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AVAILABILITY AND COST ANALYSIS OF A BATTERY SYSTEM WITH CRITICAL HUMAN ERRORS

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Abstract
The author of this research thought over a battery setup with three individual components labeled A, B, and C. While both B and C each have one unit, subsystem A has two that are linked in parallel. Failed, degraded, and good are the possible system states. Unit failure and catastrophic human mistakes are both potential causes of failure. If a unit failure occurs it is possible to repair but repair is not possible if a critical human error occurs. The repair rates follow the general distribution. Further availability and cost function of the system is evaluated in this work.

Keywords: Failure rate, Critical Human error, Availability, Cost Function.

I. Introduction

Many researchers also concluded their studies based on the assumption that important human mistakes have a healing rate roughly proportional to the normal distribution. Gupta and Gupta [1] assume a normal distribution for repair frequencies in electronic repairable redundant systems. Using a Markov model, Srinath [3] describes how to calculate the availability expression for a unitary system. Using a two-tier, single-server complicated system as an example, Gupta and Sharma [2] analyzed the human error effect on availability as well as mean time to failure. S. Narmada and M. Jacob [4] presented a stochastic model in which human error plays a critical role, and in which there are two units (one of which is a standby unit).

II. Notations

G	Overall, the system is good.
D	The system's degraded state.
F_1	Both units of subsystem A have failed while subsystem B and C are good.
F_2	Subsystem B has failed while subsystem C and A are good.
F_3	Subsystem C has failed while subsystem A and B are good.
F	Unrepairable failed state of the system due to critical human error.
λ_A	The constant failure rate of a unit of subsystem A.
λ_B	The constant failure rate of a unit of subsystem B.
λ_C	The constant failure rate of a unit of subsystem C.
λ_{hG}	The constant failure rate of the system is due to critical human error when the system is in a good state.
λ_{hD}	The constant failure rate of the system is due to critical human error when the system is in a degraded state.
$S_i(r), \phi_i(r)$	Probability distributive function and hazard rate for repair time of the system.
$i = F_1, r = x$	Repair of the system in failed state F_1, repair is completed in elapsed repair time x.
$i = F_2, r = y$	Repair of the system in failed state F_2, repair is completed in elapsed time y.
$i = F_3, r = z$	Repair of the system in failed state F_3, repair is completed in elapsed time z.
$P_i(t)$	Probability of the system in state i at time t. where $i = G, D, F_1, F_2, F_3, F$.

$\overline{P}_i(s)$	Laplace transform of $P_i(t)$
Davis formula	$S_i(r)=\phi_i(r)\exp\left[-\int_0^r \phi_i(r)\,dr\right]$

