



APPLICATION OF WAVELET MULTI-RESOLUTION ANALYSIS FOR GEAR FAULT DIAGNOSIS

Mrs. Ami Barot, Assistant Professor, Dept. of Mechanical Engineering, PVG's COET & GKPWIOM. Pune

Dr. Pravin Kulkarni, Associate Professor, Dept. of Mechanical Engineering, PVG's COET & GKPWIOM. Pune

Abstract

The gearbox is one of the most important elements in high-speed machinery. The gears are subjected to fatigue loading and progressive faults are developed. At a certain stage, catastrophic failure may occur if proper attention is not given. This can be avoided by periodic assessment of health called as condition monitoring and fault diagnosis. This technique is very important to avoid the hazardous and catastrophic failure of whole system. This paper presents the methodology for the fault diagnosis of gears based on Fast Fourier Transform and Wavelet Transform. In order to obtain useful information from the raw vibration signal from the gear, db2 mother wavelet is adopted to decompose the vibration signal. Based on multi resolution analysis technique, signals are decomposed to a level based on the sampling frequency and desired frequency to be focused upon for different deteriorated stages of the gear.

Keywords: Condition Monitoring, Gearbox, Multi Resolution Analysis, Scalogram, Wavelet Transform.

Introduction

Gears are considered as the most critical rotating elements because of their wide industrial applications [1,2]. Unexpected failure of gear, bearings or any rotating component can be result in to the catastrophic failure of the whole system leading to loss of production involving heavy cost. Condition monitoring is emerging field for the fault diagnosis of machinery. Vibration based signal analysis is the most common and effective method for fault diagnosis of rotating components for the predictive and preventive maintenance of the machinery. They can be easily applied in industry by placing vibration sensors like accelerometer on the gearbox or bearing housing [3,4]. Faults and damages represent a disturbing quantity or impulse. Any significant change in the vibration signal mirrors the presence of the disturbance in the system [5, 6] and in variably the same disturbance will be noticed in time-frequency plot [7]. These graphical impressions can be treated as colour contours but more they are representative of kinematic behaviour in gears [8, 9]. For dynamic signal analysis, the wavelet transform has been increasingly applied for system health monitoring. Detailed reviews on application of wavelet transform in machine condition monitoring have been presented by Peng and Chu [10]. The Wavelet Transform is one of the most important methods for signal processing and it is especially suitable for non-stationary vibration measurements obtained from accelerometer sensors. The WT can diagnose the abnormal change in the measured data [11].

The paper is organized as follows:

In section 2, the concept of wavelet transform along with wavelet decomposition is reviewed. Section 3 discusses the experimentation and data acquisition system for obtaining the gear vibration data. Section 4 presents the results of wavelet transform and Fourier transform. The last section concludes the paper.

Basic concepts of Wavelet Transform

In the signal processing, wavelet theory is used over the years as a tool. A wavelet transform (WT) is the decomposition of a signal into a set of basic functions consisting of contractions, expansions, and translations of a mother function $\psi(t)$, called the wavelet. Wavelets are mathematical functions



that cut up data into different frequency components, and each component is studied with a resolution matched to its scale. They are suitable for analysing physical situations where the signal contains discontinuities and sharp spikes. Wavelets offer timescale information of the signal. The commonly used wavelet algorithms are continuous wavelet transform (CWT), discrete wavelet transform (DWT), and wavelet packet transform (WPT).

Continuous Wavelet Transform

Continuous Wavelet Transform (CWT) is employed to process these signals to extract features for the further faults classification because wavelets are well appropriate for approximating these signals. CWT is the sum over all time of the signal multiplied by scaled, shifted versions of the mother wavelet. CWT for any scale b and position a can be obtained by convolving the signal $x(t)$ and a dilated and translated version of the mother wavelet. By shifting the wavelet in time, the signal is localized in time, and by changing the values of scale, the signal is localized in frequency. So, wavelet coefficients are well localized both in time and frequency, and suitable for feature extraction. CWT is a measure of similarity between the basis wavelets and the analyzed signal. The higher the coefficient is, the more the similarity. Therefore, the value of the similarity will depend on waveform of the mother wavelet. The continuous wavelet transform is dot product of $x(t)$ with translate and dilate of a wavelet ψ . ψ is wavelet translated by b and dilated by a .

$$CWT(b, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^* \left(\frac{t-b}{a} \right) dt \quad (1)$$

where $\psi^*(t)$ stands for the complex conjugation of $\psi(t)$ [12, 13].

