neutrosophic supra residual set in a fuzzy neutrosophic supra Baire Space (X_N, T^*), then $1 - B$ is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proof:

Let $fn\, int^*(B_N)$ be a fuzzy neutrosophic supra residual set in (X_N, T^*). Since (X_N, T^*) is a fuzzy neutrosophic supra Baire Space, by theorem (2.10), $fn\, cl^*[fn\, int^*(B_N)] = 1$ in (X_N, T^*). This implies that $1 - fn\, cl^*[fn\, int^*(B_N)] = 0$ and hence $fn\, int^*[fn\, cl^*(1 - B_N)] = 0$ in (X_N, T^*). This implies that $1 - B_N$ is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proposition 3.10

If $fn[int^*(B_{N_i})]$'s $(i = 1\ to\ \infty)$ are fuzzy neutrosophic supra residual sets in a fuzzy neutrosophic supra Baire Space (X_N, T^*), then $fn\, cl^*[\wedge_{i=1}^{\infty}(B_{N_i})] = 1$ in (X_N, T^*).

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Proof:

Let $fn[int^*(B_N)]$'s $(i = 1\ to\ \infty)$ be fuzzy neutrosophic supra residual sets in (X_N, T^*). Since (X_N, T^*) is fuzzy neutrosophic supra Baire Space by proposition (3.9), $1 - B_{N_i}$'s are fuzzy neutrosophic supra nowhere dense sets in (X_N, T^*) and hence $fn\ int^*[\vee_{i=1}^{\infty}(1 - B_{N_i})] = 0$ in(X_N, T^*). This implies that $fn\ int^*[1 - \vee_{i=1}^{\infty}(B_{N_i})] = 0$. Then $1 - fn\ cl^*[\vee_{i=1}^{\infty}(B_{N_i})] = 0$ in (X_N, T^*) and thus $fn\ cl^*[\vee_{i=1}^{\infty}(B_{N_i})] = 1$ in (X_N, T^*).

4. FUZZY NEUTROSOPHIC SUPRA BAIRE SPACE FUNCTIONS

Definition 4.1

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. Then (X_N, T^*) is called fuzzy neutrosophic supra Baire Space if $fn\ int^*[\vee_{i=1}^{\infty}(A_{N_i})] = 0$, where A_{N_i}'s are fuzzy neutrosophic supra nowhere dense sets is (X_N, T^*)

Definition 4.2

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is said to be fuzzy neutrosophic supra open if the image of every fuzzy neutrosophic supra open set in $(X_N, T^*)$is fuzzy neutrosophic supra open in (Y_N, S^*).

Definition 4.3

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy continuous if $f^{-1}(A_N)$ fuzzy neutrosophic supra open in (X_N, T^*) for each fuzzy neutrosophic supra open set is A_N in (Y_N, S^*).

Definition 4.4

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy neutrosophic supra semi-continuous if $f^{-1^*}(A_N)$ is fuzzy neutrosophic supra semi-open in (X_N, T^*) for each fuzzy neutrosophic supra open set is A_N in (Y_N, S^*).

Definition 4.5

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy neutrosophic supra somewhat continuous if $A_N \in S^*$ and $f^{-1}A_N \neq 0$ implies that there exist a fuzzy neutrosophic open set B_N in (X_N, T^*). Such that $B_N \neq 0$ and $B_N \leq f^{-1^*}(A_N)$.

Definition 4.6

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy neutrosophic supra somewhat open if $A_N \in T^*$ and $B_N \neq 0$ implies than there exist a fuzzy neutrosophic supra open set B_N in (Y_N, S^*). Such that $B_N \neq 0$ and $B_N \leq f(A_N)$.

Theorem 4.7

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. Then the following are equivalent.

1) (X_N, T^*) is a fuzzy neutrosophic supra Baire Space,
2) $fn(int^*(A_N)) = 0$ for every fuzzy neutrosophic supra first category set in A_N in (X_N, T^*).
3) $fn(cl^*(B_N)) = 1$, for every fuzzy neutrosophic supra residual set B_N in (X_N, T^*).