III. System transition diagram

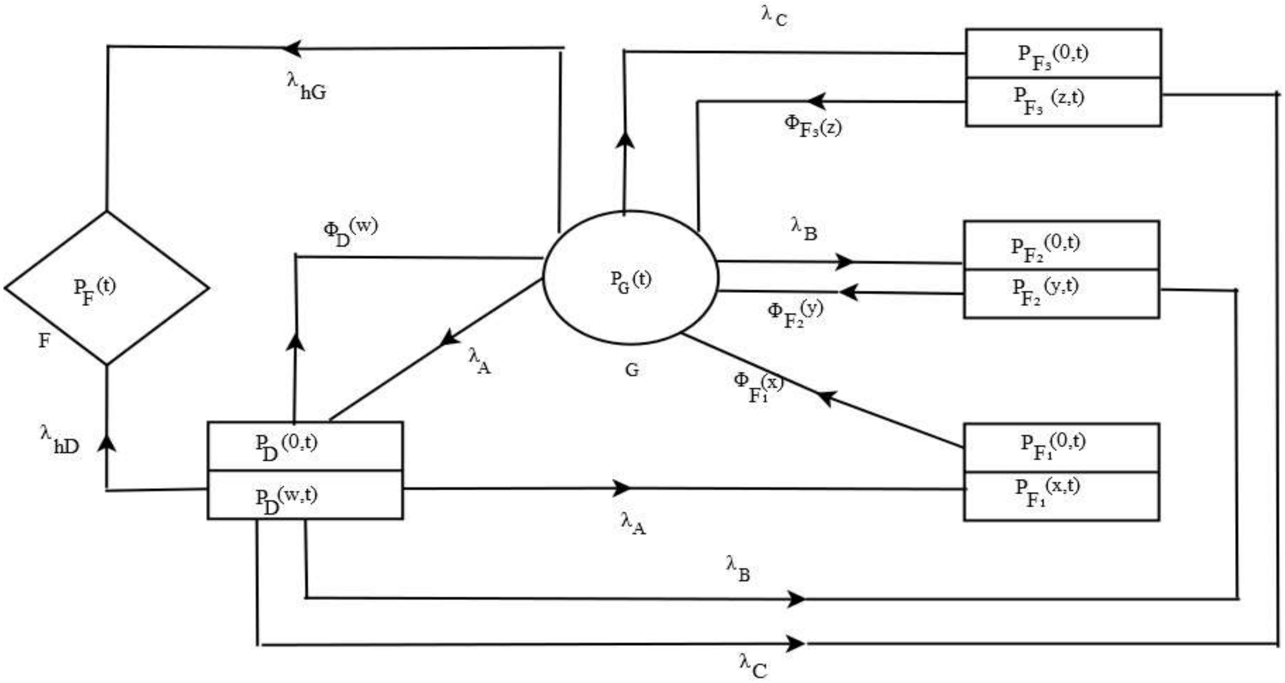

Fig. 1: State transition diagram

From the system transition diagram, the difference differential Equations are

$$\left[\frac{\partial}{\partial t}+\lambda_{hG}+2\lambda_A+\lambda_B+\lambda_C\right]P_G(t)=\int_0^\infty P_D(w,t)\phi_D(w)\,dw+\int_0^\infty P_{F_1}(x,t)\phi_{F_1}(x)\,dx$$

$$+\int_0^\infty P_{F_2}(y,t)\phi_{F_2}(y)\,dy+\int_0^\infty P_{F_3}(z,t)\phi_{F_3}(z)\,dz \quad (1)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\lambda_{hD}+\phi_D(w)+\lambda_A+\lambda_B+\lambda_C\right)P_D(w,t)=0 \quad (2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_{F_1}(x)\right)P_{F_1}(x,t)=0 \quad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_{F_2}(y)\right)P_{F_2}(y,t)=0 \quad (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_{F_3}(z)\right)P_{F_3}(z,t)=0 \quad (5)$$

$$\frac{d}{dt}P_F(t)=\lambda_{hG}P_G(t)+\lambda_{hD}P_D(t) \quad (6)$$

Boundary conditions

$$P_D(0,t) = 2\lambda_A P_G(t) \tag{7}$$

$$P_{F_1}(0,t) = \lambda_A P_D(t) \tag{8}$$

$$P_{F_2}(0,t) = \lambda_B P_D(t) + \lambda_B P_G(t) \tag{9}$$

$$P_{F_3}(0,t) = \lambda_C P_D(t) + \lambda_C P_G(t) \tag{10}$$

Initial conditions

$$P_i(0) = \begin{cases} 1 & i = G \\ 0 & i \neq G \end{cases} \tag{11}$$

IV. Solution

To solve for, we use the Laplace transform on equations (1) through (10) as well as initial conditions (11).

$$[s + \lambda_{hG} + 2\lambda_A + \lambda_B + \lambda_C]\overline{P}_G(s) = 1 + \int_0^\infty \overline{P}_D(w,s)\phi_D(w)dw + \int_0^\infty \overline{P}_{F_1}(x,s)\phi_{F_1}(x)dx$$

$$+\int_0^\infty \overline{P}_{F_2}(y,s)\phi_{F_2}(y)dy + +\int_0^\infty \overline{P}_{F_3}(z,s)\phi_{F_3}(z)dz \tag{12}$$