Above is CWT of function $x \in L2(\mathbb{R})$ w.r.t. wavelet ψ evaluated at translation b and dilation a . Equation (1) indicates that the wavelet analysis is a time-frequency analysis, or a time-scaled analysis. The analysing function or windowing function ψ must satisfy certain admissibility conditions to be considered for wavelet analysis. The dilation parameter a and translation parameter b are also referred as the scaling and shifting parameters. By changing the value of dilation parameter a , the portion of the function in vicinity of $t=b$ can be examined in different resolutions (referred as multi-resolution analysis). By changing the value of translation parameter b , the function around the point $t=b$ can be examined by the wavelet window piece by piece. It is possible to reconstruct the original function from its wavelet transform. The inversion formula [14] is given by:

$$x(t) = \frac{1}{c_\psi} \iint w(a, b) \psi_{(a,b)}(t) \frac{dad b}{a^2} \quad (2)$$

where $c_\psi = \int_{-\infty}^{+\infty} \frac{|\psi(\omega)|^2}{|\omega|} d\omega < \infty$

Using above equation, the original signal can be reconstructed without any loss of data. Scaling parameter a is positive real and translation parameter b is positive or negative. At high frequencies, the wavelet reaches at a high time resolution but a low frequency resolution, whereas, at low frequencies, high-frequency resolution and low time resolution can be obtained.

Discrete Wavelet Transform

The CWT is defined at all points in the plane and corresponds to a redundant (extra) representation of the information present in the function. This redundancy requires a large amount of computation time. Instead of continuously varying the parameters, we analyse the signal with a small number of scales with varying number of translations at each scale. The discrete wavelet transform may be viewed as a “discretization” of the CWT through sampling specific wavelet coefficients. A critical sampling of the CWT given by equation (1) is obtained via $a=2^{-j}$ and $b=k2^{-j}$, where j and k are integers representing

the set of discrete dilations and translations respectively. Upon this substitution, discrete wavelet transform (DWT) is obtained and is given by:

$$W(j, k) = \int_{-\infty}^{+\infty} x(t) 2^{j/2} \psi(2^j t - k) dt \quad (3)$$

The term critical sampling denotes the minimum number of coefficients sampled from CWT to ensure that all the information present in the original function is retained by the wavelet coefficients [15]. The DWT computes the wavelet coefficients at discrete intervals (integer power of two) of time and scales. In discrete wavelet transform, the signal is decomposed into a tree structure of low and high pass filters. Each step transforms the low pass filter into further lower and higher frequency components as shown in Figure 1.

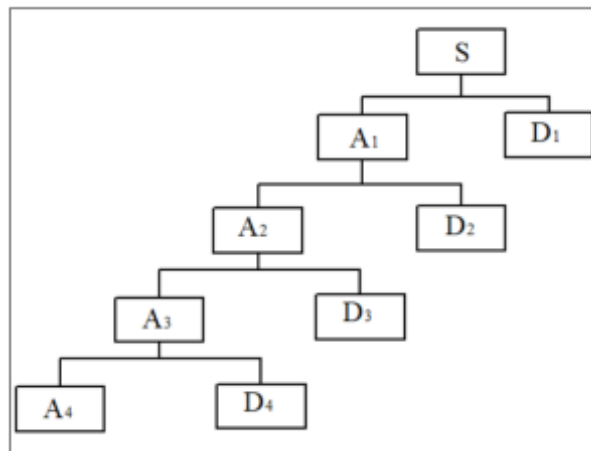


Figure 1: DWT decomposition tree of four-level [16]

Experimentation and Instrumentation

The vibration signatures are collected from gear of an experimental set up as shown in Figure 2. The shaft of the experimental setup is driven by an AC motor through a gear coupling. Spur gears are in mesh having teeth of 24 and 26 for driver and driven gear respectively. Single stage gear box is supported by four bearings, 2 bearings on each shaft. To obtain variation in speed, a variable frequency drive (VFD) is used. Radial vibration of the bearing was recorded using single axis accelerometer (100 mv/g) with NI 9234 sound and vibration module and USB Compact DAQ Chassis-cDaq 9171. Vibration signals are acquired at speed of 1230 rpm of system for different three cases of gear which is healthy, chipped and missing tooth of gear.



Figure 2: Experimental setup

Fast Fourier Transform of the gear vibration signal

FFT technique is applied to observe the distribution of frequency contents from the single stage gearbox setup. Three faults of gear tooth are considered for the study. These are healthy, chipped and missing gear of driven gear. Rotational speed is 1230 rpm, 2000 samples are recorded with a sampling

frequency of 20000 Hz. To investigate the progression of incipient fault, we focus upon the fault frequency which is calculated as,

Gear mesh frequency = Speed in RPM X no of teeth /60 = 1230 X 26/60 = 533 Hz.