Proof,

Let f^* be a function from the fuzzy neutrosophic supra topological space (X_N, T^*) to the (Y_N, S^*), Underwhat conditions on "f^*" may we assert that if (X_N, T^*) is a fuzzy neutrosophic supra

Baire Space, then (Y_N, S^*) is a fuzzy neutrosophic supra Baire Space. If may be notised that the fuzzy neutrosophic supra continuous image of a fuzzy neutrosophic supra Baire Space may fail to be a fuzzy neutrosophic supra Baire Space. For, consider the following example.

Example 4.8

Let $X = \{a, b\}$. The fuzzy neutrosophic sets A_N, B_N and C_N are defined on X as follows

$$A_N = \left(X, \left(\frac{a}{0.7}, \frac{b}{0.7}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}\right)\right)$$

$$B_N = \left(X, \left(\frac{a}{0.6}, \frac{b}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right)\right)$$

$$C_N = \left(X, \left(\frac{a}{0.6}, \frac{b}{0.7}\right), \left(\frac{a}{0.7}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right)\right)$$

Then,

$T^* = \{0, A_N, B_N, C_N, A_N \vee B_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, 1\}$ and $S^* = \{0, A_N, B_N, A_N \vee B_N, A_N \wedge B_N, 1\}$ are fuzzy neutrosophic topologies on X. Now the fuzzy neutrosophic supra nowhere dense sets in (X_N, T^*) are $1 - A_N$ are $1 - (A_N \vee B_N)$ and $[(1 - A_N) \vee (1 - (A_N \vee B_N))] = 1 - A_N$ and $fn\,int^*(1 - A_N) = 0$. Hence (X_N, T^*) is a fuzzy neutrosophic supra Baire Space. Then fuzzy neutrosophic supra nowhere dense sets is (X_N, S^*) are $(1 - A_N), (1 - B_N)$ and $1 - (A_N \vee B_N)$ this implies that $[(1 - A_N) \vee (1 - B_N) \vee (1 - (A_N \vee B_N))] = 1 - (A_N \wedge B_N)$ and $fn\,int^*[1 - (A_N \vee B_N)] = A_N \wedge B_N \neq 0$. Hence (X_N, S^*) is not a fuzzy neutrosophic supra Baire Space.

Define a function $f^*: (X_N, T^*) \longrightarrow (X_N, S^*)$ by $f^*(a) = a$ and $f^*(b) = b$. Clearly f^* is a fuzzy neutrosophic supra continuous function from the fuzzy neutrosophic supra Baire Space. (X_N, T^*) to the fuzzy neutrosophic supra topological space. (X_N, S^*) which is not a fuzzy neutrosophic supra Baire Space.

Example 4.8

Let $X = \{a, b, c\}$. Then fuzzy sets A_N, B_N and C_N are defined on X as follows:

$$A_N = \left(X, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right)\right)$$

$$B_N = \left(X, \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right)$$

$$C_N = \left(X, \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.7}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.6}\right)\right)$$

Then,

$T^* = \{0, A_N, 1\}$ and $S^* = \{0, A_N, B_N, A_N \vee B_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, 1\}$

Are fuzzy neutrosophic topological on X. Now the fuzzy neutrosophic supra nowhere dense sets in (X_N, T^*) are $1 - A_N, 1 - B_N$ and $1 - (A_N \vee B_N) = 1 - A_N$ and $fn\,int^*(1 - A_N) = 0$. Hence (X_N, T^*) is a fuzzy neutrosophic supra Baire Space.

The fuzzy neutrosophic supra nowhere dense sets in (X_N, S^*) are $1 - A_N, 1 - B_N, 1 - C_N$, $1 - (A_N \vee B_N)$ and $1 - (B_N \vee C_N)$ and

$(1 - A_N) \vee (1 - B_N) \vee (1 - C_N) \vee (1 - (A_N \vee B_N)) \vee (1 - (B_N \vee C_N)) = 1 - A_N \wedge B_N$.

Now, $fn\,int^*(1 - (A_N \wedge B_N)) = B_N \wedge C_N \neq 0$. Hence (X_N, T^*) is not a fuzzy neutrosophic supra Baire Space.

Define a function $f^*: (X_N, T^*) \longrightarrow (X_N, S^*)$ by $f^*(a) = a$ and $f^*(b) = b$ and $f^*(c) = c$ clearly f^* is a fuzzy neutrosophic supra open function from the fuzzy neutrosophic supra Baire space (X_N, T^*) to the fuzzy neutrosophic supra topological space (X_N, S^*) which is not a fuzzy neutrosophic supra Baire Space.