$$\left(s + \frac{\partial}{\partial w} + \lambda_{hD} + \phi_D(w) + \lambda_A + \lambda_B + \lambda_C\right)\overline{P}_D(w,s) = 0 \tag{13}$$

$$\left(s + \frac{\partial}{\partial x} + \phi_{F_1}(x)\right)\overline{P}_{F_1}(x,s) = 0 \tag{14}$$

$$\left(s + \frac{\partial}{\partial y} + \phi_{F_2}(y)\right)\overline{P}_{F_2}(y,s) = 0 \tag{15}$$

$$\left(s + \frac{\partial}{\partial z} + \phi_{F_3}(z)\right)\overline{P}_{F_3}(z,s) = 0 \tag{16}$$

$$s\overline{P}_F(s) = \lambda_{hG}\overline{P}_G(s) + \lambda_{hD}\overline{P}_D(s) \tag{17}$$

$$\overline{P}_D(0,s) = 2\lambda_A \overline{P}_G(s) \tag{18}$$

$$\overline{P}_{F_1}(0,s) = \lambda_A \overline{P}_D(s) \tag{19}$$

$$\overline{P}_{F_2}(0,t) = \lambda_B \overline{P}_D(s) + \lambda_B \overline{P}_G(s) \tag{20}$$

$$\overline{P}_{F_3}(0,s) = \lambda_C \overline{P}_D(s) + \lambda_C \overline{P}_G(s) \tag{21}$$

Integrating (13) to (16) and using (17) to (21) we have

$$\overline{P}_D(w,s) = 2\lambda_A \overline{P}_G(s)\exp[-(s + \lambda_{hD} + \lambda_A + \lambda_B + \lambda_C)w]\exp[-\int_0^w \phi_D(w)dw] \tag{22}$$

$$\overline{P}_{F_1}(x,s)=\lambda_A\overline{P}_D(s)\exp[-sx]\exp[-\int_0^x\phi_{F_1}(x)dx] \tag{23}$$

$$\overline{P}_{F_2}(y,s)=\lambda_B\left[\overline{P}_G(s)+\overline{P}_D(s)\right]\exp[-sy]\exp[-\int_0^y\phi_{F_2}(y)dy] \tag{24}$$

$$\overline{P}_{F_3}(z,s)=\lambda_C\left[\overline{P}_G(s)+\overline{P}_D(s)\right]\exp[-sz]\exp[-\int_0^z\phi_{F_3}(z)dz] \tag{25}$$

Making use of (22) to (25) in (12), we have after simplification

$$\begin{aligned}\left[s+\lambda_{hG}+2\lambda_A+\lambda_B+\lambda_C\right]\overline{P}_G(s)&=1+2\lambda_A\overline{P}_G(s)\overline{S}_D(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C)+\lambda_A\overline{P}_D(s)\overline{S}_{F_1}(s)\\&+\lambda_B\left[\overline{P}_G(s)+\overline{P}_D(s)\right]\overline{S}_{F_2}(s)+\lambda_C\left[\overline{P}_G(s)+\overline{P}_D(s)\right]\overline{S}_{F_3}(s)\end{aligned} \tag{26}$$

Now $\overline{P}_D(s)=\int_0^\infty\overline{P}_D(x,s)dx$

By simplifying (22) we get

$$\overline{P}_D(s)=2\lambda_A\overline{P}_G(s)\left[\frac{1-\overline{S}_D(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C)}{s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C}\right] \tag{27}$$

Using (27) in (26), then we get

$$\overline{P}_G(s)=\frac{1}{A_1(s)} \tag{28}$$

Here,

$$\begin{aligned}A_1(s)&=(s+\lambda_{hG}+2\lambda_A+\lambda_B+\lambda_C)-2\lambda_A\overline{S}_D(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C)\\&-2\lambda_A^2\left[\frac{1-\overline{S}_D(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C)}{s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C}\right]\overline{S}_{F_1}(s)-\lambda_B\left[1+2\lambda_A\left[\frac{1-\overline{S}_D(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C)}{s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C}\right]\right]\overline{S}_{F_2}(s)\\&-\lambda_c\left[1+2\lambda_A\left[\frac{1-\overline{S}_D(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C)}{s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C}\right]\right]\overline{S}_{F_3}(s)\end{aligned} \tag{29}$$