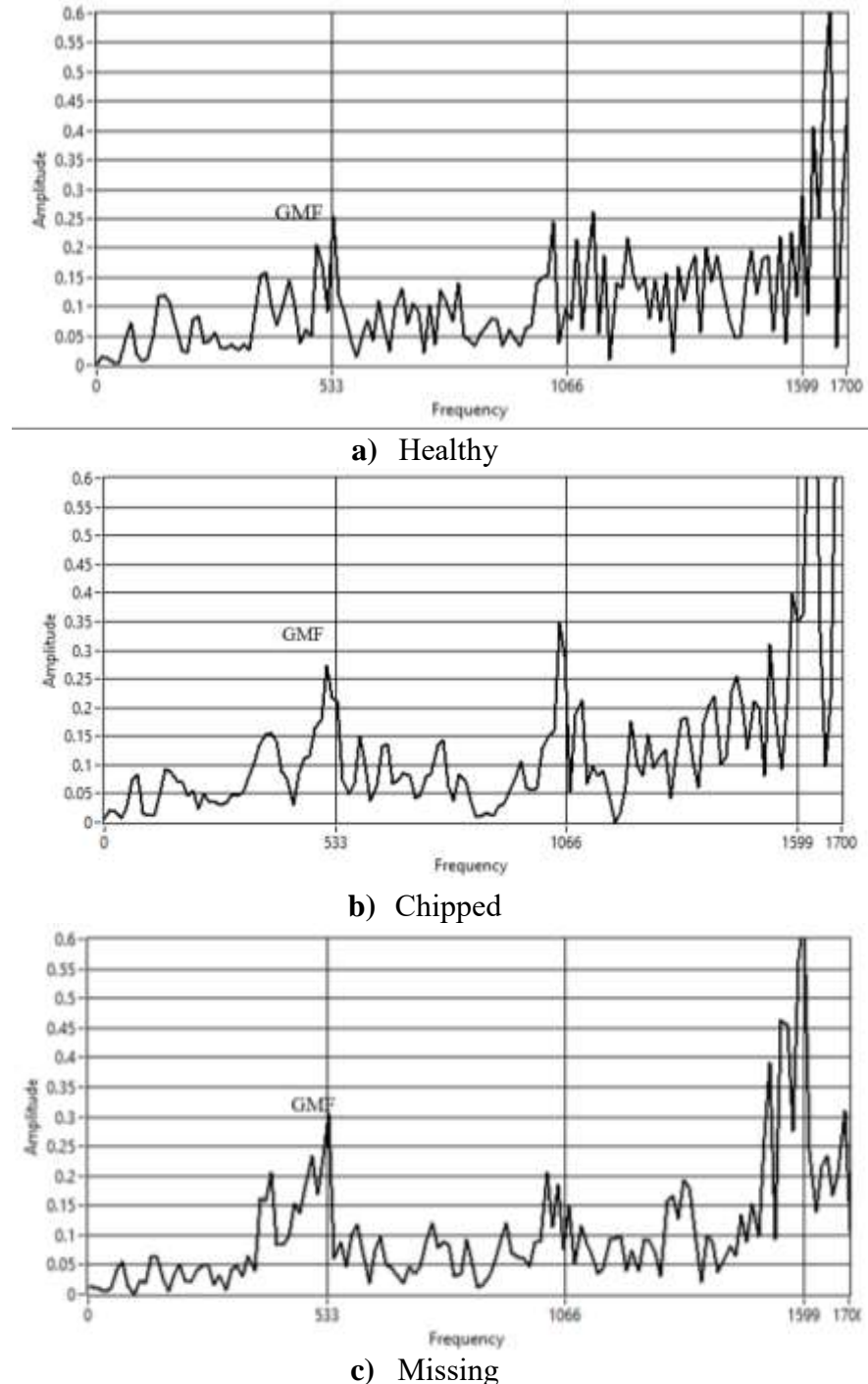


Figure 3: FFT for (a) healthy, (b) chipped and (c) missing gear tooth

From FFT shown in the Figure 3, it is observed that low frequency components altered much in different cases. High frequency components of all faults are insignificant. For two gears in mesh having one peak at gear mesh frequency that is observed in all the three cases. As the fault initiates on the gear tooth and progresses, the vibration level increases and there is remarkable increase in the amplitude of vibration. Amplitude of the frequency is increasing from healthy gear to broken tooth and it is highest for missing gear tooth. For instance, when healthy gear is compared with the faulty condition of the

gears, it contains fewer local peaks. GMF peak also represents the severity of the fault. Further if the comparison is between the chipped and missing teeth, missing teeth having more peaks and also having more amplitude in the lower frequency range.

From the FFT spectra of the gear faults, it appeared to be more essential to apply fourth level of discrete wavelet decomposition. At fourth level of approximation coefficients, the low frequency band between 0-625 Hz exist reflecting the fault characteristics frequency.

Multi-resolution Analysis

The discrete wavelet transform is well suited for multiresolution analysis. The DWT decomposes high-frequency components of a signal with fine time resolution but coarse frequency resolution and decomposes low-frequency components with fine frequency resolution but coarse time resolution. The DWT based multiresolution analysis helps in better understanding a signal and is useful in feature extraction applications such as peak detection and edge detection. Multiresolution analysis also helps in removing unwanted components in the signal such as noise.