Proposition 4.1

If a function $f^*:(X_N,T^*)\rightarrow(Y_N,S^*)$ from a fuzzy neutrosophic supra topological space (X_N,T^*) into another fuzzy neutrosophic supra topological space (Y_N,S^*) is fuzzy neutrosophic supra continuous, 1-1, onto and fuzzy neutrosophic supra open function, then for any fuzzy set A_N in (X_N,T^*). $fn\ int^*cl^*\ [f^*(A_N)]\le f^*[int^*(cl\ (A_N))]$.

Proof,

Let A_N be a fuzzy set in (X_N,T^*). Then $A_N\le fn\ cl^*(A_N)$ in (X_N,T^*) implies that $1-fn\ cl^*(A_N)\le(1-A_N)$. Then $f^*(1-fn\ cl^*(A_N))\le f^*(1-A_N)$.

Since f is a fuzzy neutrosophic supra open function and $1-fn\ cl^*(A_N)$ is a fuzzy neutrosophic supra open set is (Y_N,S^*). Such that $f^*(1-fn\ cl^*(A_N))\le f^*(1-A_N)$. But $fn\ int\ f^*(1-A_N)$ in the largest fuzzy neutrosophic supra open in (Y_N,S^*). Such that $fn\ int^*[f^*(1-A_N)]\le f^*(1-A_N)$.

Hence we have that $f^*(1-fn\ cl^*(A_N))\le fn\ int^*f^*(1-A_N)$.

Since f^*in 1-1, and onto, $f^*(1-A_N)=1-f^*(A_N)$. Hence $f^*(fn\ cl^*(A_N))\le fn\ int^*(1-f^*(A_N))$ which implies that $1-f^*(fn\ cl^*(A_N))\le\left(1-fnint^*(f^*(A_N))\right)$. Then$fn\ cl^*[f^*(A_N)]\le f^*[fncl^*(A_N)]$, which implies that $\left(fn\ int^*cl^*(f^*(A_N))\right)\le(fn\ int^*(f^*[fn\ cl^*(A_N)]))$.

Therefore we have $f^{-1^*}[fn\ int^*cl^*(f^*(A_N))]\le f^{-1^*}[fn\ int^*f^*(fn\ cl^*(A_N))]$. -------------- (1)

Now $fn\ int^*(f^*[fn\ cl^*(A_N)])$ is an fuzzy neutrosophic supra open set in (Y_N,S^*). Since f^* is fuzzy neutrosophic supra continuous. $f^{-1^*}\left[fn\ int\left(f^*(fn\ cl^*(A_N))\right)\right]$ is an fuzzy neutrosophic supra open set in (X_N,T^*). Hence $f^{-1^*}\left[fn\ int\left(f^*(fncl^*(A_N))\right)\right]=int\ f^{-1^*}\left[fn\ int\left(f^*(fncl^*(A_N))\right)\right]\le fn\ int^*(cl^*(A_N))$ [Since f^* is 1-1].

That is $f^{-1^*}\left[fn\ int^*\left(f(fn\ cl^*(A_N))\right)\right]\le fn\ int^*(cl(A_N))$ ------------------ (2)

From (1) & (2), we have $f^{-1^*}[fn\ int^*cl^*[f^*(A_N)]]\le fn\ int^*(cl^*(A_N))$ which implies that $f^*\ f^{-1^*}(fn\ int^*\ cl^*[f^*(A_N)])\le f^*\left(fn\ int^*(cl^*(A_N))\right)$.

Since f^* is onto, we have $(fn\ int^*\ cl^*[f^*(A_N)])\le f^*\left(fn\ int^*(cl^*(A_N))\right)$.

Proposition 4.2

If a function $f^*:(X_N,T^*)\rightarrow(Y_N,S^*)$ from a fuzzy neutrosophic supra topological space (X_N,T^*) into another fuzzy neutrosophic supra topological space (Y_N,S^*) is fuzzy neutrosophic supra continuous and fuzzy neutrosophic supra open function, then for any fuzzy set A_N in (X_N,T^*) $f^*\left(fn\ int^*(cl^*\ (A_N))\right)\le fn\ int^*(cl^*[\ f^*(A_N)])$.