Using (29) in (27) then we get

$$\overline{P}_D(s)=\frac{A_2(s)}{A_1(s)} \tag{30}$$

Here

$$A_2(s)=2\lambda_A\left[\frac{1-\overline{S}_D(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C)}{s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C}\right] \tag{31}$$

Also using (23), (24), (25) and on simplification, we have

$$\overline{P}_{F_1}(s)=\int_0^\infty\overline{P}_{F_1}(x,s)dx=\frac{A_2(s)}{A_1(s)}A_3(s) \tag{32}$$

Where $A_3(s)=\lambda_A\left[\frac{1-\overline{S}_{F_1}(s)}{s}\right]$ (33)

$$\overline{P}_{F_2}(s)=\int_0^\infty \overline{P}_{F_2}(y,s)dy=\frac{1+A_2(s)}{A_1(s)}A_4(s) \tag{34}$$

Where $A_4(s)=\lambda_B\left[\frac{1-\overline{S}_{F_2}(s)}{s}\right]$ (35)

$$\overline{P}_{F_3}(s)=\int_0^\infty \overline{P}_{F_3}(z,s)dz=\frac{1+A_2(s)}{A_1(s)}A_5(s) \tag{36}$$

Where $A_5(s)=\lambda_C\left[\frac{1-\overline{S}_{F_3}(s)}{s}\right]$ (37)

Using (28) and (30) in (17), we have

$$\overline{P}_F(s)=\frac{\lambda_{hG}+\lambda_{hD}A_2(s)}{sA_1(s)} \tag{38}$$

V. Evaluation of Laplace transforms of up and down state probabilities:

At time t, probabilities of operational availability as well as non-availability, as expressed by their Laplace transform, are as follows:

$$\overline{P}_{up}(s)=\overline{P}_G(s)+\overline{P}_D(s)=\frac{1+A_2(s)}{A_1(s)} \tag{39}$$

$$\overline{P}_{down}(s)=\overline{P}_{F_1}(s)+\overline{P}_{F_2}(s)+\overline{P}_{F_3}(s)+\overline{P}_F(s)=\frac{1}{A_1(s)}\left[\begin{array}{l}A_2(s)A_3(s)+\{1+A_2(s)\}A_4(s)\\+\{1+A_2(s)\}A_5(s)+\frac{1}{s}\{\lambda_{hG}+\lambda_{hD}A_2(s)\}\end{array}\right] \tag{40}$$

VI. Ergodic behaviour of the system

Utilizing Abel's corollary theorem,

$$\lim_{s\to 0} s\overline{F}(s)=\lim_{t\to\infty}F(t)=F(say);$$

The time-independent probability is achieved if and only if the following condition holds:

$$P_{up}=\lim_{s\to 0} s\overline{P}_{up}(s)=\lim_{s\to 0}\frac{s\left[1+A_2(s)\right]}{A_1(s)}=0\,(\text{since, } A_1(0)\neq 0) \tag{41}$$

$$P_{down}=\lim_{s\to 0} s\overline{P}_{down}(s)=\frac{\lambda_{hG}+\lambda_{hD}.A_2(0)}{A_1(0)} \tag{42}$$

VII. Particular case

Given an exponential distribution for the time required for repairs:

Taking $\overline{S}_i(s)=\frac{\phi_i}{s+\phi_i}$, where $i=D,F_1,F_2,F_3$ in equations (28)-(38) then we get

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$$\overline{P}_G(s)=\frac{1}{B_1(s)} \tag{43}$$