In discrete wavelet transform, the signal is decomposed into a tree structure of low and high pass filters. Each step transforms the low pass filter into further lower and higher frequency components as shown in Figure 4. The frequency band of each filter depends on the decomposition level. The low pass filter produces approximation coefficients and high pass filter produces detail coefficients. The high frequency components are not analysed further.

For example, if

N_t = Total length of signal

j = DWT decomposition level

F_s = Sampling frequency

The frequency band of each filter depends on the decomposition level. The high frequency components are not analyzed further. The low pass filter produces approximation coefficients and high pass filter produces detail coefficients. For example, if N_t =Total length of signal, j =DWT decomposition level, F_s =Sampling frequency, then each vector contains $N_t/2^j$ coefficients. Approximation corresponds to Frequency band $[0, F_s/2^{j+1}]$ while detail covers the frequency range $[F_s/2^{j+1}, F_s/2^j]$ [17].

At any decomposition level, the signal can be expressed as the sum of approximation and detail coefficients as follows:

$$S = A_j + \sum D_i (i \leq j) \tag{4}$$

where, A_j = Approximation coefficients at j^{th} level

D_i = Detail coefficients

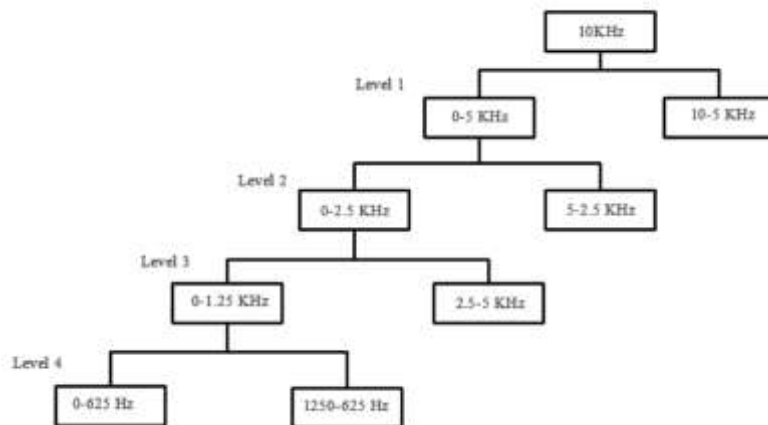


Figure 4: Multiresolution analysis at level 4

In LabVIEW, front panel shows the graphical representation of signal and the block diagram shows virtual instrumentation (VI) part. The block diagram is the programming part for signal acquisition and signal processing as shown in Figure 5.

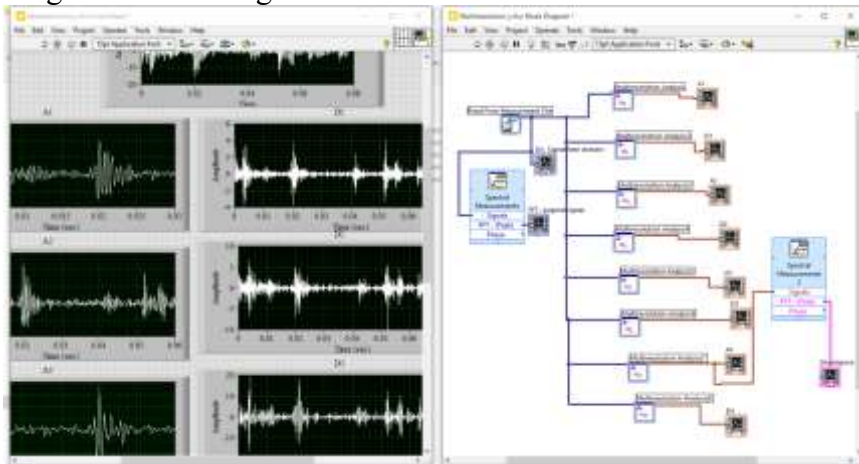


Figure 5: Multi resolution analysis in LabVIEW

Following are the graphs [Figure 6] for multi resolution analysis of healthy gear tooth up to 4th level.

