



NOVEL METHOD OF K_0, K_1, K_2, K_3 CONSTANTS EVALUATION OF MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY EQUATION OF ELECTRICAL STEEL BASED ON TEXTURE FACTOR FOR IDEAL FIBRES

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Abstract

Texture Factor, A^* and Magnetic Crystalline Anisotropy Energy Density* K_0, K_1, K_2, K_3 Constants are important parameters for Pure Iron. While the former indicates volume density of crystals having preferred Orientation, latter indicates the easy and hard magnetization directions. Evaluation of these parameters for Pure Iron and Electrical Steel enables in reduction of core losses and improving the electrical energy efficiency in Transformers, Rotating Machines. In this research article, an attempt is made to compute Magneto-Crystalline Anisotropy Energy Density for pure iron based on Texture Factor for Ideal fibers.

Keywords: Texture Factor, Magnetic Crystalline Anisotropy Energy Density, Core losses

I. Introduction

The Magneto Crystalline Anisotropy constants K_1, K_2 values determine the extent to which a material is easily magnetizable. Their value depends on Chemical Composition, Crystal Structure, and Thermo-Mechanical Processing history of the given material. In material science, Texture Factor is an important microstructural parameter which directly determines the anisotropy degree of most physical properties of a polycrystalline material at the macro scale. Its characterization is thus of fundamental and applied importance, and should ideally be performed prior to any physical property measurement or modeling. Neutron diffraction is a tool of choice for characterizing crystallographic textures. The obtained information is representative of a large number of grains, leading to a better accuracy of the statistical description of texture. Texture factor constants K_1, K_2, K_3 values determines the preferred orientations of grains, the Overall Texture Factor is quantitative measurement of texture. The value signifies extent of presence of standard texture viz. Cube Texture (T.F = 22.5), Goss Texture (T.F = 35.6), Gamma Texture (T.F = 38.68) in the given material.

1.1 STANDARD EQUATIONS:

$$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 ;$$

$$A^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 + K_4 (\sum \alpha_1^2 \alpha_2^2)(\prod \alpha_1^2) + K_5 (\sum \alpha_1^2 \alpha_2^2)^3 + K_6 (\prod \alpha_1^2)^2 ;$$

$$E^* = 0.355A^* + (0.163 - 0.013A^*)[\text{wt}\% \text{Si}] - 1.898$$

II. ESTIMATION OF MAGNETIC ANISOTROPY CONSTANTS K_0, K_1, K_2, K_3 CONSTANTS EVALUATION OF FOR ELECTRICAL STEELS:

Magneto Crystalline Anisotropy Energy is generally expressed by an expansion into direction cosines $\alpha_1, \alpha_2, \alpha_3$ of the magnetization with respect to the crystal axes.

$$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 \quad [1];$$

[uvw]	a	b	c	α_1	α_2	α_3	E
<100>	0	90^0	90^0	1	0	0	K_0
<110>	45^0	45^0	90^0	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$K_0 + K_1/4$
<111>	54.7^0	54.7^0	54.7^0	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/\sqrt{3}$	$K_0 + K_1/3 + K_2/27$



From REF 1, we have $E^* = 0.355A^* + (0.163 - 0.013A^*)[\text{wt}\% \text{Si}] - 1.898$

FOR A^* for Θ fiber $\langle 100 \rangle // \text{ND}$ is 22.5 $\Rightarrow E^* = -0.5345 [\text{wt}\% \text{Si}] + 6.0895$

FOR A^* for fiber $\langle 110 \rangle // \text{ND}$ is 35.6 $\Rightarrow E^* = -0.9406 [\text{wt}\% \text{Si}] + 10.74$

FOR A^* for Υ fibre $\langle 111 \rangle // \text{ND}$ is 38.68 $\Rightarrow E^* = -1.03608 [\text{wt}\% \text{Si}] + 11.8334$

$$E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2 \dots [I];$$

FOR [100] directions, $\alpha_1=1, \alpha_2=0, \alpha_3=0 \Rightarrow E = K_0$

\Rightarrow FOR [100] directions, $E^* = 6.0895 \Rightarrow K_0 = -0.5345 [\text{wt}\% \text{Si}] + 6.0895$