Here

$$B_1(s)=(s+\lambda_{hG}+2\lambda_A+\lambda_B+\lambda_C)-\frac{2\lambda_A\phi_D}{(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C+\phi_D)}-\frac{2\lambda_A^2\,\phi_{F_1}}{(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C+\phi_D)(s+\phi_{F_1})}$$

$$-\frac{\lambda_B\,\phi_{F_2}}{s+\phi_{F_2}}\left(1+\frac{2\lambda_A}{(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C+\phi_D)}\right)--\frac{\lambda_C\,\phi_{F_3}}{s+\phi_{F_3}}\left(1+\frac{2\lambda_A}{(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C+\phi_D)}\right) \tag{44}$$

$$\overline{P}_D(s)=\frac{B_2(s)}{B_1(s)} \tag{45}$$

Where $B_2(s)=\dfrac{2\lambda_A}{(s+\lambda_{hD}+\lambda_A+\lambda_B+\lambda_C+\phi_D)}$ (46)

$$\overline{P}_{F_1}(s)=\frac{B_2(s)B_3(s)}{B_1(s)} \tag{47}$$

Where $B_3(s)=\dfrac{\lambda_A}{(s+\phi_{F_1})}$ (48)

$$\overline{P}_{F_2}(s)=\frac{\left[1+B_2(s)\right]B_4(s)}{B_1(s)} \tag{49}$$

Where $B_4(s)=\dfrac{\lambda_B}{(s+\phi_{F_2})}$ (50)

$$\overline{P}_{F_3}(s)=\frac{\left[1+B_2(s)\right]B_5(s)}{B_1(s)} \tag{51}$$

Where $B_5(s)=\dfrac{\lambda_C}{(s+\phi_{F_3})}$ (52)

VIII. EVALUATION OF INVERSE LAPLACE TRANSFORM OF $\overline{P}_{up}(s)$ AND $\overline{P}_{down}(s)$

Setting $\phi_D=0.8,\phi_{F_1}=0.7,\phi_{F_2}=0.6,\phi_{F_3}=0.5,\lambda_A=0.06,\lambda_B=0.04,\lambda_C=0.02,\lambda_{hG}=0.1,\lambda_{hD}=0.2$ in (43) to (52) and simplifying then we get,

$$\overline{P}_{up}(s)=\frac{(s+1.24)(s+0.7)(s+0.6)(s+0.5)}{s^5+3.2s^4+3.7736s^3+2.01068s^2+0.456856s+0.02856} \tag{53}$$

Taking inverse Laplace transform of (53), we have

$$\overline{P}_{up}(t)=-0.03078199368e^{-1.210861627t}+0.08674136347e^{-0.7320654906t}+0.06079753448e^{-0.6447289223t}$$
$$+0.02134717620e^{-0.5153803594t}+0.8618959195e^{-0.09696360084t} \tag{54}$$

Table 1

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Time t	$P_{up}(t)$	$P_{down}(t)$
0	1.000000000	0.000000000
1	0.859448218	0.140551782
2	0.751648199	0.248351801
3	0.666520996	0.333479004
4	0.596531718	0.403468282
5	0.536965476	0.463034524
6	0.485006006	0.514993994
7	0.438954457	0.561045543
8	0.397739187	0.602260813
9	0.360637804	0.639362196
10	0.327125986	0.672874014

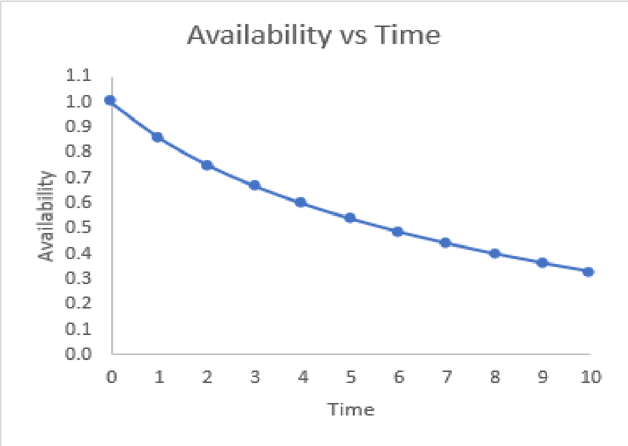

Fig. 2: Availability vs Time

Also $\overline{P}_{up}(t)+\overline{P}_{down}(t)=1$

IX. Cost function analysis

During $(0,t]$ system's s-expected up time is $E(t)=\int_0^\infty P_{up}(t)\,dt$

In $(0,t]$ the service facility's s-expected busy period is $\mu_B(t)=t$.