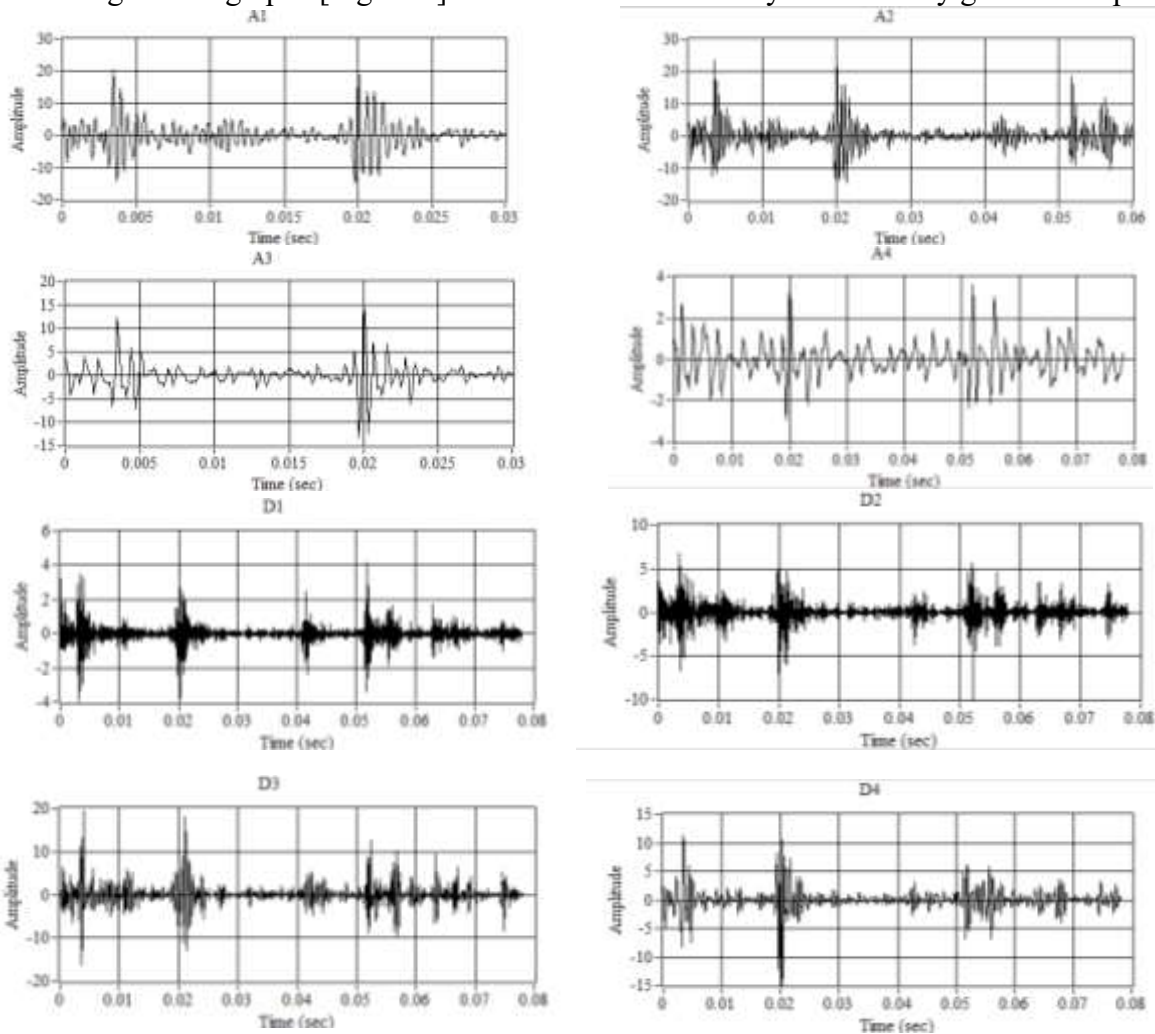


Figure. 6 Approximation and Details at level 4 for healthy gear tooth

The sampling frequency of the signal is 20000 Hz. The desired frequency i.e., Gear mesh frequency (GMF) is 533 Hz at a speed of 1230 rpm. The time domain signal is transformed to frequency domain as shown in Figure 7. Figure 8 is the spectrum of approximation at 4th level i.e. A₄