FOR [110] directions, $\alpha_1=1/\sqrt{2}, \alpha_2=1/\sqrt{2}, \alpha_3=0$

FOR [110] directions, $E^* = -0.9406 [\text{wt}\% \text{Si}] + 10.74$

$\Rightarrow E^* = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + K_1/4 + K_3/16$

$\Rightarrow -0.9406 [\text{wt}\% \text{Si}] + 10.74 = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + K_1/4 + K_3/16$

$\Rightarrow 4.6505 - 0.4061 [\text{wt}\% \text{Si}] = K_1/4 + K_3/16$

$\Rightarrow 4K_1 + K_3 = 74.408 - 6.4976 [\text{wt}\% \text{Si}] \dots [II]$

FOR [111] directions, $\alpha_1=1/\sqrt{3}, \alpha_2=1/\sqrt{3}, \alpha_3=1/\sqrt{3}$

FOR [111] directions, $E^* = -1.03608 [\text{wt}\% \text{Si}] + 11.8334$

$\Rightarrow -1.03608 [\text{wt}\% \text{Si}] + 11.8334 = K_0 + K_1/3 + K_2/27 + K_3/9$

$\Rightarrow -1.03608 [\text{wt}\% \text{Si}] + 11.8334 = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + K_1/3 + K_2/27 + K_3/9$

$\Rightarrow -13.54266 [\text{wt}\% \text{Si}] + 155.0853 = 9K_1 + K_2 + 3K_3$

$\Rightarrow 9K_1 + K_2 + 3K_3 = -13.54266 [\text{wt}\% \text{Si}] + 155.0853 \dots [III]$

$\Rightarrow 3(3K_1 + K_3) + K_2 = 155.0853 - 13.54266 [\text{wt}\% \text{Si}]$

\Rightarrow Consider $3K_1 + K_3 = 50 \dots [IV]$

$\Rightarrow 150 + K_2 = 155.0853 - 13.54266 [\text{wt}\% \text{Si}]$

$\Rightarrow K_2 = 5.0853 - 13.54266 [\text{wt}\% \text{Si}] \dots [V]$

FROM $4K_1 + K_3 = 74.408 - 6.4976 [\text{wt}\% \text{Si}] [II]$, $3K_1 + K_3 = 50 \dots [IV]$ we have

$\Rightarrow K_1 + 50 = 74.408 - 6.4976 [\text{wt}\% \text{Si}]$

$\Rightarrow K_1 = 24.408 - 6.4976 [\text{wt}\% \text{Si}]$

$\Rightarrow 3K_1 + K_3 = 50$

$\Rightarrow K_3 = 50 - 3K_1$

$\Rightarrow K_3 = 50 - 3(24.408 - 6.4976 [\text{wt}\% \text{Si}])$

$\Rightarrow K_3 = 50 - 73.224 + 19.4928 [\text{wt}\% \text{Si}]$

$\Rightarrow K_3 = -23.224 + 19.4928 [\text{wt}\% \text{Si}]$

$\Rightarrow K_0 = -0.5345 [\text{wt}\% \text{Si}] + 6.0895$

$\Rightarrow E^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2$

Substituting K_0, K_1, K_2, K_3

$\Rightarrow E^* = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + (24.408 - 6.4976 [\text{wt}\% \text{Si}]) (\sum \alpha_1^2 \alpha_2^2) + (5.0853 - 13.54266 [\text{wt}\% \text{Si}]) (\prod \alpha_1^2) + (-23.224 + 19.4928 [\text{wt}\% \text{Si}]) (\sum \alpha_1^2 \alpha_2^2)^2$

$\Rightarrow E^* = -0.5345 [\text{wt}\% \text{Si}] + 6.0895 + (13.126 - 1.98338 [\text{wt}\% \text{Si}]) (\sum \alpha_1^2 \alpha_2^2) - (28.7605) (\prod \alpha_1^2) + (21.904 + 1.43592 [\text{wt}\% \text{Si}]) (\sum \alpha_1^2 \alpha_2^2)^2$

$\Rightarrow E^* = 6.0895 + (24.408) (\sum \alpha_1^2 \alpha_2^2) + (5.0853) (\prod \alpha_1^2) - 23.224 (\sum \alpha_1^2 \alpha_2^2)^2 + [-0.5345 [\text{wt}\% \text{Si}] - 6.4976 [\text{wt}\% \text{Si}]) (\sum \alpha_1^2 \alpha_2^2) - 13.54266 [\text{wt}\% \text{Si}] (\prod \alpha_1^2) + 19.4928 [\text{wt}\% \text{Si}] (\sum \alpha_1^2 \alpha_2^2)^2$