Therefore, function for the anticipated net gain is

Expected total revenue function is defined as $G(t)=$

$$G(t)=C_1\int_0^t P_{up}(t)dt-C_2t=C_1\begin{bmatrix}0.02542156180e^{-1.210861627t}-0.1184885295e^{-0.7320654906t}\\-0.09429937510e^{-0.6447289223t}-0.04142023616e^{-0.5153803594t}\\-8.888860480e^{-0.09696360084t}\end{bmatrix}-C_2t \quad (55)$$

Whereas C_1 is defined as revenue per unit up time while C_2 is defined as repair cost per unit time.

X. Numerical computation

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Availability analysis

in equation (54), Setting $t = 0,1,2,.......,10$ Table 1 obtained.

Cost analysis

Table 2 presents the expected profit in equation (55), Setting $C_1 = 1, t = 0,1,2,.......,10$, for $C_2 = 1, 0.5, 0.10, 0.05$.

Table 2

Time t	Expected profit G(t)			
	$C_2 = 1$	$C_2 = 0.5$	$C_2 = 0.1$	$C_2 = 0.05$
0	0	0	0	0
1	-0.073422023	0.426577976	0.826577976	0.876577976
2	-0.270161408	0.729838592	1.529838592	1.629838592
3	-0.562603261	0.937396739	2.137396739	2.287396739
4	-0.932105631	1.067894369	2.667894369	2.867894369
5	-1.366086219	1.133913781	3.133913781	3.383913781
6	-1.855651091	1.144348909	3.544348909	3.844348909
7	-2.394111592	1.105888408	3.905888408	4.255888408
8	-2.976133804	1.023866196	4.223866196	4.623866196
9	-3.597264035	0.902735965	4.502735965	4.952735965
10	-4.253662927	0.746337073	4.746337073	5.246337073

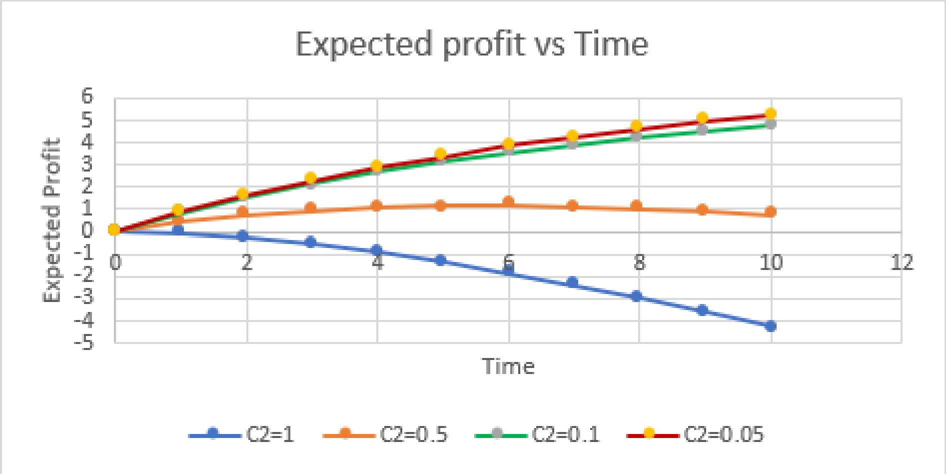

Fig. 3: Expected Profit vs Time

XI. Results' interpretation

System's availability at time t is shown in Fig. 2. Time-availability graph the system's availability declines with time, and we can also see that it declines extremely slowly over a lengthy period of time. Therefore, the system is accessible for use for an extended duration.

In Fig. 3 we see the interval profit expectation for a constant value of revenue per time unit. The expected profit vs time graph shows a precipitous drop in profits at high service costs $C_2 \geq 1$; and a sustained rise in profits at low service costs $C_2 \leq 0.1$.

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