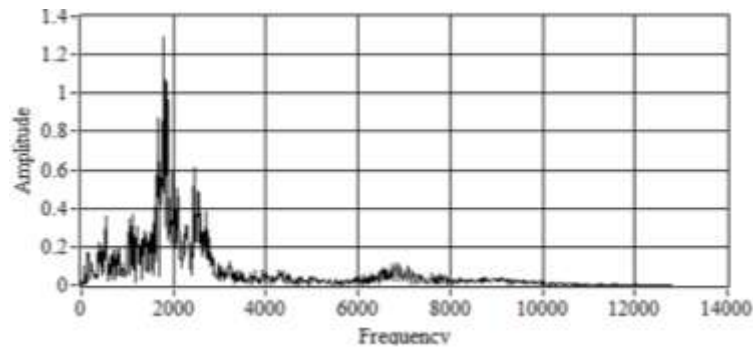


Figure 7: Spectrum of Healthy gear at a speed of 1230 rpm

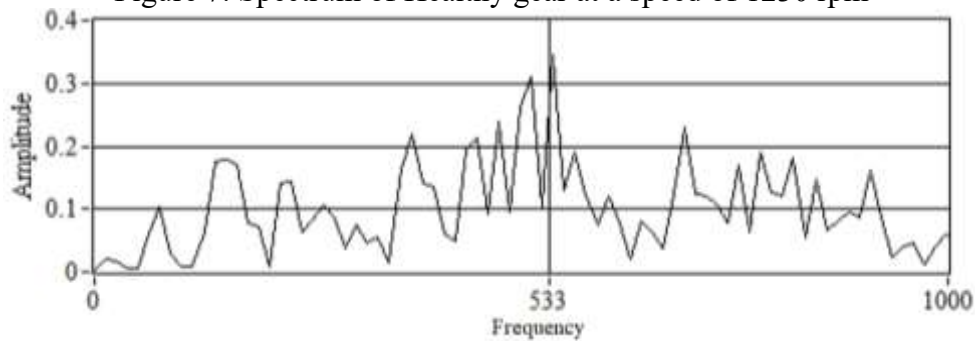


Figure 8: Spectrum of Approximation A4 for Healthy gear at a speed of 1230 rpm

A₄ contains frequency components of vibration signal from 0 to 625 Hz. The gear mesh frequency at the speed of 1230 rpm lies in this range. This is called as multi resolution analysis i.e., resolution matched to its scale. Similarly, multi resolution analysis was carried out for different fault stages of the gear tooth such as chipped and missing tooth. The results of the analysis are shown in Figure 9 and Figure 10. The spectrum for chipped gear tooth indicates the gear mesh frequency and associated peak which confirms the deteriorated state of gear tooth. Similarly, for missing tooth spectrum shows side bands around gear mesh frequency

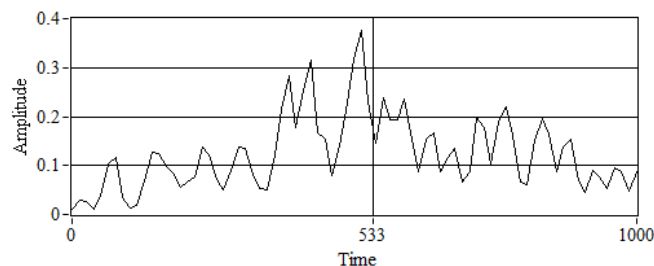


Figure 9: Spectrum of Approximation A4 for chipped gear at a speed of 1230 rpm

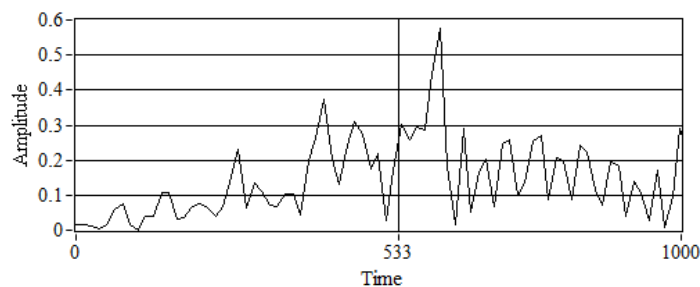


Figure 10: Spectrum of Approximation A4 for missing gear at a speed of 1230 rpm

Time frequency analysis

Wavelet theory has been proved to be a useful tool in the study of time series. Specifically, the scalogram allows the detection of the most representative scales (or frequencies) of a signal. A

scalogram is the absolute value of the continuous wavelet transform coefficients of a signal. In view of this, in the next stage of analysis, refinement over Fourier Transform was done by performing advanced signal processing i.e., Time-Frequency analysis. This technique is most suited for characterizing and manipulating signals whose statistics vary in time, such as transient signal. The technique of Fourier Transform can be extended to obtain frequency spectrum of any slowly growing signal. By visually representing signals at various scales and various frequencies through CWT, hidden features can be seen in the frequency–time domain