$\Rightarrow E^* = E^*_{\text{IRON}} + E^{**}$

⇒ $E^{**} = [-0.5345 \text{ [wt\%Si]} - 6.4976[\text{wt\%Si}]) (\sum \alpha_1^2 \alpha_2^2) - 13.54266[\text{wt\%Si}] ([\alpha_1^2] + 19.4928[\text{wt\%Si}] (\sum \alpha_1^2 \alpha_2^2)^2]$

⇒ Above Equation is very important Magneto-Crystalline Energy Density Equation for Electrical Steels

CRYSTALLOGRAPHIC DIRECTION	MAGNETO-CRYSTALLINE ANISOTROPY ENERGY DENSITY
[100] $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0$	$E^*_{[100]} = 6.0895 - 0.5345 \text{ [wt\%Si]}$
[110] $\alpha_1 = 1/\sqrt{2}, \alpha_2 = 1/\sqrt{2}, \alpha_3 = 0$	$E^*_{[110]} = 10.74 - 0.9406 \text{ [wt\%Si]}$
[111] $\alpha_1 = 1/\sqrt{3}, \alpha_2 = 1/\sqrt{3}, \alpha_3 = 1/\sqrt{3}$	$E^*_{[111]} = 11.8334 - 1.03608 \text{ [wt\%Si]}$

S.N O.	Standard Crystallographic Directions	Magnet o- Crystalline Anisotropy Values ,E* For Pure Iron	Magneto- Crystalline Anisotropy Values ,E*	Magneto - Crystalline Anisotropy Values ,E* for Fe- 0.51%Si	Magneto - Crystalline Anisotropy Values ,E* for Fe- 1.38%Si	Magnet o- Crystalline Anisotropy Values ,E* for Fe- 2.8%Si	Magnet o- Crystalline Anisotropy Values ,E* for Fe- 3.2%Si
1	[100]	$E^*_{[100]} = 6.0895$	$E^*_{[100]} = -0.5345 \text{ ([wt\%Si])} + 6.0895$	5.816905	5.351892	4.5929	4.3791
2	[110]	$E^*_{[110]} = 10.74$	$E^*_{[110]} = -0.9406 \text{ [wt\%Si]} + 10.74$	10.260294	9.441972	8.10632	7.73008
3	[111]	$E^*_{[111]} = 11.8634$	$E^*_{[111]} = -1.03608 \text{ [wt\%Si]} + 11.8634$	11.334992	10.4336096	8.962376	8.547944

2.1 Discussion

From Ref¹, The <100>//ND fibre accounts for the lowest anisotropy energy since the flux lines, distributed homogenously in a plane of the rotating laminated sheet, have an easiest magnetization direction with the in-plane rotated cube texture components. On the contrary, the Υ and the <011>//ND fiber orientations have relatively high anisotropy energy and as such, the occurrence of these components in electrical steels is undesirable.

III. ESTIMATION OF TEXTURE K_0, K_1, K_2, K_3 CONSTANTS FOR ELECTRICAL STEELS

⇒ $E^* = 0.355A^* + (0.163 - 0.031A^*)[\text{wt\%Si}] - 1.898$ FROM REF 1,

⇒ $A^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 ([\alpha_1^2]) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2$

⇒ $[0.355 - 0.031[\text{wt\%Si}]]A^* = -0.163[\text{wt\%Si}] + 1.898 - 0.5345 \text{ [wt\%Si]} + 6.0895 + (24.408 - 6.4976[\text{wt\%Si}]) (\sum \alpha_1^2 \alpha_2^2) + (5.0853 - 13.54266[\text{wt\%Si}]) ([\alpha_1^2]) + (-23.224 + 19.4928[\text{wt\%Si}]) (\sum \alpha_1^2 \alpha_2^2)^2$