Proof,

Let A_N be any fuzzy set in (X_N,T^*). Then $fn\ int^*(cl^*\ (A_N))$ is a fuzzy neutrosophic supra open set is (X_N,T^*). Since f^* is a fuzzy neutrosophic supra open function, $f^*\left(fn\ int^*(cl^*\ (A_N))\right)$ is a fuzzy neutrosophic supra open set is (Y_N,S^*). Now $f^*\left(fn\ int^*(cl^*\ (A_N))\right)\le f^*(fn\ cl^*(A_N))$. Since f^* is a continuous, $f^*(fn\ cl^*\ (A_N))\le fn\ cl^*(f^*(A_N))$.

Hence we have $f^*\left(fn\ int^*(cl^*\ (A_N))\right)\le fn\ cl^*(f^*(A_N))$. But $fn\ int^*(cl^*\ f^*(A_N))\le fn\ cl^*(f(A_N))$.

Therefore, we have $f^*\left(fn\ int^*(cl^*\ (A_N))\right)\le fn\ int^*(cl^*[\ f^*(A_N)])$.

Proposition 4.3

If a function $f^*:(X_N,T^*)\rightarrow(Y_N,S^*)$ from a fuzzy neutrosophic supra topological space (X_N,T^*) into another fuzzy neutrosophic supra topological space (Y_N,S^*) is fuzzy neutrosophic supra continuous and fuzzy neutrosophic supra open 1-1 and onto function, then for any fuzzy set A_N in (X_N,T^*),

$f^*\left(fn\,int^*(cl^*(A_N))\right) = fn\,int^*(cl^*[f(A_N)])$.

Proof,

Let A_N be any fuzzy set in (X_N, T^*). Since f^* is fuzzy neutrosophic supra continuous, 1-1, onto and fuzzy neutrosophic supra open function, by Proposition (4.1), we have

$(fn\,int^*\,cl^*[f^*(A)]) \leq f^*\left(fn\,int^*(cl^*(A))\right)$ ----------------- (1)

Since f is a fuzzy neutrosophic supra continuous and a fuzzy neutrosophic supra open function, by Proposition 4.2, we have $f^*\left(fn\,int^*(cl^*(A_N))\right) \leq fn\,int^*(cl^*[f^*(A_N)])$ ----------------- (2)

From (1)&(2), we have $f^*\left(fn\,int^*(cl^*(A_N))\right) = fn\,int^*(cl^*[f(A_N)])$

Proposition 4.4

Let the function $f^*:(X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is fuzzy neutrosophic supra continuous fuzzy neutrosophic supra open 1-1 and onto function, then for any fuzzy set A_N in (X_N, T^*), A_N is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*) if and only if $f^*(A_N)$ is a fuzzy neutrosophic supra nowhere dense set in (Y_N, S^*).

Proof,

Let A_N be a fuzzy neutrosophic supra nowhere dense set is (X_N, T^*). Then, $fn\,int^*(cl^*(A_N)) = 0$. Since f is *fuzzy neutrosophic supra* continuous, fuzzy neutrosophic supra open, 1-1 and onto function, by Proposition (4.3), we have, $f^*\left(fn\,int^*(cl^*(A_N))\right) = fn\,int^*(cl^*[f^*(A_N)])$. Then, $fn\,int^*(cl^*(A_N)) = f^*(0) = 0$. Hence $f^*(A_N)$ is a fuzzy neutrosophic supra nowhere dense set in (Y_N, S^*).

Conversely, Let $f^*(A_N)$ be a fuzzy neutrosophic supra nowhere dense set in (Y_N, S^*).

Then, $fn\,int^*(cl^*(A_N)) = 0$. Hence $f^*\left(fn\,int^*(cl^*(A_N))\right) = fn\,int^*(cl^*[f^*(A_N)])$ implies that $f^*\left(fn\,int^*(cl^*(A_N))\right) = 0$, Therefore $f^{-1^*}f^*\left(fn\,int^*(cl^*(A_N))\right) = f^{-1^*}(0) = 0$. Since f^* is 1-1, $fn\,int^*(cl^*(A_N)) = 0$, Hence A_N is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

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