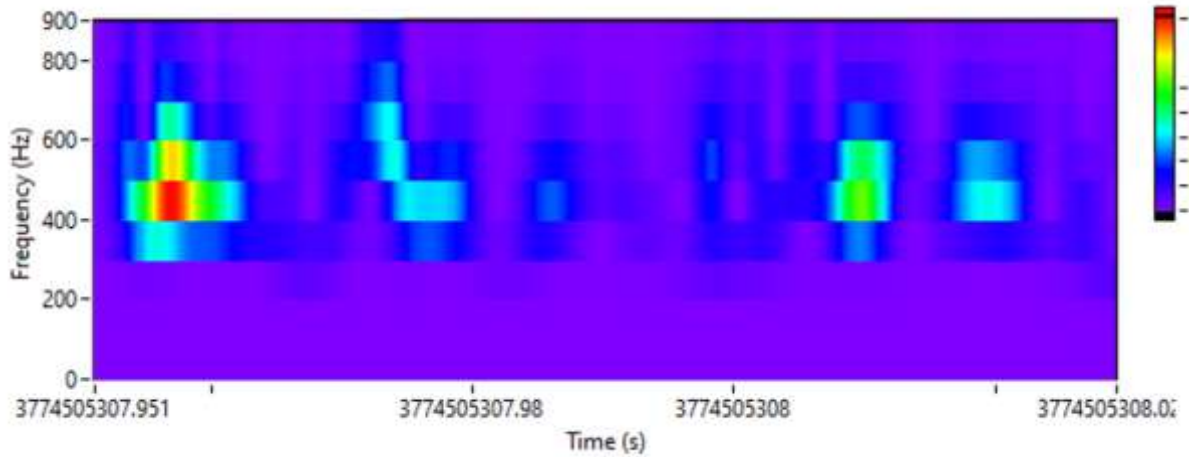


Figure 11: Time frequency representation of healthy gear at 1230 rpm

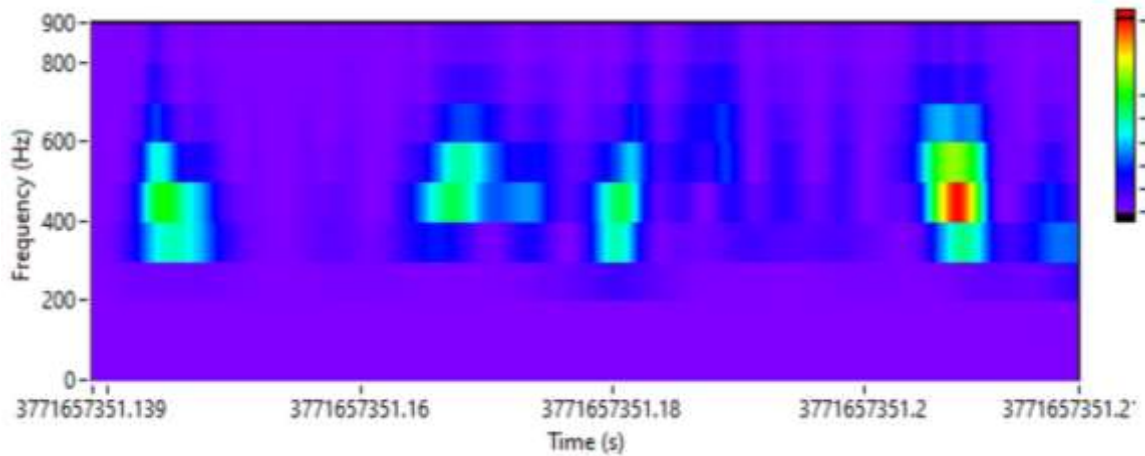


Figure 12: Time frequency representation of chipped gear at 1230 rpm

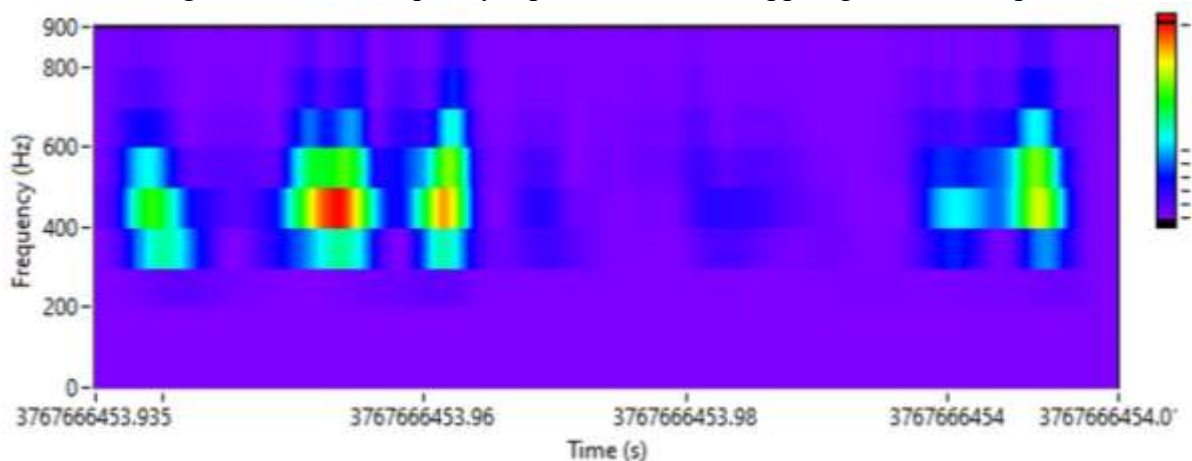


Figure 13: Time frequency representation of missing gear at 1230 rpm



Figure 11, 12 and 13 show scalogram for various cases of fault such as no fault(healthy), chipped and missing. As discussed in the earlier section, time frequency representation is most suited technique for characterizing and manipulating signals whose statistics vary in time, such as transient signal. Theoretical gear mesh frequency as calculated is 533 Hz at a speed of 1230 rpm. The colour contour in scalogram shown above strongly endorses this.

Results and Discussions

As discussed in earlier sections, the technique of Fourier transform is applied on gear tooth signals at a speed of 1230 rpm. The results of investigation are shown in Figure 3 for different cases such as healthy, chipped and missing tooth. the gear mesh frequency is characterized by a strong peak at this frequency. It is observed that with the deterioration of gear tooth, the level of amplitude increases giving rise to high level of vibrations.