$$\Rightarrow [0.355 - 0.031[\text{wt}\% \text{Si}]]A^* = -0.6975[\text{wt}\% \text{Si}] + 7.9875 + (24.408 - 6.4976[\text{wt}\% \text{Si}]) (\sum \alpha_1^2 \alpha_2^2) + (5.0853 - 13.54266[\text{wt}\% \text{Si}]) (\prod \alpha_1^2) + (-23.224 + 19.4928[\text{wt}\% \text{Si}]) (\sum \alpha_1^2 \alpha_2^2)^2$$

$$\Rightarrow A^* = K_0 + K_1 (\sum \alpha_1^2 \alpha_2^2) + K_2 (\prod \alpha_1^2) + K_3 (\sum \alpha_1^2 \alpha_2^2)^2$$

$$\Rightarrow K_0 = \frac{-0.6975[\text{wt}\% \text{Si}] + 7.9875}{[0.355 - 0.031[\text{wt}\% \text{Si}]]}; K_1 = \frac{(24.408 - 6.4976[\text{wt}\% \text{Si}])}{[0.355 - 0.031[\text{wt}\% \text{Si}]]}; K_2 = \frac{(5.0853 - 13.54266[\text{wt}\% \text{Si}])}{[0.355 - 0.031[\text{wt}\% \text{Si}]]}; K_3 = \frac{(-23.224 + 19.4928[\text{wt}\% \text{Si}])}{[0.355 - 0.031[\text{wt}\% \text{Si}]]}$$

S.NO.	TEXTURE FACTOR CONSTANTS	Fe + 0.51%Si	Fe + 1.38%Si	Fe + 2.8%Si	Fe + 3.2%Si
1.	K ₀	22.5	22.5	22.5	22.5
2.	K ₁	62.1899	49.45651	23.171961	14.13479
3.	K ₂	-5.370018	-43.57046	-122.424116	-149.53562
4.	K ₃	-39.15997	11.773954	116.91215	153.0608

$$\Rightarrow A^*_{0.51\% \text{Si}} = 22.5 + 62.1899 (\sum \alpha_1^2 \alpha_2^2) - 5.370018 (\prod \alpha_1^2) - 39.15997 (\sum \alpha_1^2 \alpha_2^2)^2$$

$$\Rightarrow A^*_{1.38\% \text{Si}} = 22.5 + 49.45651 (\sum \alpha_1^2 \alpha_2^2) - 43.57046 (\prod \alpha_1^2) + 11.773954 (\sum \alpha_1^2 \alpha_2^2)^2$$

$$\Rightarrow A^*_{2.8\% \text{Si}} = 22.5 + 23.171961 (\sum \alpha_1^2 \alpha_2^2) - 122.424116 (\prod \alpha_1^2) + 116.91215 (\sum \alpha_1^2 \alpha_2^2)^2$$

$$\Rightarrow A^*_{3.2\% \text{Si}} = 22.5 + 14.13479 (\sum \alpha_1^2 \alpha_2^2) - 149.53562 (\prod \alpha_1^2) + 153.0608 (\sum \alpha_1^2 \alpha_2^2)^2$$

S.NO.	CRYSTALLOGRAPHIC DIRECTIONS	DIRECTIONAL COSINE RELATIONSHIP	TEXTURE FACTOR FOR A* IDEAL FIBER	TEXTURE FACTOR FOR A* _{0.51% Si}	TEXTURE FACTOR FOR A* _{1.38% Si}	TEXTURE FACTOR FOR A* _{2.8% Si}	TEXTURE FACTOR FOR A* _{3.2% Si}
1	<100>direction	α ₁ =1, α ₂ =0, α ₃ =0	22.5 Θ fibre <100>// ND	22.5	22.5	22.5	22.5
2	<110>direction	α ₁ =1/√2, α ₂ =1/√2, α ₃ =0	35.6 fibre <110>// ND	35.6	35.6	35.6	35.6
3	<111> direction	α ₁ =1/√3, α ₂ =1/√3, α ₃ =1/√3	38.68 Υ fibre <111>// ND	38.68	38.68	38.68	38.68

5. Conclusions

Magneto-Crystalline Anisotropy Energy Density value is least for [100] directions, and higher for [110] & [111] directions. Therefore [100] directions are easy directions of magnetization for pure iron and [111] hardest direction for magnetization of pure iron, [110] direction is harder direction for UGC CARE Group-1,



magnetization of pure iron. Texture Factor Equation results are consistent with the standard results and conforms to the value of ideal fibres.

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