In the next stage, wavelet based multi resolution analysis technique is applied to extract useful information from the signal. The aim of multi resolution analysis is to decompose the signals in approximation and detail coefficients and in a way to match the resolution at appropriate scale. The appropriate approximation is focused upon to trace out the expected gear mesh frequency. The approximation and details are shown in Figure 6 at level 4 for healthy gear tooth. Similar investigation is carried out for the other condition of gear tooth such as chipped and missing. Further, the spectrum of approximation for each case is analysed to examine the presence of gear mesh frequency as shown in Figure 8, 9 and 10.

In the final stage of analysis, wavelet-based time frequency analysis was carried out and conclusion is drawn based on scalogram. Figure 11, 12 and 13 shows scalogram for various cases such as healthy, chipped and missing tooth. Time frequency graph is most suited technique for nonstationary signals. Theoretical gear mesh frequency as calculated is 533 Hz at a speed of 1230 rpm.

Conclusion

Wavelet based signal analysing and processing technique is effectively applied for gear tooth vibration signal. The technique is suitable for non-stationary signals. The multiresolution analysis and time frequency analysis gives precise results to identify the faults.

References

- [1] Jiang X, Wang J, Shi J et al (2019) A coarse-to-fine decomposing strategy of VMD for extraction of weak repetitive transients in fault diagnosis of rotating machines. *Mech Syst Signal Process* 116:668–692
- [2] Liu H, Huang W, Wang S, Zhu Z (2014) Adaptive spectral kurtosis filtering based on Morlet wavelet and its application for signal transients detection. *Signal Process* 96:118–124
- [3] A Aherwar (2012) An investigation on gearbox fault detection using vibration analysis techniques: A review, *Australian Journal of Mechanical Engineering*, 10:2, 169-183
- [4] J. Wang, S. Li, Y.u. Xin, Z.(2019) An, Gear fault intelligent diagnosis based on frequency-domain feature extraction, *J. Vib. Eng. Technol.* 7 (2) 159– 166, <https://doi.org/10.1007/s42417-019-00089-1>.
- [5] S. A. Adewusi and B. O. Al-Bedoor, (2001)“Wavelet analysis of vibration signals of an overhang rotor with a propagating transverse crack,” *Journal of Sound and Vibration*, vol. 246, no. 5, pp. 777–793.
- [6] A. G. Bruce, D. L. Donoho, et al.,(1994) “Smoothing and robust wavelet analysis,” in *Proceedings of the Computational Statistics 11th Symposium*, pp. 531–547, Vienna, Austria.
- [7] J. I. Taylor, (1994) *The Vibration Analysis Handbook*, Vibration Consultants.
- [8] S. J. Derrek,(1999) *Gear Noise and Vibration*, Marcel Dekker, New York, NY, USA.



- [9] V. C. Chen and H. Ling, (2002) Time-Frequency Transforms, Artech House Publishers, Boston, Mass, USA.
- [10] Peng ZK, Chu FL (2004) Application of the wavelet transform in machine condition monitoring and fault diagnostics: a review with bibliography. Mech Syst and Sig Proc 18:199–221
- [11] Hocine Bendjama, Salah Bouhouche, and Mohamed Seghir Boucherit, (2012) “Application of Wavelet Transform for Fault Diagnosis in Rotating Machinery”, International Journal of Machine Learning and Computing, Vol. 2, No. 1.
- [12] P. G. Kulkarni 1 & A. D. Sahasrabudhe2, (2016) “Investigations on mother wavelet selection for health assessment of lathe bearings”, The International Journal of Advanced Manufacturing Technology ISSN 0268-3768, Int J Adv Manuf Technol springer
- [13] P. G. Kulkarni, A. D. Sahasrabudhe, (2013) “Application Of Wavelet Transform For Fault Diagnosis of Rolling Element Bearings”, INTERNATIONAL JOURNAL OF SCIENTIFIC & TECHNOLOGY RESEARCH VOLUME 2, ISSUE 4, ISSN 2277-8616, 138- 148
- [14] HaiQiu, Jay Lee, Jing Lin, Gang Yu, (2006) Wavelet filter-based weak signature detection method and its application on rolling element bearing prognostics, Journal of Sound and Vibration, vol. 289, pp. 1066-1090.
- [15] K. P. Soman, K. I. Ramachandran, (2005) Insight into Wavelets- From Theory to Practice. Prentice Hall of India Private Limited.
- [16] N. G. Nikolaou, I. A. Antoniadis, (2002) “Rolling element bearing fault diagnosis using wavelet packets,” NDT& E International, vol. 35, pp. 197-205.
- [17] Pravin Kulkarni Ami Barot, (2020) “A Wavelet Based Multi-resolution Technique for Analysis of Vibration Signals”, IJSTE - International Journal of Science Technology & Engineering | Volume 7 | Issue 4, ISSN (online): 2349-784X